A COURSE OF

LECTURES

IN

Natural Philoſophy.

By the Late

RICHARD HELSHAM, M. D.

Profeſſor of PHYSICK and NATURAL PHILOSOPHY
in the UNIVERSITY of *DUBLIN*.

PUBLISHED BY

BRYAN ROBINSON, M. D.

The SECOND EDITION.

LONDON:

Printed for J. NOURSE, at the *Lamb*, without
Temple-Bar.

M.D.CC.XLIII.

THE CONTENTS.

LECTURE I.

THE

THE

PREFACE.

THAT the reader may be duly prepared for the perusal of the following Treatise, it will be necessary that he first acquaint himself with the genuine Method and Rules of Philosophizing, as they have been delivered by Sir ISAAC NEWTON.

His Method of Philosophizing is thus laid down in his *Opticks* *.

" As in Mathematicks, so in natural Philo-
" sophy, the investigation of difficult things
" by way of *Analysis*, ought ever to precede
" the method of composition. This *Analysis*
" consists in making experiments and obser-
" vations, and in drawing general conclusions
" from them by induction and admitting of
" no objections against the conclusions, but
" such as are taken from experiments or other
" certain truths. And although the arguing
" from experiments and observations by in-
" duction, be no demonstration of general
" conclusions; yet it is the best way of ar-
" guing which the nature of things admits
" of, and may be looked upon as so much

* *Opt. p.* 380.

" the ſtronger, by how much the induction
" is more general. And if no exception oc-
" cur from *Phænomena*, the concluſion may
" be pronounced generally. But if at any
" time afterwards, any exceptions ſhall occur
" from experiments, it may then be pro-
" nounced with ſuch exceptions as ſhall occur.
" By this way of *Analyſis*, we may proceed
" from compounds to ingredients, and from
" motions to the forces producing them; and
" in general from effects to their cauſes,
" and from particular cauſes to more general
" ones, till the argument ends in the moſt
" general. This is the method of *Analyſis*:
" And the *Syntheſis* conſiſts in aſſuming the
" cauſes diſcovered, and eſtabliſhed as prin-
" ciples, and by them explaining the *Phæ-*
" *nomena* proceeding from them, and proving
" the Explanations."

His Rules of Philoſophizing, delivered in his *Principles* *, are theſe four.

RULE I.

" *More cauſes of natural things are not to be*
" *admitted, than are both true and ſufficient*
" *for explaining their* Phænomena.

" Thus Philoſophers ſay; nature does no-
" thing in vain, and in vain that is done by
" more cauſes, which can be done by fewer.
" For nature is ſimple, and delights not in
" ſuperfluous cauſes of things.

RULE

* *Philoſ. Natur. Princip. Mathem. p.* 387.

RULE II.

"*Of natural effects therefore of the ſame*
"*kind the ſame cauſes are to be aſſigned, as far*
"*as it can be done.*

"As of reſpiration in a man and in a "beaſt; of the deſcent of ſtones in *Europe* "and in *America*; of light in a culinary fire "and in the ſun; of the reflexion of light in "the earth and in the planets.

RULE III.

"*The qualities of bodies which cannot be* "*increaſed and diminiſhed, and which agree* "*to all bodies in which experiments can be* "*made, are to be reckoned as qualities of all* "*bodies whatſoever.*

"For the qualities of bodies are not known "but by experiments; and therefore, as ma-"ny are to be reckoned general as generally "agree with experiments, and thoſe which "cannot be diminiſhed cannot be taken "away. Certainly dreams are not to be de-"viſed at pleaſure contrary to the tenor of "experiments; nor muſt we depart from the "analogy of nature, ſince ſhe is wont to be "ſimple, and always conſonant to herſelf. "The extenſion of bodies is not known but "by the ſenſes, nor is it perceived in all bo-"dies: but becauſe it agrees to all bodies "which are perceiveable, it is affirmed of all "whatſoever. We experience many bodies

" to be hard. But the hardneſs of the whole " ariſes from the hardneſs of the parts, and " thence with good reaſon we conclude the " undivided parts not only of thoſe bodies " which are perceived, but alſo of all others " to be hard. We gather all bodies to be " impenetrable, not by reaſon, but by ſenſe. " We find the bodies we handle to be im- " penetrable, and thence conclude impene- " trability to be a property of all bodies what- " ſoever. That all bodies are moveable, and " by certain forces (which I call *vires iner-* " *tiæ)* perſevere in motion or reſt, we gather " from theſe ſame properties in bodies which " are ſeen, Extenſion, hardneſs, impenetrabi- " lity, mobility, and *vis inertiæ* of the whole, " ariſe from the extenſion, hardneſs, impene- " trability, mobility, and *vires inertiæ* of the " parts; and thence we conclude that all the " leaſt parts of all bodies are extended, and " hard, and impenetrable, and moveable, " and endued with *vires inertiæ*. And this " is the foundation of all Philoſophy. Far- " ther we know from the *Phænomena*, that " the parts of bodies which are divided, " and mutually contiguous to one another, " may be ſeparated from one another, and it " is certain from Mathematicks, that the " undivided parts may by reaſon be diſtin- " guiſhed into leſs parts. But whether thoſe " parts diſtinct, and not yet divided, can " by the powers of nature be divided and " ſeparated from one another, is uncertain. " But if it ſhould appear, even by one ſingle

" ex-

" experiment, that by breaking a hard and " solid body, any undivided particle suffered " a division; we might conclude by the " force of this rule, that not only the di- " vided parts were separable, but that the " undivided parts might be divided *in infi-* " *nitum.*

" *Lastly*, If it be universally evident by ex- " periments and astronomical observations, " that all bodies round the earth gravitate " towards the earth, and that in proportion " to the quantity of matter in each, and that " the moon gravitates towards the earth in " proportion to it's quantity of matter, and in " like manner our sea gravitates towards the " moon, and that all the Planets mutually " gravitate towards one another, and that " there is a similar gravity of Comets to- " wards the sun; we must pronounce by " this rule, that all bodies gravitate mutual- " ly towards one another. For the argu- " ment from the *Phænomena* will be stron- " ger for an universal gravity, than for the " impenetrability of bodies, concerning which " in the heavenly bodies we have no experi- " ment, no observation at all.

RULE IV.

" *In experimental Philosophy propositions* " *collected from the* Phænomena *by induction,* " *are to be deemed, notwithstanding contrary* " Hypo-

" Hypothefes, *either accurately or very nearly*
" *true, till other* Phænomena *occur, by which*
" *they may be rendered either more accurate or*
" *liable to exceptions.*

" This ought to be done, left argu-
" ments of induction fhould be deftroyed
" by *Hypothefes.*

This Method and thefe Rules have been carefully obferved by our Author in thefe LECTURES, which, from the clearnefs and diffufivenefs of the ftile, and the eafy and juft manner of reafoning, are, in my opinion, better fitted for the inftruction of youth, than any thing which I have feen on this fubject.

I have added a few *Problems* by way of APPENDIX.

LECTURE I.

OF ATTRACTION.

AS natural philoſophy is a ſcience in it's own nature entertaining and delightful, and withal conducive in many inſtances to the eaſe and convenience of life; it is not to be wondered that there have been men in all ages who have laid themſelves out in the improvement and cultivation of it. But it is a matter of no ſmall ſurpriſe to think how inconſiderable a progreſs the knowledge of nature had made in former ages, when compared with the vaſt improvements it has received from the numberleſs diſcoveries of later times; inſomuch that ſome of the branches of natural philoſophy, which at this day is almoſt complete in all it's parts, were utterly unknown before the laſt century. If we look into the reaſon of this, we ſhall find it to be chiefly owing to the wrong meaſures that were taken by philoſophers of former ages in their purſuits after natural knowledge: for they diſregarding experiments, the only ſure foundation whereon to build a rational philoſophy, buſied themſelves in framing hypotheſes, for the ſolution of natural appearances, which as they were creatures of the brain, without any foundation in nature, were generally ſpeaking ſo lame and defective, as in many caſes not to anſwer thoſe very phænomena for whoſe ſakes they had been contrived. Whereas the philoſophers of later times, laying aſide thoſe falſe lights, as being of no other uſe than to miſguide the underſtanding

LECT. I.

in

LECT. I. in it's ſearches into nature, betook themſelves to experiments and obſervations; and from thence collected the general powers and laws of nature; which with a proper application, and the aſſiſtance of mathematical learning, inabled them to account for moſt of the properties and operations of bodies; and to ſolve many difficulties in the natural appearances, which were utterly inexplicable on the foot of hypotheſis. By this means has natural philoſophy, within the compaſs of one century, been brought out of the greateſt darkneſs and obſcurity into the cleareſt light; and this has been chiefly owing to the unparalleled abilities, and indefatigable induſtry of that great and accurate philoſopher Sir ISAAC NEWTON; who, to his great honour, has in his principles of natural philoſophy, and his incomparable treatiſe of light and colours, cleared more difficulties, and diſcloſed more and more important truths relating to nature, than are to be met with in the voluminous writings of all that went before him. To illuſtrate ſome of theſe truths by experiments is the deſign of this courſe, which conſiſts of four parts. In the firſt are conſidered ſolid bodies and their properties. In the ſecond water and watery fluids. In the third the elaſtic fluid of air. And in the laſt the ſubtile fluid of light. But before I proceed to theſe particulars, it will be neceſſary to ſay ſomething concerning certain principles, forces, or powers, wherewith all parts of matter, of what kind ſoever, ſo far as experience reaches, ſeem to be endued; and whereby they act upon one another for producing a great part of the phænomena of nature.

Such is firſt that power whereby the minute particles of matter do in ſome circumſtances tend towards one another, which is commonly called attraction; the cauſe whereof is in a great meaſure unknown, though the thing itſelf is manifeſt from experiments. Exp. 1. For if two poliſhed plates of braſs be laid

laid one upon another, having their contiguous sides smeared with oil, they will cohere in *vacuo*, and with such firmness that when they are suspended, the force of gravity in the lower plate will not suffice to separate and pull them asunder. LECT. I.

That the cohesion of these plates is to be attributed to the mutual attractions of their contiguous parts, cannot I think admit of a doubt, since the pressure of the outward air on their external surfaces, (to whose force this effect might otherwise have been attributed) is in this case taken off.

The use of the oil is to fill up the minute cavities in the surfaces, and by so doing to prevent the lodgment of air between the plates; which upon the removal of the outward air would expand itself by reason of it's elasticity, and thereby force the plates asunder.

The forementioned attraction is in like manner collected from the following experiments. Exp. 2.

If two plane polished plates of glass be laid together, so that their sides be parallel, and at a very small distance from one another, and their lower edges be dipped in water, the water will rise up between them; and the less the distance of the glasses is, the greater will the height be to which the water rises. If the distance be about the hundredth part of an inch, it will rise to the height of about an inch; and if the distance be greater or less in any proportion, the height will be reciprocally proportional to the distance very nearly.

The reason why the water ascends between the plates is, that those parts of the surfaces of the glasses which lie next above the surface of the water, and are contiguous thereto, attract the water, and by that means cause it to ascend; and this ascent continues till the weight of the elevated water becomes equal to the force of the attracting surfaces, and then the motion ceases, the water tending as

LECT. I. much downward by the force of it's own gravity, as it doth upward by the attraction of the glaſſes.

The reaſon why the water riſes to heights which are inverſly as the diſtances of the glaſſes, is this: the abſolute attractive force of the glaſſes, whereby the water is raiſed, continues unvaried whatever be the diſtance of the glaſſes; for the height and length of the glaſs ſurfaces, whoſe attractions influence the aſcent of the water, are always the ſame, and conſequently the attractive force muſt be ſo too; and for that reaſon will conſtantly ſupport the ſame weight of water; but the quantity, and conſequently the weight of the elevated water, will always be the ſame, if it's height be reciprocally as it's baſe, that is in this caſe, as the diſtance of the plates; for the length of the baſe being equal to the length of the plates, it continues unvaried; and therefore the baſe will ever be as it's breadth, that is as the interval between the plates.

Exp. 3. If the glaſs plates inſtead of being ſet parallel to one another, be made to meet at one of their ends, and kept at a little diſtance at the other; and their lower edges be then dipped in water, ſpirit of wine, or any other convenient liquor; the inward ſides of the plates being firſt moiſtened with a clean cloth dipped in the liquor; the liquor will riſe between the plates; and the upper ſurface of the elevated liquor will form a curve, the heights of whoſe ſeveral points above the ſurface of the ſtagnating liquor will be to one another reciprocally as their perpendicular diſtances from the concourſe of the plates. Fig. 1. For the illuſtration of which, let AE be the ſurface of the ſtagnating liquor wherein the lower edges of the plates are immerſed, AH the concourſe of the plates, and F, G, I, K, L the curve formed by the ſurface of the elevated liquor; from any points in the curve as G, I, K, L taken at pleaſure, let fall the right lines GB, IC, KD, LE perpendicular to AE, and thoſe lines will expreſs the

 heights

heights of the reſpective points of the curve above the ſurface of the ſtagnant liquor; whilſt AB, AC, AD, AE denote the perpendicular diſtances of the ſame points from the concourſe of the glaſſes; now thoſe heights and diſtances are to one another in a reciprocal proportion: for if we ſuppoſe the lines GB, IC, KD, LE to be ſo many pillars of liquor conſiſting of four ſides, two of which are terminated by the plates, and the other two by the contiguous liquor; and if thoſe ſides which lie next the plates be of an equal but exceedingly ſmall breadth in all the pillars, then will the attracting ſurfaces of the plates which ſupport thoſe pillars be likewiſe equal, and conſequently the quantities ſupported, that is the pillars muſt be ſo too. But in order to have them equal, their heights muſt be reciprocally proportional to their baſes; which baſes, inaſmuch as they are ſuppoſed to be equally broad, muſt be as their lengths, that is, as the intervals between the glaſſes in thoſe parts where the pillars are taken; and therefore the heights of the pillars muſt be reciprocally as the intervals between the plates; but from the nature of ſimilar triangles the intervals between the glaſſes at different diſtances from the concourſe are to one another directly as thoſe diſtances; whence it follows, that the heights of the pillars are to one another reciprocally as their reſpective diſtances from the concourſe of the plates; that is, if GB be double of IC, then is AC double of AB.

From what has been ſaid it is plain that the curve formed by the upper ſurface of the elevated liquor muſt be an hyperbola; for from the nature of the hyperbola the external ordinates are reciprocally as the abſciſſæ; wherefore if AB, AC, AD, AE, be taken for the abſciſſæ; then will BG, CI, DK, EL, be the reſpective ordinates; and conſequently the curve which paſſes through the points G, I, K, L is an hyperbola.

LECT. I. Exp. 4. As water or any other proper fluid aſcends between poliſhed plates of glaſs by the force of their attractions; ſo does it likewiſe in ſlender pipes of glaſs open at both ends; for if ſuch tubes be dipped at one end into water, ſpirit of wine, or any other convenient fluid, the liquor will riſe within the pipes to a conſiderable height, and this experiment (as alſo thoſe before made) ſucceeds in the very ſame manner in *vacuo*, as in the open air, for the liquor conſtantly aſcends to the ſame height in both.

That the aſcent of liquor in theſe ſmall tubes, as alſo between poliſhed plates of glaſs, is to be attributed to ſome power in the glaſs ſtrongly acting on the liquor, and not to the preſſure either of the ſtagnating liquor or incumbent atmoſphere, is evident from this conſideration; that as much of the liquor remains ſuſpended in the pipes, and between the plates, when they are lifted out of the ſtagnating fluid, either in *vacuo* or the open air, as was elevated above the ſurface of the fluid, while they were immerſed therein: and therefore whatever cauſe concurred to the elevating of the liquor while the plates and pipes were therein immerſed, and expoſed to the air; the ſame contributes as powerfully to keep it up, when the ſtagnating liquor is removed, and the preſſure of the atmoſphere taken off, and conſequently muſt be ſome power inherent in the glaſs.

The heights to which the liquor riſes in ſlender pipes, are to one another reciprocally as the diameters. For the power which raiſes the liquor in a ſlender pipe, being the attractive force of that part only of the internal concave ſurface which lies next above the liquor, and conſtitutes a ring of an indefinitely ſmall height, which height is ever the ſame whatever be the diameter of the ring, becauſe the diſtance to which the attractive force of glaſs reaches is unvaried; and the attractive force of ſuch an annual ſurface being as the number of attracting parts whereof it is compoſed, that is, as the ſurface, which

which becaufe it's height is given is as the periphery, that is, as the diameter, the attractive force of the pipe muft be as the Diameter. Wherefore if in comparing the forces of two fuch pipes we make F to denote the attractive force of the larger, and f the attractive force of the fmaller, and alfo D and d to denote their diameters; we fhall have this analogy, *viz.* $F : f :: D : d$, that is, the force of the larger pipe is to that of the fmaller as the diameter of the larger to the diameter of the fmaller: but thefe forces are likewife to one another in the fame ratio with the quantities of liquor which they keep fufpended, for they continue to elevate the liquor till fuch time as the weights, and confequently the quantities of liquor drawn up, become a balance to the attracting forces. Wherefore if H be put for the height of the liquor in the pipe, whofe diameter is D, and h for it's height in the pipe whofe diameter is d; then will H multiplied into the fquare of D be as the quantity of liquor in the larger pipe; and h multiplied into the fquare of d as the quantity of liquor in the fmaller pipe; whence we have this fecond analogy $F : f :: H \times D^2 : h \times d^2$; and by fubftituting D and d in the room of F and f, to which they are proportional, as appears from the firft analogy, we fhall have $D : d :: HD^2 : hd^2$; and then multiplying extremes and means, and throwing off fimilar quantities, we fhall have $HD = hd$, and by refolving this equation into an analogy, we fhall have $H : h :: d : D$, that is, the height to which the liquor rifes in the larger pipe is to the height to which it rifes in the fmaller, as the diameter of the fmaller pipe to that of the larger; fo that the heights of the liquor are reciprocally proportional to the diameters of the pipes.

LECT. I.

By virtue of this attractive force, wherewith fmall pipes are indued, plants receive nourifhment from the earth; the flender tubes, whereof their roots are compofed, fucking in various juices according to

their

LECT. I. their different natures and conſtitutions. From the ſame attractive force it is that ſponges take in water: and that water aſcends in loaf ſugar, when any part of it is dipped therein; thoſe parts of the ſugar which lie next above the water attracting, and thereby raiſing the ſame. And here it muſt be obſerved that the water riſes by the action of thoſe particles alone which are contiguous to, and lie next above the ſurface of the elevated water; thoſe particles which are at any the leaſt ſenſible diſtance above the water, being too far removed to influence the water by their attractions: and what has been thus obſerved of ſugar, is likewiſe true of poliſhed plates, ſlender pipes, and every other attracting body, by vertue of whoſe attractions fluids are raiſed. For if thoſe parts of attracting ſurfaces which are at any ſenſible diſtance above the ſurface of the fluid, do in any meaſure contribute to the aſcent; it is evident that the fluid, *cæteris paribus*, muſt riſe to a greater height when the attracting ſurfaces are continued to a conſiderable height above the elevated fluid, than when they terminate at a very little diſtance above the ſame. But the contrary appears from experiment. For if two poliſhed plates of glaſs ſet parallel to one another, at the diſtance of about the hundredth part of an inch, be immerſed in water ſo far that only an inch and one tenth be ſuffered to remain above the water, the water will riſe up between them to the height of about an inch; and if the ſurface of the ſtagnating water be then depreſſed by drawing off ſome of the water, the elevated water will likewiſe deſcend between the plates, ſo as ſtill to preſerve the height of about an inch and no more.

Exp. 5.

Exp. 6. If a poliſhed plate of glaſs be laid parallel to the horizon, and another plate of the ſame kind be laid thereon, ſo as that they may touch at one of their ends, and be kept at a very ſmall diſtance at the other; being firſt moiſtened on their inward ſides with

with a clean cloth or feather dipped in oil of oranges; and if a drop of the oil be placed between the plates at that end where they are at some distance from each other, so as that it may be touched by both the plates, it will begin to move towards the concourse of the glasses, and will continue to go on with an accelerated motion till it arrives at the concourse. And if during the motion of the drop, that end of the glasses where they meet, and towards which the drop moves, be lifted up, the drop will nevertheless continue it's motion, and of consequence must be attracted; but as the end of the glasses is raised higher and higher, the drop will ascend more and more slowly, till at last, upon a certain elevation of the plates the motion ceases, the gravity of the drop, wherewith it tends downward, becoming equal to the attractive force which draws it upward; as appears from this, that upon giving the plates the least degree of elevation beyond what is necessary to stop the drop, it straightway begins to descend, it's gravity in that case overcoming the attraction.

By the help of this phænomenon may the force be determined, wherewith the drop is attracted at all distances from the concourse of the glasses. For that part of a body's gravity whereby it is carried down an inclined plane, is to it's absolute weight, as the sine of the angle of the plane's elevation, to the radius, or as the perpendicular height of the plane to the length thereof; and therefore may be denoted by the perpendicular height applied to the length; and where the length of the plane is given, that force will be every where as the sines of the angles of elevation, or the perpendicular altitudes of the plane; as shall be made appear when I come to treat of the descent of bodies on inclined planes. If therefore the sines of such elevations of the plates as are necessary to stop the motion of the drop, be taken at two different distances of the drop from the concourse of the plates; those sines will de-

LECT. I. note the refpective gravities of the drop, and confequently the attractive forces, wherewith the plates act upon the drop at each of thofe diftances. Thus for inftance, if the diftances of the drop from the concourfe of the glaffes be as one and two; and the fine of the elevation neceffary to ftop the motion of the drop when at the fmaller diftance be as four, and when at the greater diftance as one; the gravity of the drop, wherewith it endeavours to defcend at the forementioned diftances of one and two, will be as four and one. For the illuftration of which, let A B and A C reprefent the plates at different elevations; F and G the places where the drop ftands upon thofe elevations; then will B D and C E denote the forces of gravity wherewith the drop endeavours to defcend along the plates in the points F and G, which forces are equal to the attractions of the glaffes in thofe points; and if BF and C G the diftances of the drop from the concourfe of the plates be as one and two, and B D and C E as four and one; then is the attractive power wherewith the glaffes act upon the drop at F, to the force wherewith they act upon it at G, as four to one, that is, reciprocally as the fquares of the diftances of the drop from the concourfe of the glaffes; and this is nearly the cafe, as will appear from the experiment.

Fig. 2.

Exp. 6.

Though the drop be attracted by forces that are in the reciprocal duplicate ratio of the diftances of the drop from the concourfe of the glaffes; yet are the attractions within the fame quantities of attracting furface in the reciprocal fimple ratio only of thofe diftances: for as the drop moves towards the concourfe of the glaffes, it muft fpread and touch each glafs in a larger furface; and this fpreading is always proportional to the leffening of the interval between the glaffes; and of confequence from the nature of fimilar triangles, it is likewife proportional to the diminution of the diftance from the concourfe.

course. So that the force which acts upon the drop is increased as the drop approaches the concourse in the simple reciprocal ratio of the distance, on account of the inlargement of the attracting surface in that proportion; and therefore in a given quantity of attracting surface the force must be in the reciprocal simple ratio of the distance from the concourse; that is to say, any given portion of the glass surfaces, taken at the distance of one inch from their concourse, must act with twice the force that it does at the distance of two inches, and with thrice the force that it does at the distance of three inches, and so on. Hence it will be found that the attractive force of one and the same slender pipe of a conical figure is given; or in other words, that the attractive force wherewith a conical pipe is indued at any one distance from the vertex of the cone, is equal to the attractive force of the same, at any other distance from the vertex; so that the attractive force of a conical pipe is in every part equal throughout the whole length of the pipe; and may be expressed by the diameter of a circular section of the pipe, taken at any distance from the vertex, applied to that distance. For the attraction in any part of such a pipe, is as the quantity of attracting surface in that part multiplied into the absolute force; but the quantity of attracting surface in any part is as the diameter of that part, and the absolute force is reciprocally as the distance from the vertex; wherefore if A be put to denote the distance of any part from the vertex, and D the diameter, $\frac{D}{A}$ will express the attraction of that part; but from the nature of similar triangles, the diameters of the circular sections of a cone, taken at different distances from the vertex, are to one another as the distances, consequently $\frac{D}{A}$ is a standing quantity.

LECT. I.

 Wherefore

Wherefore since the attractive force in every part of a conical tube is denoted by a quantity which is invariable, it follows that the force is so too; so that in this respect conical pipes do not differ from those of a cylindrical form; but herein lies the difference, that in very slender pipes where the diameters are equal, the attractions of such as are conical do far surpass the attractions of those which are cylindrical. And indeed so exceeding great does this attractive force become with respect to the quantity of attracting surface in that part of a conical pipe, where the diameter is but one part of an inch divided into ten millions (if such minuteness may be supposed) that if the attraction of a cylindrical tube, whose diameter is an inch, were as great with respect to it's quantity of attracting surface, it would be able to support a column of water an inch in diameter and upwards of three miles in height. For let us suppose a conical tube, whose base is an inch in diameter, to be continued till the diameter is so far diminished as to equal only one part of an inch divided into ten millions; it is evident from what was just now said, that the whole attractive force of such a pipe, where it's diameter is an inch, is equal to the whole attractive force of the same, where the diameter is but the ten millioneth part of an inch; consequently if a portion of the larger attracting surface be taken equal to the smaller attracting surface, the force of that will be to the force of this, as the force of the smaller surface divided by the number of parts in the larger surface, to the force of the smaller surface, that is, as one divided by ten millions to one. If therefore a conical, or indeed a cylindrical tube an inch in diameter (for where the diameter is so large there is scarcely any difference) was indued with an attractive force as great in proportion to it's quantity of attracting surface, as is a conical tube of the ten millioneth part of an inch in diameter, it's force would be ten millions of times

greater

greater than it is, and of consequence would raise the water ten millions of times higher than it doth at present: but it has been found by experience, that in a cylindrical tube of an inch in diameter, the water will rise to the height of about the fiftieth part of an inch, and therefore if the force by which it rises was augmented in the forementioned proportion, it must rise to the height of two hundred thousand inches, which being divided by sixty three thousand three hundred and sixty, the number of inches in a mile, gives three and a little more in the quotient.

The quantities of liquor supported by the attractions of slender conical pipes are to one another, as the diameters of the little circular surfaces of the elevated liquor, applied to the respective distances of the same circular surfaces from the vertices of the several cones whereof the pipes are portions. For it has been proved that the attractive forces of conical pipes are as those quantities; and therefore the weights which they support must be so too. Hence it follows that the less the proportion is, which the distance of the elevated liquor's surface from the vertex of the cone bears to the diameter of the same surface, or which amounts to the same thing, the faster the sides of the pipe converge, the stronger is it's attractive force, and the greater the quantity of liquor which is supported.

The firm union and strong cohesion of the particles of solid bodies seems to arise from this force, wherewith they mutually attract each other; which as it appears to be exceeding strong in the immediate contact of the particles, so it is found by experience to reach but a very little way beyond the same with any sensible effect. At very small distances indeed it is sufficient to raise up liquors, as also to produce the many odd and surprizing appearances which are to be met with in chymical operations, and which without the assistance of this and some other principles, which I shall hereafter have

LECT. I. occaſion to mention, are utterly inexplicable. For want of a due knowledge of theſe powers chemiſts have fallen into groſs miſtakes and abſurdities in their reaſonings. Thus for inſtance, ſome who were unacquainted with the principle of attraction, have attempted to give a reaſon for the floating of the minute particles of ſolid bodies in menſtruums ſpecifically lighter than themſelves; by ſaying that there is an inteſtine motion in the parts of the menſtruums, by vertue whereof the particles of the ſolid bodies are driven perpetually from place to place, and by that means are kept from falling: not conſidering that Sir ISAAC NEWTON has demonſtrated, in the nineteenth propoſition of the ſecond book of his principles, that fluids have not naturally any inteſtine motion; but that, ſetting aſide all external cauſes of motion, the particles of fluids are as perfectly at reſt as thoſe of ſolid bodies. There is indeed during the time of the ſolution a conſiderable motion, but as this is occaſioned by the mutual attraction between the menſtruum and the body, by means of which attraction the parts of the fluid are driven with great force between the parts of the ſolid, ſo as to looſen and divide them one from another; as ſoon as the ſolution is over the motion ceaſes, and all the parts are at reſt again, and the particles of the diſſolved body are kept ſuſpended by their cloſe adheſion to the parts of the menſtruum, and not by any imaginary motion, wherewith they are toſſed to and fro in the manner of a ſhuttle-cock; and in truth, could ſuch an inteſtine motion be allowed, as it muſt be made in all manner of directions, it would be as apt, nay more apt, conſidering the conſpiring gravity of the particles, to precipitate and caſt them down, than to raiſe and keep them up.

Were it not beſide my preſent purpoſe, I could produce many more inſtances of falſe reaſonings in the writings of chemiſts, occaſioned by their ignorance of the true principles of nature; but as chemiſtry

miſtry is at preſent out of my province, I ſhall reſt contented with the ſingle inſtance which I have given.

LECTURE II.

OF ATTRACTION.

HAVING in my former Lecture proved from experiments, that there is a power in nature whereby the parts of matter, which are brought ſo near as to touch, do in ſome circumſtances mutually attract each other; I ſhall now treat of ſuch kinds of attraction as extend themſelves to conſiderable diſtances beyond the point of contact, and on that account affect the mind more ſtrongly, ſo as to convince it more fully of the reality of ſuch a principle. Of this kind is, Firſt, that attraction which obtains between glaſs and glaſs. Secondly, that of electricity. Thirdly, the attraction of magnetiſm. And laſtly, that of gravity; of all which in their order.

LECT. II.

And firſt, if a glaſs bubble be ſet to float on water contained in a glaſs veſſel, at a ſmall diſtance from the ſide of the veſſel, it will, from a ſtate of reſt, begin to move towards the ſide of the veſſel; and it's motion will be continually accelerated, ſo as to make it, upon it's arrival at the ſide of the veſſel, to ſtrike the ſame with ſome force.

Exp. 1.

Perhaps it may be thought, that the motion of the bubble ariſes from ſome declivity of the water towards the ſides of the veſſel: but whoever obſerves the ſurface of the water will find, that it riſes all about the ſides of the glaſs, ſo as to become of a concave figure, and for that reaſon may retard, but can by no means promote the motion of the bubble; and this riſing of the liquor about the ſides of the veſſel, is to be attributed to the ſame cauſe with the motion of the bubble, namely, the attraction of the glaſs.

LECT. II.

The acceleration obſervable in the bubble's motion ariſes from two cauſes; the firſt is, the continued and uninterrupted action of the attractive force of the glaſs; for if we ſuppoſe the time of the bubble's motion to be divided into a number of equal parts, as for inſtance ten; and if the attraction of the glaſs be ſuppoſed to make equal impreſſions on the bubble in each of thoſe parts of time, it is plain, that whatever be the motion which is excited in the bubble by the impreſſion of attraction in the firſt portion of time, the ſame will be doubled in the ſecond, tripled in the third, and ſo on continually through the ſeveral portions of time; for the motion produced in the firſt portion of time is not loſt, and therefore by the addition of as much more in the ſecond portion of time it becomes double, and in the third triple, and ſo on. Now if inſtead of ten parts we ſuppoſe the time of the motion to be divided into numberleſs parts indefinitely ſmall, in each of which the attraction of the glaſs makes equal impreſſions on the bubble as before; the motion will be continually accelerated, though the attractive force of the glaſs ſhould continue the ſame at all diſtances of the bubble; but the attractive force acts more ſtrongly the nearer the bubble approaches, on which account the motion is more and more accelerated the nearer the bubble comes to the glaſs.

By electrical attraction, I mean that kind of attraction which is excited in bodies when their parts are heated by friction, and which doth not diſcover it ſelf by any ſenſible effect when the bodies are cold. Of this ſort are the attractive forces, which amber, roſin, ſealing-wax, and indeed moſt ſulphurous ſubſtances when heated by rubbing, have been found to exert towards chaff, feathers, leaf-gold, lamp-black, and many other light ſubſtances. But as the attractions of theſe bodies have fallen within the notice of vulgar eyes, I think it needleſs to make any experiment for the proof thereof; but choofe

choose rather to lay before you some experiments LECT. II.
which plainly shew this power to obtain in glass, and that to a very notable degree, though it has not till of late been commonly observed. And first,

If a cylindrical tube of flint glass be rubbed briskly with brown paper, or woollen cloth till it acquires some degree of heat, and be then held near to small pieces of gold or brass leaf; they will begin to move, and some of them will fly towards the tube with great swiftness, and fix themselves upon it so as to adhere thereto, being acted upon by the attractive force of the glass: whilst others, during their ascent towards the tube, will, before they can reach the same, be driven backward with great violence, as will likewise some of those which touch the glass, being actuated by another force, very different from that of attraction, which I shall endeavour to explain to you hereafter. The hotter the tube is made by rubbing, the farther doth it's power reach, so as in some cases to act upon the leaf at the distance of a foot or more.

This electrical attraction of glass doth in like manner appear from the following experiments.

Exp. 2. If over a globe of glass fixed on an axis, whose position is horizontal, a parcel of woollen threads be suspended from a semicircular wire, so as that their lower ends may be distant an inch or a little more from the globe, they will, suitably to the nature of all heavy bodies, hang down perpendicular to the horizon, and parallel to each other; if then the globe be moved pretty briskly round it's axis, the threads will immediately change their position, so as to have their ends bent a little upward, pointing that way towards which the motion tends; the rotatory motion of the globe being communicated to the circumambient air wherein the threads hang, and by means thereof in some measure to the threads themselves. Let then an hand be applied to the lower part of the globe, so as to rub the same, and

LECT. II. as ſoon as it grows warm from the friction, the threads, which before were crooked, will dart themſelves out into ſo many ſtrait lines, all pointing towards the center of the globe; but as ſoon as the attrition ceaſes, and the globe cools, they quit this direction, and return to their former poſition; whence it evidently appears that they are attracted by the glaſs, ſince they are made to point towards it's center, notwithſtanding the contrary directions that were given them, by the motion of the air and the force of gravity. In this and the two following experiments there is one remarkable circumſtance, which though it does not concern the matter in hand, yet, becauſe I ſhall have occaſion to have recourſe to it hereafter, I ſhall, to prevent the repetition of experiments, take notice of it here. And it is this; if while the threads are extended, and acted upon by the attraction of the globe, a finger be moved towards the extremity of any of them, they will immediately recede and fly from the touch, and this they will do upon every approach of the finger.

Exp. 3. If the axis of the globe, inſtead of being parallel to the horizon, be placed perpendicular thereto, and the ſemicircular wire which ſupports the threads be in the plane of a circle parallel to the horizon, the threads muſt, by reaſon of their gravity, hang down in lines parallel to the axis of the globe; yet as ſoon as the motion and attrition are given to the globe as before, the threads will be given to raiſe and extend themſelves towards the center of the globe, and appear like ſo many rays converging towards that center in a plane parallel to the horizon: ſo that in this caſe, the attractive force of the glaſs does not only draw the threads out of the parallel poſition they have to each other, but likewiſe raiſes them up in a poſition parallel to the horizon, notwithſtanding the force of gravity, which is conſtantly acting upon them, to carry them down.

If the threads, inftead of being placed without the globe, be fixed to the axis at the center, and be of fuch a length as to reach within about an inch of the furface; when the globe is turned round, they will bend backward, contrary to the direction of the motion; becaufe the included air, though it does in fome meafure partake of the rotation of the globe, yet doth it not move with equal fwiftnefs, and for that reafon, muft refift the rotation of the threads, and bend them backward. When the threads are in this ftate, if the attraction of the glafs be excited by attrition, as in the two laft experiments, they will ftraightway extend themfelves towards the concave furface of the globe, conftituting, as it were, fo many rays iffuing from the center, and diverging from one another in a regular manner. LECT. II. Exp. 4.

The reafon why the threads, in all thefe experiments, are ftretched into lines, tending either to or from the center of the globe, feems to be this. Whatever be the force wherewith the globe acts on the threads, the direction of it muft be perpendicular to the furface of the globe; confequently, in the fame direction muft the threads move; but from the nature of the globe thofe, and thofe lines only, are perpendicular to it's furface, which either iffue from or tend towards the central point. Exp. 5.

Having faid thus much concerning electrical attraction, I now proceed to that of magnetifm. Many and furprifing are the properties both of the loadftone and magnetical needle, which however I fhall not here confider; my intent at prefent being only to fhew from experiment the law of magnetical attraction; or in other words, to fhew in what proportion the attractive power of the loadftone varies, according to the different diftances of the iron which it attracts. And in order to this, let a loadftone be fufpended at one end of a balance, and counterpoifed by weights at the other; let a flat piece of iron be placed beneath it, at the diftance of four Exp. 6.

tenth

LECT. II. tenth parts of an inch, the ſtone will immediately deſcend, and adhere to the iron: let the ſtone again be removed to the ſame diſtance, and a weight of four grains, and four tenth parts of a grain, be thrown into the ſcale at the other end of the balance; this weight will be an exact counterbalance to the attractive force, and prevent the deſcent of the ſtone; but if any part of the weight be taken out, the attraction will prevail, and carry the ſtone down. If the ſtone be placed at half the former diſtance, that is to ſay, at the diſtance of two tenth parts of an inch above the iron, the weight neceſſary to hinder it's deſcent will be about ſeventeen grains and an half, that is four times as much as before. Conſequently, the attractive force of the ſtone at the ſingle diſtance from the iron, is to the ſame at the double diſtance as four to one, that is reciprocally as the ſquares of the diſtances.

Perhaps it may be objected, that Sir ISAAC NEWTON (to whoſe judgment in natural affairs the utmoſt regard is due) has ſaid that the power of the loadſtone decreaſes nearly in the triplicate ratio of the increaſe of the diſtance. But whoever conſiders his words in the fifth corollary of the ſixth propoſition of the third book of his principles, where he mentions this law, will find that he ſpeaks of it with diffidence, as a thing which he rather gueſſed at from ſome rude obſervations, than collected from accurate experiments, for his words are, *Et in receſſu a magnete decreſcit in ratione diſtantiæ non duplicatâ, ſed fere triplicatâ, quantum ex craſſis quibuſdam obſervationibus animadvertere potui.* So that notwithſtanding this objection, I ſhall ſtill venture to affirm, the law of magnetical attraction to be ſuch as makes it act with forces which are in the reciprocal duplicate ratio of the diſtance. Becauſe this law is deduced from an experiment made with ſufficient exactneſs, and which does not ſeem liable to any exception.

Though

Though the principle of gravity, which comes next to be treated of, be diffused throughout the solar system, and may probably be extended so far as to reach the other systems of the universe; yet shall I consider it at present with respect only to the globe of earth, which we inhabit; the parts whereof would by reason of the diurnal rotation be apt to fly asunder, were they not kept together by the influence of this principle; whereby likewise all bodies on or near the surface of the earth, are made to tend towards it's center. This power at equal distances from the center of the earth is always proportional to the quantity of matter in the body whereon it acts; for all bodies, the light as well as heavy, being let fall from the same height descend with equal swiftness, provided they meet with no resistance from the air, as will appear from the following experiment. Let a piece of gold and a feather be let fall from the top of an exhausted receiver at the same instant of time, and they will both arrive at the bottom at the same time very nearly. LECT. II. Exp. 7.

The reason why the feather doth not reach the bottom quite so soon as the gold, is, that the receiver cannot be perfectly exhausted, and therefore the small portion of air which remains within, though very much rarified, gives some small resistance to the descending bodies, which suitably to the nature of all resistance must retard the lighter body more than the heavier; and consequently cause some little difference in the times of the descent, which otherwise would be exactly equal. This then being the case, it evidently follows, that the forces of gravity, whereby bodies descend, must at equal distances from the center be as the quantities of matter in the descending bodies; for if a certain force of gravity be requisite to carry down a certain quantity of matter with a certain swiftness, then is double the force necessary to carry down a double quantity of matter with the

LECT. II. the fame fwiftnefs; and triple the force to carry down a triple quantity, and fo in proportion, whatever be the quantity of matter: fo that the weights of bodies, at equal diftances from the center of the earth, are always proportional to the quantities of matter which they contain; and therefore, the quantity of matter in any body may be meafured by it's weight.

The gravity of a body, at any place beneath the furface of the earth, has been proved by Sir I. NEWTON to be directly as the diftance from the center; that is, fuppofing the earth's radius to be four thoufand miles, a body, which on the furface of the earth weighs a pound, will within the earth, at the diftance of two thoufand miles from the center, weigh only half a pound, at the diftance of one thoufand miles only a quarter, and fo on till at the center it lofes all it's gravity.

It has been likewife proved, that the force of gravity on the furface of the earth, and all diftances beyond it, is in the reciprocal duplicate ratio of the diftance from the center; that is, if a body weighs a pound at the furface of the earth, whofe diftance from the center is four thoufand miles, it will at double that diftance weigh only a quarter of a pound, and at triple the diftance, only the ninth part of a pound, and fo on, whatever be the diftance the force of gravity will be reciprocally as the fquare of the diftance. For is it not highly rational that the power of gravity, whatever it be, fhould exert it felf more rigoroufly in a fmall fphere, and weaker in a greater, in proportion as it is contracted or expanded; and if fo, feeing that the furfaces of fpheres are as the fquares of their *radii*, this power at feveral diftances muft be as the fquares of thofe diftances reciprocally. Though, ftrictly fpeaking, this be the law of gravity, yet where the diftances from the furface are inconfiderable with refpect to the earth's radius,

the force of gravity may be looked upon as equal at all thoſe diſtances; thus for inſtance, the gravity of a body at the diſtance of half a mile from the earth may be looked upon as equal to the gravity thereof at the diſtance of a quarter of a mile; or at the very ſurface; becauſe the difference is ſo ſmall, that if it be rejected it will not occaſion any error in calculations. And indeed on this ſuppoſition are founded moſt of the reaſonings of GALLILÆO, TORRICELLIUS, HUYGENS, and other naturaliſts concerning the deſcent of heavy bodies; and by the help of the ſame ſuppoſition have the ſeveral theorems been formed relating to the acceleration of falling bodies, the ſpaces deſcribed, the times of the fall, and the velocities thereby acquired; as I ſhall now ſhew you.

If the force of gravity whereby a body deſcends remains unvaried, the motion of a body falling by ſuch a force will be accelerated, and that uniformly; that is the velocity will increaſe, and the increments thereof in equal times will be equal. For let us ſuppoſe the time of the deſcent to be divided into a number of equal parts indefinitely ſmall, in each of which by ſuppoſition, the force of gravity makes equal impreſſions on the body to carry it down; whatever velocity therefore the body receives from the impreſſion of gravity in the firſt portion of time, it muſt receive as much in every other portion; ſince therefore ſetting aſide all outward lets and obſtacles the effect of every impreſſion remains, the velocity given in the firſt portion of time, will be doubled in the ſecond, tripled in the third, quadrupled in the fourth, and ſo on continually through the ſeveral portions of time. So that the velocity of a body falling by the force of gravity will conſtantly increaſe in the ſame proportion with the time of the deſcent. Or in other words, the motion of a body carried down by the force of gravity will be uniformly accelerated; and the ve-

LECT. II. locities acquired will be as the times of the defcent from the beginning of the fall.

From what has been faid it follows, that if a Fig. 3. right line as A B be fuppofed to denote the time of a body's fall, and another right line as BC fet at right angles to the former, to exprefs the velocity acquired by the falling body in the time denoted by A B. The triangle A B C being completed, and another right line as D E drawn parallel to B C, then will D E denote the velocity acquired by the falling body in a portion of time, which is to the time denoted by A B, as A D to A B. For from the nature of fimilar triangles, A B is to A D as B C to D E; but B C expreffes the velocity acquired where the time is as A B, confequently, fince the velocities are as the times of the defcent, D E will exprefs the velocity acquired in the time denoted by A D.

And what has been thus proved of the line D E, is in like manner true of any other right line, as F G, or H I, drawn within the triangle parallel to the bafe; for F G and H I will exprefs the velocities acquired in the times denoted by A F and A H.

The fpaces defcribed by bodies falling from a ftate of reft by the force of gravity are to one another as the fquares of the times from the beginning Fig. 4. of the fall. In the triangle A B C, let A B exprefs the time of a body's fall, and B C the velocity acquired at the end of the fall, let A B be divided into a number of equal parts indefinitely fmall; and from each of thofe divifions fuppofe lines, as D E drawn parallel to B C; it is evident from what has been faid, that thofe lines will exprefs the velocities of the falling body in the feveral refpective points of time; which velocities, inafmuch as the body is given and the portions of time are indefinitely fmall, will be as the refpective fpaces defcribed in thofe times: but the fum of the fpaces defcribed in all the fmall portions of time is equal to

to the ſpace deſcribed from the beginning of the fall; and the ſum of all the lines, as DE taken indefinitely near each other conſtitute the area of the triangle. And therefore the ſpace deſcribed by a falling body in the time expreſſed by A B, and where the velocity acquired at the end of the fall is denoted by B C, will be as the area of the triangle A B C. And for the ſame reaſon the ſpace deſcribed by a falling body in the time expreſſed by A D will be as the area of the triangle A D E. But from the nature of ſimilar triangles theſe areas are to one another as the ſquares of their homologous ſides; that is, as AB^q to AD^q, or as BC^q to DE^q. But A B and A D expreſs the times of the fall, and B C and D E the velocities acquired by the fall; wherefore the ſpaces deſcribed by a falling body are as the ſquares of the times from the beginning of the fall, or as the ſquares of the velocities at the end of the fall. And what has been thus demonſtrated from the nature of gravity is likewiſe confirmed by experiments. For if a weight of eleven hundred grains be let fall from the height of three inches, ſo as to ſtrike one end of a balance; its force will be juſt ſufficient to raiſe a pound weight at the other end of the balance to the height of about the eight or tenth part of an inch; whereas if the ſame body be required to raiſe a weight of two pounds to the ſame height, it muſt be let fall from the height of twelve inches; and if the weight to be raiſed be three pounds, then muſt the moving body fall from the height of twenty ſeven inches, for leſſer heights will not ſuffice, as will appear from the experiment.

LECT. II.

Exp. 8.

The forces wherewith the deſcending body ſtrikes the end of the balance are meaſured by the weights that are raiſed; which in this caſe are as one, two, and three; but the forces wherewith one and the ſame body ſtrikes, are as the velocities of the body, wherefore in the caſe before us the velocities acquired

by the falling body are as one, two, and three; but the heights from which it descends in order to acquire those velocities are as one, four, and nine; that is as the squares of the velocities.

Exp. 9. If this experiment be repeated with a body double in weight to the former, to wit, with one of twenty two hundred grains; the weights raised by the strokes will be two, four, and six pounds, to wit, double the former.

From this experiment appears the truth of that rule, which collects the quantity of motion in any body by multiplying the velocity of the body into it's quantity of matter. For the force of a stroke is, *cæteris paribus*, always proportional to the quantity of motion in the striking body; consequently in like circumstances the motions of bodies may be measured by the force of their strokes; but it has appeared from the experiment that where the striking body is as unity, and the velocities wherewith it moves at the times of the strokes; as one, two and three; the forces of the respective strokes are likewise as one, two and three. But where the body is as two, the strokes are as two, four and six: that is, in both cases the strokes are as the products arising from the multiplication of the quantities of matter in each body into the respective velocities; wherefore the quantities of motion are as those products. Whence as a corollary it follows, that if the weight of one body multiplied into it's velocity gives an equal product to what arises from the multiplication of the weight of another body by it's velocity, the motions of those two bodies are equal; and this will ever be where the weights of the bodies are reciprocally proportional to their velocities. Thus when the body whose weight was as unity, was let fall from the height of twelve inches, and thereby acquired a velocity which was as two; it raised a two pound weight, which was likewise raised by the body whose weight was as two, when by fall-

ing

ing from the height of three inches, it had acquired a velocity which was as unity. LECT. II.

From what has been proved concerning the ſpaces deſcribed by falling bodies it follows, that if the time of a body's fall be divided into a number of equal parts, the ſpaces through which it falls in each of thoſe parts of time taken ſeparately and in their order, beginning from the firſt, are as the odd numbers taken likewiſe in their order, beginning from unity. For inſtance, if the time of the fall be four ſeconds, the ſpace deſcribed in the firſt of thoſe ſeconds will be as one, in the ſecond as three, in the third as five, and in the fourth as ſeven; for where the times of the fall are as one, two, three and four; the ſpaces deſcribed are as one, four, nine and ſixteen; and therefore if from the ſpace deſcribed in two ſeconds, to wit, four, be ſubducted the ſpace deſcribed in the firſt ſecond, to wit, one, the remainder, to wit, three, will be the ſpace deſcribed in the next ſecond. And if from nine, which is the ſpace deſcribed in three ſeconds, be taken four, which is the ſpace deſcribed in two ſeconds, the remainder, which is five, will be the ſpace deſcribed in the third ſecond. In like manner ſubducting nine, the ſpace deſcribed in three ſeconds, from ſixteen, which is the ſpace deſcribed in four ſeconds, the remainder, to wit, ſeven, will be the ſpace deſcribed in the fourth ſecond; and ſo on according to the number of parts into which the time of the fall is divided.

From what has been ſaid it likewiſe follows, that the velocity acquired by a falling body at the end of the fall is ſuch as with an equable motion would in the ſame time in which the body fell, carry it through a ſpace double that of the fall. That the truth of this may be made appear, it is neceſſary that ſome things be premiſed concerning the ſpaces deſcribed by bodies carried with an equable motion. And firſt, if the velocity of a body moving uniformly be given,

 the

LECT. II. the ſpace deſcribed will be as the time of the motion; for if a body with a given velocity moves through a certain ſpace a foot, for inſtance, in a ſecond of time, it will in two ſeconds, with the ſame velocity, move through two feet, and through three feet in three ſeconds, and ſo on, whatever be the time, the ſpace deſcribed will be proportional thereto. On the other hand, if the time be given, the ſpace deſcribed will be as the velocity; for if a body in a given time moves through the ſpace of a foot with a certain velocity, with double the velocity it will paſs through the ſpace of two feet, and with triple the velocity through the ſpace of three feet, and ſo on, whatever be the velocity, the ſpace deſcribed will be in the ſame proportion. But if neither the time of a body's motion, nor the velocity wherewith it moves be given, the ſpace deſcribed will be as the time and velocity conjointly; for if a body moving with a certain velocity runs through a certain ſpace in a certain time, it follows from what has been ſaid, that if the time be increaſed or diminiſhed in any proportion, in the ſame alſo will the ſpace be increaſed or diminiſhed, ſuppoſing the velocity to remain the ſame, but if that likewiſe be changed, it is plain that the ſpace will be changed in the ſame proportion; and therefore univerſally the ſpace deſcribed by a body moving equally is as the time and velocity conjointly. For which reaſon, if in the rectangle, one ſide, as A B, be ſuppoſed to denote the time wherein a body moves equally, and B C the velocity wherewith it moves, the rectangle A B C D will be as the ſpace deſcribed; but the triangle A B C of the ſame figure, is as the ſpace deſcribed by a falling body in the time denoted by A B, and B C is as the velocity acquired at the end of the fall; and the rectangle A B C D is double the triangle A B C, conſequently the velocity acquired by a falling body is ſuch as will carry the body with an equable motion in the time of the fall through double the ſpace of the fall.

Fig. 5.

As

As the motion of bodies falling from a ſtate of reſt is uniformly accelerated; ſo likewiſe the motion of bodies thrown upward is uniformly retarded; for the ſame force of gravity, which conſpires with the motion of deſcending bodies, acts in direct oppoſition to the motion of ſuch as aſcend; and therefore in whatever manner it accelerates the one, in the very ſame manner muſt it retard the other. Whence it follows, that if a body be thrown directly upward, the time of its riſe will be equal to that wherein a body falling freely from a ſtate of reſt, acquires the ſame velocity wherewith the body is thrown up. For ſince the action of gravity is conſtant and uniform in whatever time it generates any velocity in a falling body, in the ſame time muſt it deſtroy that velocity in a riſing body; and therefore the time of the riſe muſt be equal to that of the fall. It likewiſe follows that the height to which a body thrown upward riſes is equal to that from which a body falling freely does at the end of the fall acquire a velocity equal to that wherewith the body is thrown up. For ſince the times in which the velocity of the falling body is generated, and that of the riſing body is deſtroyed, are equal: and ſince of the two equal velocities one is generated and the other deſtroyed by the conſtant uniform action of one and the ſame power; it is manifeſt that whatever be the ſpace through which the falling body moves in order to acquire it's velocity, the riſing body muſt aſcend through an equal ſpace in order to loſe it's velocity; that is it muſt riſe to the ſame height from which the other falls.

The force of gravity at the ſurface of the earth is ſuch as, ſetting aſide the reſiſtance of the air, makes a body falling from a ſtate of reſt to deſcend through a ſpace of ſixteen feet and an inch in a ſecond of time. For the time wherein a pendulum performs it's ſmalleſt vibrations is to the time in which a body falls through half the length of the pendulum as the

LECT. II.

circumference of a circle to it's diameter (as ſhall be ſhewn when I come to treat of the pendulum) wherefore ſince the ſpaces deſcribed by falling bodies are as the ſquares of the times, and ſince the diameter of a circle expreſſes the time which a body takes to fall through half the length of a pendulum vibrating ſeconds, when the circumference expreſſes a ſecond; it follows that as the ſquare of the diameter is to the ſquare of the circumference, ſo is half the length of the pendulum to the ſpace through which a body falls in a ſecond of time. So that putting D to denote the diameter of a circle, which is as unity, P the periphery which is as 3,1416, L the length of the pendulum vibrating ſeconds which is $39\frac{1}{8}$ inches; and S to denote the ſpace ſought; we ſhall have this analogy $D^2 : P^2 :: \frac{L}{2} : S$. Conſequently $S = \frac{P^2 \frac{1}{2} L}{D_2}$ or rejecting the diviſor as being equal to unity $S = P^2 \frac{1}{2} P = 193$ inches or ſixteen feet and an inch.

Before I quit this ſubject I muſt obſerve to you that bodies do not every where deſcend at the rate of ſixteen feet and an inch in a ſecond of time, but in ſuch places only as are in or near the latitude of forty nine degrees; in places more diſtant from the line the deſcent is quicker, and more ſlow in thoſe leſs diſtant. For the force of gravity is leſs towards the æquator than towards the poles, as has been collected from obſervations made on pendulums; for they have been found to vibrate more ſlowly near the line than in places farther removed; inſomuch that a pendulum which in the latitude of Paris vibrates ſeconds, muſt be ſhortened one ſixth of an inch French meaſure in order to it's vibrating ſeconds under the line. And the length of a pendulum which in the latitude of Paris performs it's vibrations in a ſecond, is to the length of a pen-

dulum

dulum whose vibrations are performed in the same time under the line as 220 to 219. Since therefore the forces of gravity which actuate pendulums that vibrate in equal times are to one another as the lengths of the pendulums (as shall be shewn when I come to treat of pendulums) it is evident that the force of gravity in the latitude of Paris is to the same force under the line as 220 to 219. And indeed it has appeared from a great number of observations that the force of gravity is least at the æquator, and that it continually increases as we recede from thence and approach the poles, under which it is greatest of all. And the chief cause of this difference is the rotation of the earth about it's axis, whereby all bodies on or near the surface of the earth are endued with a centrifugal force, which acts in opposition to that of gravity, and of course must lessen the same; and the diminution of gravity arising from this cause must be greatest under the æquator, and grow less and less in the approach to the poles: and that for two reasons, first, because the centrifugal force is greatest at the æquator, and from thence towards the poles is continually diminished so as at last to vanish in the polar points. For all parts of the earth's surface with the bodies thereto adjacent revolve in the same time either in the æquator or in circles parallel thereto; but the æquator is the largest of all those circles, and the others grow less and less as they are more and more distant from the æquator. Now the centrifugal forces of bodies revolving in the same time in different circles being to one another as the *radii* of the circles (as shall be shewn when I come to treat of those forces) it follows that the centrifugal force must be greatest at the æquator, and thence be continually diminished towards the poles. To illustrate this, let A B be the axis of the earth, CK the radius of the æquator, DI, EH and FG the radii of so many circles parallel to the æquator, Fig. 6.

 the

LECT. II. the centrifugal forces in the points K, I, H, G, are as thoſe radii; ſo that the centrifugal force is greateſt in the point K, that is at the æquator, and at I it is leſs than at K, and at H leſs than at I, and leſs again at G, and ſo on till at length it vaniſhes at the polar point where there is no rotation. Whence it is evident that the force of gravity muſt be ſmalleſt under the line, and muſt increaſe towards the poles, inaſmuch as the force which acts in oppoſition to it is greateſt under the line and leſſens in the approach to the poles. The force of gravity muſt likewiſe be leſs under the æquator than in any other place, becauſe under the line the centrifugal force acts in direct oppoſition to the force of gravity; whereas in other places it acts in an oblique direction to that of gravity, and of conſequence muſt act leſs powerfully againſt it. Thus in the point K the force of gravity pulleth from K towards C whilſt the centrifugal force pulleth directly contrary from C towards K; whereas in the point L gravity pulleth from L towards C, whilſt the direction of the centrifugal force is from O towards L. Let the centrifugal force in the point L be expreſſed by the line LM, and to CL continued to N let fall the perpendicular MN. The force LM, according to the known method of reſolving forces, of which I ſhall ſpeak hereafter, may be reſolved into two forces denoted by the lines NM, and LN; whereof the latter only acts in oppoſition to gravity as pulling directly againſt it; the other no way affecting the ſame: conſequently, ſuppoſing the centrifugal force at L to be the ſame as at K, yet will the force of gravity be leſs diminiſhed by it at L than at K, becauſe at L part only of the centrifugal force reſiſts that of gravity, whereas at K the whole centrifugal force acts in oppoſition thereto.

From what has been ſaid it follows, that the force whereby gravity is leſſened in the æquator is to the force whereby it is leſſened in any other part of the

earth's

earth's surface as the square of radius to the square of the sine of the complement of latitude. For the centrifugal force in the point K, the whole of which acts in opposition to gravity, is to the centrifugal force in the point L, as CK or CL to OL; but the whole centrifugal force in L is to that part of it which opposes gravity, as LM to LN, that is, because the triangles LNM and COL are similar, as CL to OL; wherefore the centrifugal force or the force which opposes gravity in the point K is to that part of the centrifugal force which opposes gravity in the point L in the duplicate ratio of CL to OL, that is, as the square of radius to the square of the sine of the complement of latitude.

LECTURE III.

OF REPULSION AND CENTRAL FORCES.

AS experience has convinced us that there are Powers in nature, whereby not only the larger systems and collections, but likewise the smaller parcels and particles of matter are in some cases made to tend to one another; the same experience will inform us of other powers in nature, whereby the parts of matter do in some circumstances recede and fly from each other. For if the disagreeing pole of a loadstone be moved towards a magnetical needle floating on water, the needle will recede; and the nearer the stone is brought to it, with the greater violence and precipitation will it fly off; the repelling power, like the attractive, exerting itself with greater vigor at smaller distances.

LECT. III.

Exp. 1.

This repelling power is likewise evident from the experiments which were made relating to electrical attraction: for it was observable that upon holding the glass tube, when heated by friction, nigh small pieces of brass-leaf; some of those pieces which by the

LECT. III. the attraction had been raised towards the tube, were, before they could reach it, driven back again with great precipitation: and of those which adhered to the tube some were thrown off with a velocity much greater than could possibly arise from the force of gravity in such light bodies, and consequently must have been driven down by some repelling power in the glass. And in the experiments of the glass-globe and woollen threads; when the threads were, by the attractive force of the globe, made to extend themselves towards it's surface, upon moving one's finger towards them, they were observed to recede and fly off, and that at considerable distances from the finger; which plainly argues a repelling power interceding the finger and the threads, when under the circumstances of those experiments. From this power it is, that the leaves of the sensitive plant shrink and retire from the touch of an approaching hand. And to the same power we are to attribute the elasticity of the air; as also the shaking off of the particles of light from the sun and other luminous bodies.

Besides the forementioned principles of attraction and repulsion, whereby nature seems to perform most of her operations, and which for that reason are very properly stiled active principles; there is another of a passive nature, commonly called the *vis insita* and *vis inertiæ* of matter, a force arising from the inertness or inactivity of matter; which force in any body is proportional to it's quantity of matter. From this force result three passive laws of motion, usually called by modern naturalists the three LAWS OF NATURE.[a]

The

[a] By virtue of the *vis inertiæ* it is, that the motion of a body produced by a force impressed upon it, is measured by the quantity of matter in the body and it's velocity, taken together. For the body by it's *vis inertiæ*, resists the force impressed upon it which causes it's motion, in proportion to it's quantity of matter; and consequently, to produce a given tendency in the body forward,

The firſt of theſe laws is, That every body, in proportion to it's quantity of matter, perſeveres in it's preſent ſtate, whether it be of reſt or uniform motion ſtraight forward in a right line. For as every particle of matter is with reſpect to itſelf perfectly unactive, it is utterly impoſſible it ſhould produce any alteration in it's own ſtate; for which reaſon (ſetting aſide all impreſſions from external cauſes) if it be at reſt, it muſt continue ſo for ever; or if in motion, it muſt for ever continue it's motion without any change either as to direction or velocity: ſo then the continuation of motion in bodies projected, (the cauſe whereof very much perplexed the naturaliſts of old) is to be attributed to the paſſive nature of matter, which makes it as impoſſible for a body of it ſelf to ſtop it's own motion when once begun, as it is for it to move it ſelf originally, or of it ſelf to change it's figure.

As a conſequence of this law it follows, that all motion is of it ſelf equable and rectilineal. For firſt whatever be the velocity wherewith a body begins to move, the ſame velocity muſt continue during the motion, unleſs a change be made therein by ſome cauſe from without; wherefore the body

forward, by whch it moves at a given rate or with a given velocity, the force impreſſed muſt be proportional to the reſiſtance ariſing from it's *vis inertiæ*, that is, to it's quantity of matter; and if the quantity of matter in the body, and conſequently the reſiſtance ariſing from it's *vis inertiæ*, be given, the force impreſſed will be proportional to the tendency forward which it communicates to the body, that is to it's velocity; and if neither the quantity of matter in the body, nor it's velocity be given the force impreſſed will be in a ratio compounded of the quantity of matter and velocity; that is, putting F for the force impreſſed, Q for the quantity of matter in the body, and V for it's velocity, F will be as $Q \times V$. But the motion of the body is the effect produced by the force F, and is proportional to it, that is, putting M for the motion of the body, M is as F. And therefore, by proportion of equality, M will be as $Q \times V$; or the motion of the body will be meaſured by it's quantity of matter and velocity taken together.

muſt

LECT. III. muſt in equal times move through equal ſpaces with an uniform velocity; that is, the motion muſt be equable. And as motion is by vertue of this law in it ſelf equable; ſo is it likewiſe rectilineal: for motion cannot otherwiſe be conceived than as directed and determined towards ſome place or other; and it muſt by the foregoing law keep the direction which it had at firſt, until it be hindred or put out of it's way by ſome extrinſic cauſe, that is, it muſt move on in a right line. If therefore a body moves in a curve, that curvature muſt of neceſſity proceed from ſome external force continually acting on the body; and whenever that force ceaſes to act, the body will move forward in a right line touching the curve in that point wherein the body is at the inſtant of time when the force ceaſes to act. Thus for inſtance, if a ſtone, moved about in a ſling, be ſet at liberty by ſlipping one end of the ſling; it will not continue it's circular motion, but go on in a right line touching the circle made by the circumvolution of the ſling in that point where the Fig. 8. ſtone is let go. If the circle B C D E be the curve deſcribed by the revolution of the ſling A B about the centre A; and if the ſtone be let off at the point B; it will move on in the right line B G, which touches the circle in B. For by the law, the natural tendency of the ſtone in the point B is along the line B G, though by the force of the ſling it be made to revolve in the curve. And what has been ſaid of the ſtone in the point B, is in like manner true of the ſame at any other point as C, D, or E; for in thoſe points it's tendency is along the lines C F, D H, and E K.

Another conſequence of the foregoing law is that all bodies, which revolve about a center, muſt endeavour to recede from the center; for ſince bodies, that are moved round in a curve, do of themſelves in every point of the curve tend to move in the

tangents to each point; and ſince all the parts of the tangents are more diſtant from the center of motion than are the parts of the curve, as is evident from the figure; it is manifeſt that bodies ſo moved muſt perpetually endeavour to fly off from the center of motion, which endeavour of receding is commonly called the centrifugal force; and it is oppoſed to the centripetal force, or that force which by drawing the bodies towards the center makes them to revolve in a curve. LECT. III.

Theſe two forces are by one common name called the central forces: and they are in all caſes equal the one to the other. For let us ſuppoſe a body to revolve in the orbit E A C, and that being in the point A the centripetal force ceaſes to act; it will then move forward in the direction of the tangent A B, and B C will be the ſpace through which the body recedes from the orbit by means of the centrifugal force; and if A B be in it's naſcent ſtate, the centrifugal force will be as B C; but if the centripetal force acts at A, it will make the body deſcribe the arc A C in the ſame time that it would deſcribe the tangent A B, in caſe it were not acted upon by the centripetal force; conſequently, the ſpace B C is deſcribed by means of the centripetal force; and the arc A C being in it's naſcent ſtate, the centripetal force will be as B C, and of conſequence equal to the centrifugal. Fig. 9.

In treating of theſe central forces I ſhall proceed in the following manner. Firſt, I ſhall conſider two equal bodies moving uniformly in two different circles; and thence deduce one general expreſſion for the central forces in the terms of the circle. Secondly, By ſubſtituting other proportional quantities in the place of thoſe which conſtitute the general expreſſion, I ſhall form other general expreſſions for the ſame forces. Thirdly, By a proper application of thoſe expreſſions I ſhall determine the laws

LECT. III. laws of central forces in particular cases, and at the same time confirm each law by an experiment.

Fig. 10, 11. As to the first, if two equal bodies moving uniformly in the circles marked 1, & 2, do in the same portion of time taken indefinitely small describe the nascent arches A C; and if from the points C be drawn the lines C B, perpendicular to the tangents A B, those lines will express the proportion of the central forces. For since the time in which the arches A C are described is indefinitely small, the bodies will be carried through the spaces BC, by one single impulse of each central force; for which reason the motions of the bodies through those spaces will be uniform; consequently since the time of the motion is the same, and the bodies equal, the motion will be as the spaces described, that is, as the lines B C; but forces which generate equable motions are to one another as the motions generated; that is, in this case, as the lines B C; which lines being equal to the versed sines A D of the arches A C, must be equal to the squares of the arches A C, divided by their respective diameters A E. For from the nature of the circle, the versed sine of any arch is equal to the square of the chord divided by the diameter; but as in this case the arches A C are supposed to be nascent, they do not differ from their chords; and therefore in each circle the versed sine of the arch A C, (which versed sine expresses the central force) is equal to the square of the arch divided by the diameter: consequently, the central forces are as the squares of the nascent arches applied to their respective diameters; and forasmuch as those nascent arches are to one another as any other two arches, which are described by the revolving bodies in a given time, the central forces of two equal bodies revolving uniformly in different circles, are to one another as the squares of the arches described in a given time applied to their respective diameters; or because the diameters

are

are as the radii, as the ſquares of the arches applied to their reſpective radii. Wherefore putting A to denote the arch of a circle deſcribed in a given time, D for the radius, and F for the central force; F is as $\frac{A^2}{D}$, as it ſtands in the firſt place of the firſt rank of ſymbols.

F is as $\frac{A^2}{D}$	F is as $\frac{QA^2}{D}$
F is as $\frac{V^2}{D}$	F is as $\frac{QV^2}{D}$
F is as $\frac{D}{P^2}$	F is as $\frac{QD}{P^2}$
F is as DN^2	F is as QDN^2

As the bodies are ſuppoſed to move uniformly in the circles, it is evident that the arches deſcribed in a given time are as the velocities of the revolving bodies; and therefore in the general expreſſion for the central force, the velocity of the body may be ſubſtituted in the place of the circular arch; whence putting V for the velocity of the body, F is as $\frac{V^2}{D}$, as in the ſecond place of the firſt rank of ſymbols, which is a ſecond general expreſſion for the central force.

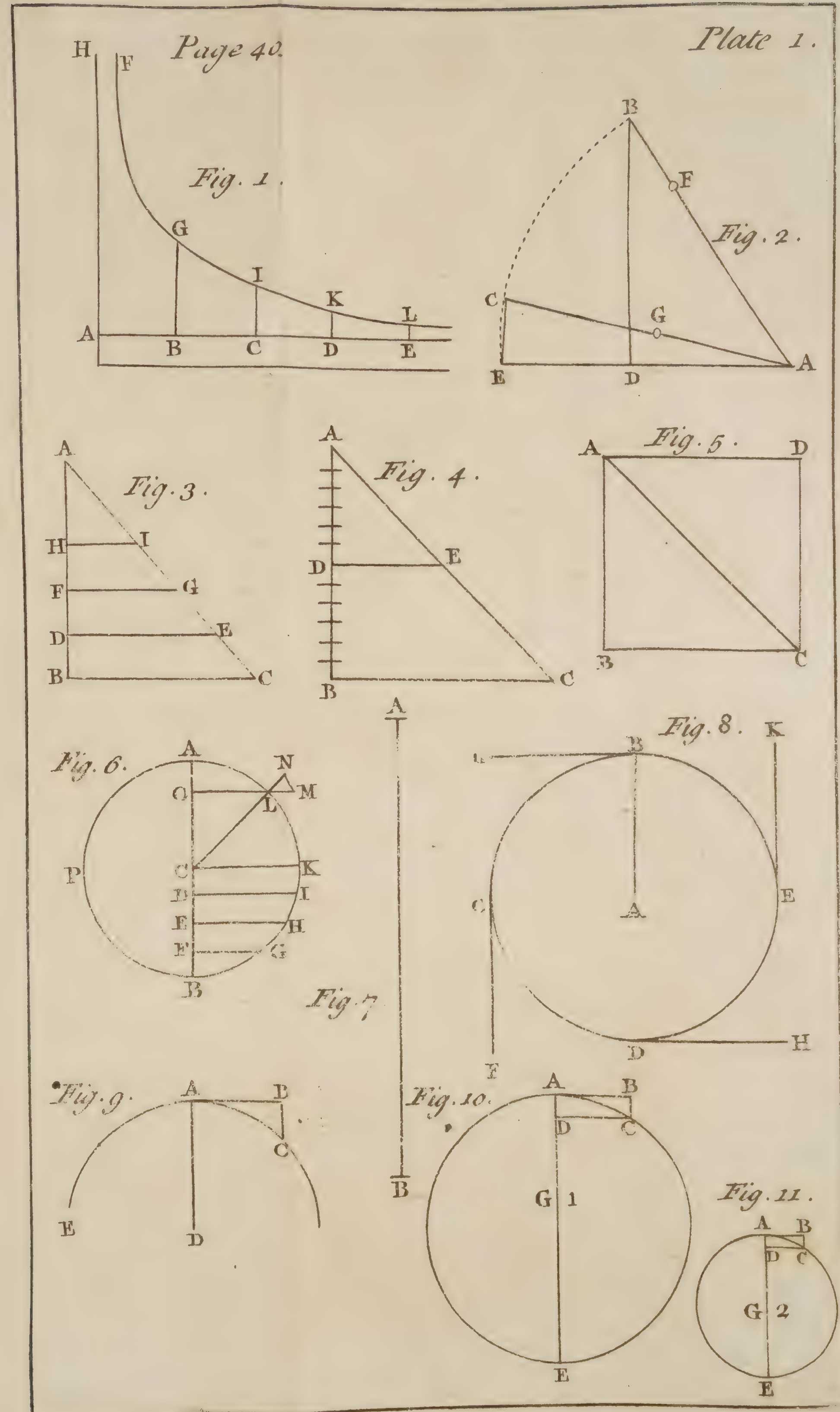
Again, the velocity of a body moving uniformly in a circle, is as the radius applied to the periodic time, or the time of one intire revolution. For if the velocity of the body be given, the periodic time muſt be proportional to the circumference of the circle, inaſmuch as a body, which with a given velocity deſcribes a certain ſpace in a certain time, will require a double or triple time, to deſcribe a double or triple ſpace; and univerſally whatever be the magnitude of the ſpace, the time in which it is

D deſcribed

LECT. III. defcribed will be proportional to it. If the circumference of the circle be given, the periodic time will be inverfly as the velocity with which the body moves; for if a body moves through a given fpace with a certain velocity in a certain time, it will with double the velocity move through the fame fpace in half the time, and with a triple velocity in one third of the time; and in general, in whatever proportion the velocity is increafed, in the fame proportion will the time be leffened; that is, the periodic time will be inverfly as the velocity. If therefore neither the circumference of the circle, nor the velocity of the body be given, the periodic time will be directly as the circumference, and inverfly as the velocity; that is, as the circumference applied to the velocity; or (becaufe the circumference is as the radius) as the radius applied to the velocity. Wherefore putting P for the periodic time of a body revolving in a circle, P is as $\frac{D}{V}$, and confequently, V is as $\frac{D}{P}$. If therefore in the fecond general expreffion $\frac{D}{P}$ be fubftituted in the place of V, we fhall have a third general expreffion for the central force, wherein F is as $\frac{D}{P^2}$, as in the third place of the firft rank of fymbols.

Again, the periodic time of a body revolving uniformly, is inverfly as the number of revolutions performed in a given time. For if the periodic time of a body be fuch, as that in a given time it can perform a certain number of revolutions; if the periodic time thereof be doubled, it will perform but half the number of revolutions in the fame time; and if the periodic time becomes thrice as great, it will perform but one third of the number of revolutions in the given time; and fo on, as the periodic time is inlarged, the number of revolutions will

Page 40. Plate 1.



will be diminiſhed in the ſame proportion, ſo that putting N for the number of revolutions in a given time, P will be as $\frac{1}{N}$. Conſequently, if in the third general expreſſion $\frac{1}{N}$ be ſubſtituted in the room of P, we ſhall have a fourth general expreſſion for the central force, wherein F is as DN^2, as it ſtands in the laſt place of the firſt rank of ſymbols.

In collecting theſe general expreſſions, I have all along ſuppoſed the quantity of matter in the revolving body to be given; and for that reaſon have not made it a part of thoſe expreſſions, inaſmuch as it may be denoted by unity; and as ſuch, whether it be taken in, or left out, it will not vary the expreſſions. But the caſe will be different, if the quantity of matter in the revolving body varies; becauſe the central forces, and conſequently the expreſſions for thoſe forces, will likewiſe vary; ſo as to be greater *cæteris paribus* in larger quantities of matter than in ſmaller. For the whole central force of any body, is made up of the forces of each particle whereof the body conſiſts; and therefore the more numerous the particles of matter are in any body, the greater will it's central force be; ſo as to be double in a double quantity of matter, triple in a triple quantity; and ſo on in proportion to the quantity of matter. In order therefore to render the expreſſions yet more general, let Q be put for the quantity of matter in the revolving body, and let it be multiplied into each of the four expreſſions, as in the ſecond rank of ſymbols.

Before I apply theſe expreſſions to the ſeveral particular caſes, I ſhall offer an experiment in confirmation of what I juſt now proved, *viz.* that the greater the quantity of matter in any body is, the greater is the central force.

LECT. III.

Let three glaſs tubes half full, one with mercury and water, another with water and ſmall leaden bullets, the third with water and a piece of cork, be ſtopped cloſe, and made faſt to an inclined plane; and let the plane be ſo fixed to a table, moveable about it's center, by means of a wheel and axle, as that the lowermoſt part of the plane may reſt upon the center of the table. As long as the table continues at reſt, the liquors and ſolids contained in the tubes will, by reaſon of their gravity, poſſeſs themſelves of thoſe parts of the tubes which lye next the center of the table, leaving the remoter parts empty: and of the two bodies included in each tube, that which is heavieſt will be neareſt the center; but upon turning the table about, the ſeveral bodies will, by reaſon of their centrifugal forces, whereby they are carried from the center of motion, fly to the uppermoſt parts of the tubes; and in each tube, the heavier body will poſſeſs the uppermoſt place, as being indued with the ſtronger centrifugal force.

Exp. 1.

If bodies moving in equal circles perform their revolutions in equal times; or in other words, if the velocities of bodies revolving in circles be equal, and their diſtance from the center likewiſe equal, their centrifugal forces are as their quantities of matter. For in the ſecond general expreſſion, ſince V and D are given, F is as Q; that is, the central force is as the quantity of matter; which is confirmed by the following experiment. Let two ſmall troughs be ſo fixed to two moveable tables, as that the centers of the troughs may lye upon the centers of the tables, and let the centers of the tables be fixed to two axles, on each of which is a grooved wheel, with equal diameters; let the two wheels be turned by means of one and the ſame chord going round them: it is manifeſt, that as the wheels are equal, they, and conſequently the tables with their affixed troughs, muſt perform their revolutions in the ſame time; and the parts of the tables

Exp. 2.

tables and troughs, whose distances from their respective centers are equal, will revolve equally swift; and so likewise must all bodies that are placed in the troughs at equal distances from the centers: so that by this contrivance, if two bodies be placed one in each trough at equal distances from the centers, they will revolve equally swift. Let then two balls, whereof one is double the other, be laid one in each trough, and let each ball be fastened to one end of a chord, whose other end passing through an hole in the center of the table is made fast to a weight, which rests upon the floor; and let the lengths of the chords be such, as that being stretched, and the weights not raised, the balls in the troughs may be equally distant from the centers. This being done, if the weights be to one another as the balls, and if the tables be turned about with such a velocity as that the centrifugal forces of the balls may be sufficient to raise the weights, they will be lifted up precisely at the same time. Whence it appears, that in this case the centrifugal forces are as the quantities of matter, inasmuch as they overcome resistances which are in that proportion.

If equal bodies moving in unequal circles perform their revolutions in equal times; or in other words, if the quantity of matter in the revolving bodies be given, as also the number of revolutions performed in a given time, their centrifugal forces are as their distances from the center. For in the fourth general expression, since Q and N are given, F is as D, that is, the force is as the distance. For the confirmation whereof, let two equal balls be placed in the troughs at distances from the centers, which are as one and two, and when the tables are turned about, that ball, whose distance from the center is double, will raise a double weight. Exp. 3.

If equal bodies move in unequal circles with equal velocities; or more generally, if the quantity of matter in the revolving bodies be given, as also the

LECT. III. velocity wherewith they revolve; their central forces are inverſly as their diſtances from the center. For in the ſecond general expreſſion, ſince Q and V are given, F is as $\frac{1}{D}$, that is, the force is inverſly as the diſtance. Before I mention the experiment whereby this law is confirmed, I muſt obſerve to you, that to the axle of one of the tables is fixed a ſecond wheel, whoſe diameter is but one half of the diameter of the other wheel; and therefore when the chord goes round the ſmaller wheel, the table muſt turn as faſt again as when it goes round the larger wheel; ſo that the table which is moved by means of the ſmaller wheel, will revolve twice in the ſame time, that the other table which is turned by means of the larger wheel, performs one revolution.

Exp. 4. This being premiſed, let two equal balls be ſo placed in the troughs, as that the diſtance of that ball which is to revolve by means of the ſmaller wheel, may be but one half of the other's diſtance from the center; in which caſe their velocities will be equal: for though the peripheries of the circles which the two balls deſcribe, are as one and two; yet will the leſſer periphery be deſcribed twice in the ſame time that the larger is deſcribed once; and therefore the ſpaces through which the bodies move in a given time will be equal, and of conſequence their velocities will be ſo too. If then two weights be made faſt to the chords of the balls in the manner of the former experiments; the tables being turned about, the ball whoſe diſtance from the center is as one, will raiſe twice the weight that is raiſed by the ball whoſe diſtance is as two; ſo that the weights raiſed, and conſequently the forces which raiſe them, will be inverſly as the diſtances of the balls from the center.

If equal bodies revolve in equal circles with unequal velocities, their central forces are as the ſquares of the velocities, or becauſe the velocities are as the

number

number of revolutions in a given time, the forces are as the ſquares of the numbers of revolutions performed in a given time. For by the fourth general expreſſion, ſince Q and D are given, F is as N^2, that is, the force is as the ſquare of the number of revolutions in a given time. To confirm this law, let two equal balls be placed in the troughs at equal diſtances from the centers; and let that table, whoſe axle has two wheels, be turned about by means of the ſmaller, ſo that it may perform two revolutions in the ſame time that the other table performs one: in this caſe the numbers of revolutions performed by the two balls in a given time being as one and two, their ſquares will be as one and four, in which proportion the weights raiſed will likewiſe be. Exp. 5.

If unequal bodies revolve in equal circles with unequal velocities, their central forces are as the products of their quantities of matter into the ſquares of their reſpective velocities; or, which is the ſame thing, as the products of their quantities of matter into the ſquares of the numbers of revolutions in a given time. For by the fourth general expreſſion, D being given, F is as QN^2. Let therefore two balls, whereof one is double the other, be placed at equal diſtances from the centers; and let the larger revolve twice in the ſame time that the ſmaller revolves once. In this caſe the quantity of matter in Exp. 6. the leſſer ball, which is as unity, being multiplied into the ſquare of it's number of revolutions in a given time, which is likewiſe as unity, gives one for the product. And the quantity of matter in the larger ball, which is as two, being multiplied into the ſquare of it's number of revolutions in the given time, which ſquare is as four, gives eight for the product: ſo that the weights raiſed by the two balls will be as one and eight.

If unequal bodies revolve in unequal circles with unequal velocities, their forces are as their quantities of matter multiplied into the ſquares of their re-

LECT. III. ſpective velocities, and that product divided by their reſpective diſtances from the centers; or what amounts to the ſame thing, their forces are as the products ariſing from the continued multiplication of their quantities of matter into their reſpective diſtances from the centers, into the ſquares of their numbers of revolutions in a given time; or to uſe the mathematical phraſe, their forces are in a ratio compounded of their quantities of matter, of their diſtances from the center, and of the ſquares of their numbers of revolutions in a given time. For by the fourth general expreſſion F is as QDN^2. To confirm this law by an experiment, let two balls, whereof one is double the other, be placed in the troughs, ſo as that the diſtance of the ſmaller from the center may be to the diſtance of the larger as two to one; and let the larger revolve twice in the ſame time that the ſmaller revolves once. In this caſe the quantity of matter in the ſmaller body, which is as one, being multiplied into the diſtance from the center, which is as two, and the product being multiplied into the ſquare of the number of revolutions performed by the ſmaller body in a given time, which is as one, gives two for the product. In like manner the quantity of matter in the larger body, which is as two, being multiplied into the diſtance from the center, which is as one, and the product of that multiplication being again multiplied into the ſquare of the number of revolutions performed by the larger body in the given time, which ſquare is as four, gives eight for the product; conſequently, the weights which are raiſed, as alſo the forces which raiſe them, are as two and eight, or one and four.

Exp. 7.

If equal bodies revolve in unequal circles in ſuch a manner as that the ſquares of their periodical times are as the cubes of their diſtances from the center, their central forces are inverſly as the ſquares of their diſtances from the center. For ſince the

quantity

quantity of matter in the revolving bodies is given, and the cubes of the diftances are as the fquares of the times; if in the third general expreffion the cube of D be fubftituted in the room of the fquare of P, F will be as D divided by the cube of D, or as one divided by the fquare of D; that is, the force will be inverfly as the fquare of the body's diftance from the center. To confirm this law, let two equal balls be placed in the troughs, fo as that the diftance of one from the center may be as two, and the diftance of the other as three and one fixth; and let that which is at the fmalleft diftance revolve twice in the fame time that the other revolves once; fo that their periodical times may be as one and two, the fquares of which being one and four, are very nearly proportional to the cubes of the diftances; for the cube of the fmaller diftance is eight, and that of the larger thirty-two very nearly; confequently, the balls muft raife weights which are to one another inverfly as the fquares of the diftances from the center; that is, the weight raifed by the ball, whofe diftance is as two, muft be to the weight raifed by the ball whofe diftance is as three and a fixth, as the fquare of the laft diftance to the fquare of the former, that is as ten to four, or five to two very nearly.

Exp. 8.

If the fquares of the periodical times be proportional to the cubes of the diftances, and the revolving bodies unequal, the central forces are directly as the quantities of matter in the bodies, and reciprocally as the fquares of their diftances from the center. For in the third general expreffion, if the cube of D be fubftituted in the room of the fquare of P, F will be as $\frac{Q}{D^2}$. If therefore all things remain as in the laft experiment, excepting that the body which is at the greater diftance from the center is to the body lefs diftant, as two to one; the weight which is raifed by the former, will be to the weight raifed by the

LECT. III. the latter, as two, to two and a half; that is, the weights raiſed, will be as the products ariſing from the multiplication of the quantity of matter in one body, into the ſquare of the other body's diſtance.

Among the ſeveral laws of central forces, that which obtains in nature, and by vertue whereof the heavenly bodies are made to revolve in their ſeveral orbits, is, where the forces are to one another inverſly as the ſquares of the diſtances of the revolving bodies from the center. For it has been found by obſervation, that all the planets, as well primary as ſecondary, revolve either in circular orbits, or ſuch as are nearly ſo. And that the ſix primary planets move about the ſun as their center, in ſuch a manner, as that the cubes of their mean diſtances from the ſun are very nearly proportional to the ſquares of their periodical times. And the ſame thing has been diſcovered, with regard to the four ſecondary planets or ſatellites that move about JUPITER, as alſo with reſpect to the other five that revolve about SATURN. And therefore the forces whereby they are retained in their orbits, muſt be in the inverſe ratio of the ſquares of their diſtances from the central bodies about which they revolve.

Exp. 9. If two bodies are, by means of their mutual attraction, made to revolve about each other, and alſo about a fixed point; and if their diſtances from that fixed point be reciprocally proportional to their quantities of matter, that is to ſay, if as much as one body exceeds the other in quantity of matter, ſo much is it's diſtance from the fixed point exceeded by the others diſtance from the ſame point; or what amounts to the ſame thing, if the product, ariſing from the multiplication of one body into it's diſtance from the fixed point, be equal to the product ariſing from the like multiplication of the other body into it's diſtance from the fixed point, their central forces are equal. For as the two bodies muſt of neceſſity perform their revolutions in the ſame time; the

the number of their revolutions in a given time is given: and therefore by the fourth general expreſſion F is as QD, that is, the central force is as the product arising from the multiplication of the quantity of matter into the diſtance from the center, or fixed point; but by ſuppoſition the product of one of the bodies into it's diſtance from the fixed point, is equal to the product of the other into it's diſtance, conſequently, their central forces are equal; for which reaſon neither of them can fly off from the fixed point, ſo as to draw the other after it; for however ſtrongly either of them endeavours to recede, by vertue of it's own centrifugal force, it is with equal ſtrength drawn the contrary way by the centrifugal force of the other. But if the diſtances of the bodies from the fixed point be not reciprocally proportional to their quantities of matter; that body, whoſe diſtance, with regard to the diſtance of the other, is greater than in the forementioned proportion, will fly off and draw the other after it: for in this caſe, the product of the former body into it's diſtance from the fixed point is greater than the product of the latter into it's diſtance; which products being as the centrifugal forces of the bodies, the former body will have a greater centrifugal force than the latter, and of courſe muſt recede from the fixed point, and drag the other after it; all which is fully confirmed by the following experiments. Let two equal balls be tied together by a ſmall chord; and let them be laid in one and the ſame trough, one at each end, ſo as that the chord being ſtretched may have it's middle point juſt over the center of the table; let then the table be turned about, and the balls will revolve about the center without flying off either way; and continue ſo to do as long as the motion of the table laſteth. And the ſame thing will likewiſe happen though one ball be double the other, provided it's diſtance from the center of the table be but one half of the diſtance of the

LECT. III.

Exp. 10.

ſmaller.

ſmaller. But when equal balls are made uſe of, if one of them be placed at a greater diſtance from the center than the other, upon turning the table it will fly off and draw the other after it. So likewiſe when unequal balls are made uſe of, ſhould that which is double the other be placed at a diſtance from the center greater than one half of the diſtance of the ſmaller, it will fly off and draw the ſmaller after it. And on the other hand, if the diſtance of the larger be leſs than half the diſtance of the ſmaller, the ſmaller will in that caſe fly off and draw the larger after it.

LECTURE IV.

OF THE COMPOSITION AND RESOLUTION OF MOTION.

LECT. IV.

THE ſecond LAW OF NATURE, reſulting from the inertneſs of matter, is; that whatever motion, or change of motion, is produced in any body, it muſt be proportional to, and in the direction of, the force impreſſed. For ſince a body cannot, by reaſon of it's inactivity, contribute to the production of it's own motion, or of any change therein, it is plain, that whatever motion or change of motion is generated in any body, it muſt intirely proceed from the force impreſſed on the body; and of conſequence, ſince effects are ever proportionate to their adequate cauſes, muſt be proportional thereto. And it muſt likewiſe be directed and determined towards the ſame part with the generating force. Wherefore if the body whereon the impreſſion is made, was in motion before the impulſe, that motion will be retarded or accelerated according as the force impreſſed oppoſes it, or conſpires therewith, or if it acts obliquely to the ſame, the direction thereof will be changed, and the body will

will move in a direction situated between the direction of it's former motion, and that of the impressed force. For instance, if a body moving from A towards B, be impelled at the point A by a force acting in the direction A C, it will move along a line as A D placed between A B and A C, the situation of which may be thus determined. Let the line A B denote the velocity wherewith the body moves in the direction A B; and let A C denote the velocity wherewith the body would move by vertue of the impulse along the line A C, supposing it had no other motion: that is, let A B be to A C as the space described by the body in a given time in the direction A B, to the space described by it in the direction A C, each of the motions being considered singly and apart; then completing the parallelogram A B D C, and drawing the diagonal A D, that diagonal is the line in which the body moves; for the proof of which, let us suppose a small inflexible wire equal in length to the line A B, to pass through the center of a ball, and that whilst the ball moves uniformly on the wire from A towards B, with a velocity which is as A B, the wire is also moved uniformly from A B towards C D, with a velocity which is as A C, and in such a manner as to be always parallel to A B, and with it's extremities to describe the lines A C and B D. Then, forasmuch as the spaces described in a given time, where the motions are uniform, are to one another as the velocities of the motions; it is evident, that in whatever time the ball moves the length of the wire, in the same time will the wire move the length of A C, to wit from A B to C D; consequently, at the end of that time the ball will be found in D, at the extreme point of the diagonal A D. From any point in the diagonal taken at pleasure as E, let the line E F be drawn parallel to D B, and from the nature of similar triangles, A F will be to F E, as A B to B D, that is, as the velocity

LECT. IV.

Pl. 2. Fig. 1.

LECT. IV.

velocity of the ball to the velocity of the wire; confequently, in the fame time that the ball moves the length of A F along the wire, the wire will move the length of F E, from A B to K L; and the point F, which is the place of the ball on the wire, will be found in E. And what has been thus proved, in relation to the two points D and E of the diagonal, may in the very fame manner be demonftrated of any other point in the fame line; wherefore the ball will, by vertue of it's own motion, and that of the wire, whereof it partakes, be carried in fuch a manner as to be always found in the diagonal A D; that is, it will, by vertue of it's compound motion, defcribe the diagonal line. This being fo, it plainly follows, that if the wire be taken away, and the ball at A have two motions impreffed upon it at once; one in the direction A B, the other in the direction A C; and if the motions impreffed, or, which is the fame thing, if the forces impreffing thofe motions be to one another in the proportion of A B to A C, the ball will, by vertue of the double impreffion, move along the diagonal A D. For as to the effect it matters not whether the motion which the ball has in the direction A C arifes from a force impreffed on it at the point A, or whether it be communicated by a wire fupporting the ball, and carrying it along with it in that direction.

Exp. 11. To confirm this by an experiment, let three ivory balls of equal fize be fufpended from three pins, by ftrings of equal lengths, and let the middle ball reft over one angle of a wooden fquare; then let each of the extreme balls be let fall feparately from the fame height, in fuch manner as to ftrike the middle ball in the direction of one fide of the fquare, and the middle ball will, by each of the ftrokes made feparately, be moved along over that fide of the fquare, which correfpondeth to the direction of the ftroke; but if the two balls

be

be at the fame inftant of time let fall from equal heights, fo as that they may ftrike the middle ball at once, and in the directions of the two fides of the fquare, the middle ball will, by the double ftroke, be driven over the diagonal of the fquare.

As a COROLLARY it follows, that a body will in the fame time defcribe the diagonal A D of a parallelogram with two forces conjoined, that are to one another as the fides A B and A C, that it would the refpective fides with each of thofe forces feparately. As alfo, that the velocity wherewith a body moves along the diagonal, is to the velocity wherewith it is carried along the fides, when acted upon by each force fingly, as the diagonal to each fide refpectively: confequently, if the two forces be given, the velocity along the diagonal, which arifes from the conjunction of both forces, will be fo much the greater, by how much the angle B A C is lefs; for as that angle is diminifhed, the diagonal, which in this cafe denotes the velocity, is lengthened, till at laft the angle vanifhing by the coincidence of the fides, the diagonal becomes equal to both the fides taken together; and the velocity of the body equal to the fum of the velocities wherewith the body would move, were each of thofe forces impreffed upon it in the fame direction. Thus the lines A B and A C being placed at three different angles, fo as to conftitute the fides of three different parallelograms, (the diagonals whereof are reprefented by the pricked lines) it is evident to fight, that as the angle B A C grows lefs, the diagonal grows longer; and that when the angle vanifhes by the coincidence of A B with A C, the diagonal A D becomes equal to A C and C D, that is, to A C and A B; and the velocity denoted by A D, is in that cafe a *maximum*, or the greateft that can arife from the conjunction of thofe two forces. On the other hand, as the angle inlarges, the velocity along the diagonal

Pl. 2. Fig. 2.

Pl. 2. Fig. 3.

LECT. IV. diagonal muſt decreaſe, till at length the angle vaniſhing by the two ſides becoming one right line, the velocity becomes equal to the difference of the velocities, ariſing from the impreſſion of each force when made ſingly and ſeparately. Thus the lines

Pl. 2. Fig. 4. A B and A C being as before placed at three different angles B A C, it is evident that the diagonals A D repreſented by the pricked lines, grow ſhorter as the angle B A C inlarges; till at laſt the angle, and with it the diagonal vaniſhing, the two ſides

Pl. 2. Fig. 5. B A and A C conſtitute one right line, as B A C, wherein the body is, as it were, carried two contrary ways, to wit, from A towards B, by the force which acts in the direction A B, and from A towards C, by the force acting in the direction A C; and the difference of the velocities, which ariſe from the impreſſions of the two forces when they act ſeparately, is the velocity wherewith the body actually moves in the direction of the ſtronger force; which velocity is a *minimum*, or the leaſt velocity that can ariſe from the joint action of thoſe two forces.

As a ſecond COROLLARY it follows, that a body may be moved through one and the ſame line by numberleſs pairs of forces acting upon it. For if

Pl. 2. Fig. 6. inſtead of the force, whoſe direction is A B, we ſuppoſe another, the direction whereof is A E; and if inſtead of the force acting in the direction A C, we ſuppoſe one to act in the direction A F, and that thoſe forces are to one another as A E to A F; then completing the parallelogram A E D F, the line A D will be the diagonal of this parallelogram, as well as of the former; and therefore the body will from the joint action of theſe two forces deſcribe the ſame line A D which it did before: and as A D may be made the diagonal of numberleſs parallelograms, it is evident that it may be deſcribed by a body acted upon by numberleſs pairs of forces in different directions. And not only ſo,

but

but it may likewife be defcribed by a body, whereon a great number of forces act at the fame time; for as the forces acting upon the body in the directions A B and A C make it to move along the diagonal A D, fo may the direction along A B arife from the directions of two other forces, and each of thofe from the directions of two others, and fo on without number. Hence we fee, that all forces and motions whatever may be refolved into innumerable forces and motions; and any fimple direct force or motion may be looked upon as compounded of innumerable oblique forces or motions. For the line and direction of the motion is the fame, whether that motion be compounded of two motions arifing from forces impreffed in the directions A B, A C, or in the directions AE, AF, or arife from the impreffion of a fingle force in the direction A D; and therefore the motion along the line A D, though it be fimple arifing from one fingle force acting in that direction; yet may it be confidered as compounded of two or more motions in other directions, fuch as AB and AC, or AE and AF, fince the very fame motion would arife from fuch a compofition.

LECT. IV.

Pl. 2. Fig. 6.

This compofition and refolution of motions and forces is of fingular ufe in mechanics; for by the help thereof, the effects of powers acting in oblique directions are readily determined, as will appear hereafter.

The third LAW OF NATURE arifing from the inertnefs of matter is, that reaction is always equal to action, and contrary thereto; or in other words, that the actions of two bodies, one upon another, are conftantly equal, and in directions contrary to each other; fo that whatever change is made in the ftate of one body, whether at reft or in motion, by the action of another; the fame change is produced in the ftate of the other by the reaction of the former; but the tendencies or directions of thofe changes are contrary ways. Thus, when one preffes

LECT. II. a ſtone with his finger directly downward, the finger is equally preſſed by the ſtone, and that directly upward. And when a horſe draws a load, he is equally drawn back by the load; for as much as he promotes the progreſs of the load, ſo much is he retarded in his own motion; that is, he is in effect drawn back; for the ſame force of muſcles and ſinews, which he exerts in order to drag on the load, would if he was freed from the incumbrance, carry him forward to a diſtance much greater than what he reaches in the ſame time whilſt tied to the load; and conſequently, as far as his progreſs falleth ſhort of that diſtance, ſo much is he in effect drawn back; and whatever motion he communicates to the load, ſo much does he loſe of his own, the load reacting upon him with the ſame force that he acts upon it; for which reaſon, if by addition of weight the load be ſo far increaſed as to require the whole ſtrength of the horſe to move it, no motion will enſue, the whole power of the horſe wherewith he endeavours to go forward, being but juſt equal to the reaction of the load whereby he is drawn back. This equality of action and reaction obtains in all kinds of attractions whatever. When a loadſtone attracts a piece of iron, it is equally attracted by it; as will appear from the following experiment. Exp. 12. Let a piece of iron and a loadſtone equal in weight, be ſuſpended by two cords of an equal length, and let the diſtance between them be ſo ſmall, as that they may not be out of the reach of each other's attraction; then will they from a ſtate of reſt, begin to move towards each other, and that with equal velocities, ſo as to meet at the middle point of their firſt diſtance if they be again ſeparated, and the loadſtone fixed, the iron being ſuſpended at the ſame diſtance from it as before, will move towards it, ſo as at length to touch it and adhere thereto. And on the other hand, if the iron be fixed, and the ſtone moveable,

the

the ſtone will approach the iron in the ſame manner, as the iron did the ſtone; all which plainly ſhews, that the attraction between the loadſtone and iron is mutual, the one drawing the other as much as it is drawn by it; ſo that the reaction of the iron upon the ſtone is exactly equal to the action of the ſtone upon the iron. LECT. IV.

The equality of action and reaction, with reſpect to attractions, is likewiſe manifeſt from hence, that if a man placed in a boat, draws another boat by means of a rope faſtened thereto; the boat wherein the man is placed will be equally drawn with the other, and the two boats will approach one another with equal quantities of motion; ſo that if they be equal in weight, and of the ſame ſize and ſhape, they will approach with equal velocities, and meet at the middle point: but if one be heavier than the other, than by how much it exceeds the other in weight, by ſo much will it be exceeded by the other in the velocity of it's motion; for inſtance, if the weight of one be to the weight of the other as one to two, then will the velocity of the former be to the velocity of the latter as two to one; that is, their velocities will be reciprocally proportional to their weights. To confirm this by an experiment, let a cord be made faſt to one end of a ſmall boat, and let it paſs over a pulley fixed to the end of another ſmall boat of the ſame ſhape and ſize, and let a weight be tied to the end of the cord, and hang in the water; this being done, let the boats be placed at ſuch a diſtance as that the cord may be ſtretched, then letting go the boats the weight will deſcend, and in deſcending draw the boat to whoſe end the cord is faſtened towards the other, and at the ſame time the other will move towards it; and when they come together, the ſpace deſcribed by the boat whoſe weight is as one, will be to the ſpace deſcribed by the boat the weight whereof is as two, as two to one; that is, if the Exp. 13.

LECT. IV. diſtance between the two boats be divided into three equal parts, that boat which is double in weight to the other, will move through one of thoſe parts in the ſame time that the lighter moves through the other two.

As action and reaction are equal with regard to attractions, ſo are they likewiſe in reſpect of ſtrokes or impulſes made by bodies one upon another; the force of two bodies, ſtriking each other equally, affecting the motions of both, and producing equal changes therein towards contrary parts. On this equality of action and reaction do the ſeveral laws which have been collected concerning the colliſion of ſolid bodies in a great meaſure depend; which laws, as they relate to bodies void of elaſticity, I ſhall now explain; in doing of which, I ſhall lay down one general PROPOSITION concerning the colliſion of ſuch bodies, whence I ſhall deduce the laws of particular caſes, and at the ſame time confirm each law by an experiment.

The PROPOSITION is as follows: *If two bodies void of elaſticity move in one right line, either the ſame or contrary ways, ſo as that one body may ſtrike directly againſt the other; let the ſum of their motions before the ſtroke when they move the ſame way, and the difference of their motions when they move contrary ways, be divided into two ſuch parts as are proportional to the quantities of matter in the bodies; and each of thoſe parts will reſpectively exhibit the motion of each body after the ſtroke.* For inſtance, if the quantities of matter in the bodies be as two and one, and their motions before the ſtroke as five and four, then the ſum of their motions is nine, and the difference is one; and therefore when they move the ſame way, the motion of that body, which is as two, will after the ſtroke be ſix, and the motion of the other three: but if they move contrary ways, the motion of the greater body after the ſtroke will

be

be two thirds of one, and of the leſſer one third of one.

For ſince the bodies are ſuppoſed to be void of elaſticity, they will not ſeparate after the ſtroke, but move together with one and the ſame velocity; and of conſequence, their motions will be proportional to their quantities of matter; and from the equality of action and reaction it follows, that no motion is either loſt or acquired by the ſtroke when the bodies move the ſame way, becauſe whatever motion one body imparts to the other, ſo much muſt it loſe of it's own; conſequently, the ſum of their motions before the ſtroke, is neither increaſed nor diminiſhed by the ſtroke, but is ſo divided between the bodies, as that they may move together with one common velocity, that is, it is divided between the bodies in proportion to their quantities of matter; but it is otherwiſe, where the bodies move contrary ways; for then the ſmaller motion will be deſtroyed by the ſtroke, as alſo an equal quantity of the greatter motion, becauſe action and reaction are equal; and the bodies after the ſtroke will move together equally ſwift, with the difference only of their motions before the ſtroke; conſequently, that difference is by means of the ſtroke divided between them in proportion to their quantities of matter.

The ſeveral particular caſes concerning the colliſion of bodies, may be reduced to four general ones. For, 1ſt, it may be that one body only is in motion at the time of the ſtroke, Or, 2dly, they may both move one and the ſame way. Or, 3dly, they may move in direct oppoſition to each other, and that with equal quantities of motion. Or, laſtly, they may be carried with unequal motions in directions contrary to each other. As the bodies may be either equal or unequal, each of theſe four general caſes may be looked upon as conſiſting of two branches; and as ſuch I ſhall conſider them,

 and

LECT. IV.

and treat of them in the order, wherein I have laid them down.

As to the firſt, if a body in motion ſtrikes another equal body at reſt, they will by the propoſition move together, each of them with one half of the motion that the body had which was in motion before the ſtroke; and ſince the quantity of motion in any body, is as the product ariſing from the multiplication of it's quantity of matter into it's velocity; the common velocity of the two bodies will be but one half of the velocity of the moving

Exp. 14.

body before the ſtroke. For the confirmation whereof, let two equal balls of clay be ſuſpended from two pins of an equal height, by threads of an equal length, and in ſuch a manner as that when they hang freely they may juſt touch one another, and that their centres and point of contact may lye in a right line parallel to the horizon. This being done, and one of the balls being at reſt, let the other be removed to any diſtance from it, and then let fall; it will in it's deſcent deſcribe the arch of a circle, and by the time it arrives at the loweſt point of the arch, that is, when it comes to touch the quieſcent ball, it will have acquired ſuch a velocity as would carry it to the ſame height from which it fell, as ſhall be ſhewn when I come to treat of pendulums; and conſequently, if the other ball was removed, would actually aſcend to that height; but upon ſtriking the other ball which is of equal ſize, it will communicate one half of it's motion to it, and they will move together with half the velocity that the moving body had at the time of the ſtroke, ſo as to aſcend to one half only of the height from which the ſtriking body fell.

That the nature of this and the other experiments relating to the colliſion of bodies, may be more readily comprehended; I ſhall lay down ſome things concerning the motion of bodies through the arches of

circles,

circles, the truth whereof ſhall be demonſtrated in my lecture upon pendulums. And Firſt, all the arches of a circle, provided they be not large, are deſcribed in equal times by bodies deſcending along them; and therefore if two bodies be let fall at the ſame time, one from C and the other from E, or from D and F, they will both arrive at the loweſt point B at one and the ſame time; and the ſtroke of the ſubſequent body upon the preceding will be made at B: and for the ſame reaſon if one be let fall from C, and the other from D or F, or one from E, and the other from D or F, they will meet and ſtrike one another at B.

LECT. IV.

Pl. 2. Fig. 7.

2dly, The velocity which a body acquires in falling through the arch of a circle, is as the chord of the arch; that is, the velocity of a body which has fallen from C to B, is to the velocity of a body that has fallen from E to B, as the chord C B to the chord E B. And here I muſt obſerve to you, that when in the following experiments I ſpeak of a body falling from, or riſing to any height, as four, ſix, or ten inches, I would be underſtood to mean it of a body's falling through or moving up an arch, whoſe chord is of ſuch a length.

3dly, The velocity wherewith a body begins to riſe up through the arch of a circle, is as the chord of the arch which the body deſcribes in it's aſcent. Thus the velocity wherewith a body begins to move from the point B towards D, if it aſcends as high as D, is as the chord B D; but if it riſes only to F, the velocity is as the chord B F. So that in the experiments the chords of the arches through which the bodies deſcend, expreſs the velocities of the bodies in the point B at the time of the ſtroke; and the chords of the arches through which the bodies aſcend after the ſtroke expreſs the velocities of the bodies immediately after the ſtroke.

Theſe things being laid down, I ſhall now proceed to determine the laws of the four general caſes.

LECT. IV.

As to the firft, it has been already fhewn, that where the moving body is equal to the quiefcent, the common velocity of the two bodies after the ftroke, is but one half of the velocity of the moving body before the ftroke; and of confequence, the motion of each body after the ftroke, is equal to one half of what the moving body had before the ftroke. But if the quiefcent body differs in fize from the moving body, then the common velocity after the ftroke will be fo much lefs than the velocity of the moving body before the ftroke, by how much the fum of the two bodies exceeds the body which was firft in motion. Thus, if the moving body be to the quiefcent as two to one, the common velocity after the ftroke will be to the velocity of the moving body before the ftroke, as

Exp. 15. two to three; wherefore if a ball of clay, falling from the height of nine inches, ftrikes another at reft, and of one half the magnitude, they will afcend together, to the height of fix inches only: and on the other hand, if the larger be quiefcent, and the fmaller falls from the height of nine inches, they will afcend to the height of three inches only; and the quantity of motion in each body immediately after the ftroke will be had, by multiplying each of them into the common velocity.

As to this and all other experiments of this nature, it muft be obferved, that they do in fome meafure vary from the theory, and that for two reafons. Firft, becaufe clay or any other body, wherewith thefe experiments can be made, is not perfectly void of elafticity. Secondly, becaufe the air refifts the motions of the balls, and by fo doing diminifhes their velocities.

As to the fecond general cafe, where both the bodies are in motion before the ftroke, and move one and the fame way. In order to find their common velocity after the ftroke, let the fum of their motions before the ftroke be divided by the fum of the bodies,

dies, and the quotient will expreſs the common velocity. Wherefore, if two equal balls of clay be let fall at the ſame time, one from the height of three inches, and the other from the height of ſix, after the ſtroke they will aſcend to the height of four inches and an half; for as in this caſe the bodies are equal, their motions are as their velocities, that is, as ſix, and three, the ſum of which being divided by two, the ſum of the bodies gives four and an half for the common velocity after the ſtroke. LECT. IV. Exp. 16.

Where the bodies are unequal, let us ſuppoſe the preceding body to be as one, and to fall from the height of three inches as before, ſo that it's quantity of motion will be as three; and let the ſubſequent body be as two, and fall from the height of ſix inches, ſo that it's quantity of motion will be twelve; and the ſum of the two motions will be fifteen, which being divided by three, the ſum of the two bodies gives five in the quotient; ſo that in this caſe, after the ſtroke the balls will aſcend to the height of five inches, and the motion of the greater will be as ten, and that of the ſmaller as five. Exp. 17.

As to the third general caſe, where the bodies move in direct oppoſition to each other, if they have equal quantities of motion, they will upon the ſtroke loſe all their motion, and continue at reſt; for by the propoſition, the bodies after the ſtroke will be carried with the difference of their motions before the ſtroke; which difference is ſuppoſed to be nothing. Wherefore, if two equal balls of clay be let fall at once from equal heights, upon the ſtroke they will ceaſe to move; and the ſame thing will happen where the balls are unequal, provided the heights from which they fall are reciprocally proportional to their quantities of matter; for inſtance, if the balls be as one and two, let the former fall from the height of ſix inches, and the latter from the height of three, and upon their meeting they will ſtand ſtill, for in this caſe, the quanti- Exp. 18. Exp. 19.

ties

LECT. IV. ties of motion, wherewith they oppofe each other, will be equal.

When two bodies meet with unequal quantities of motion, if the difference of their motions be divided by the fum of the bodies, the quotient will exprefs their common velocity after the ftroke; for by the propofition, the difference of their motions before the ftroke, is equal to the fum of their motions after the ftroke; confequently, that difference divided by the fum of the bodies muft give the velocity. Wherefore, if two equal balls of clay be let fall at the fame time, one from the height of three inches, and the other from the height of fix, after the ftroke they will afcend together to the height of an inch and an half; for fince the balls are equal, their motions will be as their velocities, that is, as fix and three, the difference whereof is three, which being divided by two, the fum of the bodies gives one and an half in the quotient. If the balls be unequal in the proportion, for inftance, of two to one; and if that which is as two falls from the height of fix inches, and the other from the height of three; after the ftroke they will afcend together to the height of three inches; for the greater ball being as two, and it's velocity as fix, it's motion is as twelve, whereas the fmaller being as one, and it's velocity as three, it's motion is likewife as three, which being fubducted from the greater motion leaves a remainder of nine; and this being divided by three, the fum of the bodies gives three for the common velocity, or the height to which the bodies will rife.

Exp. 20.

Exp. 21.

In order to difcover the quantity of motion communicated by one body to the other, I fhall lay down four rules adapted to the four general cafes. And Firft, if one of the bodies be quiefcent at the time of the ftroke, let that body be multiplied into the common velocity after the ftroke, and the product will exprefs the communicated motion. For

fince

ſince that body had no motion before the ſtroke, it is manifeſt, that whatever motion it has after the ſtroke, muſt be communicated to it by the ſtriking body; but that motion is as the product ariſing from the multiplication of the quantity of matter in the body into the common velocity, conſequently, that product expreſſes the communicated motion.

Since the body which is at reſt before the ſtroke has no motion, but what is imparted to it by the ſtriking body; and ſince the motion of the ſtriking body is by the propoſition to be divided between the two bodies in proportion to their quantities of matter; it follows, that where the ſtriking body is greater than the quieſcent, it will communicate leſs than half it's motion, and where it is equal to it, it will impart one half; and where it is leſs, more than one half: and if the quieſcent body be infinitely great with reſpect to the ſtriking body, which is in effect the caſe where the quieſcent body is fixed, ſo as not to give way to the ſtroke, the ſtriking body will impart all it's motion to the other; for as the quieſcent body is ſuppoſed to be infinitely greater than the ſtriking body, the motion, which it receives from the ſtriking body, muſt bear an infinite proportion to the motion remaining in the ſtriking body; but as the motion communicated is a finite quantity, it cannot bear an infinite proportion to the remaining motion, unleſs that remaining motion be in it's evaneſcent ſtate, and reduced to nothing.

When both the bodies are in motion before the ſtroke, and their motions are directed the ſame way, which was the ſecond general caſe; the rule for determining the quantity of motion communicated is as follows. Let the preceding body be multiplied into the common velocity after the ſtroke, and from the product let the motion which it had before the ſtroke be ſubducted, and the remainder will

LECT. IV. will be the motion communicated. For the product arising from the multiplication of the preceding body into the common velocity, gives the whole motion of that body after the ſtroke, and therefore, if from thence be taken the motion which it had before and independent of the ſtroke, the remainder muſt be the motion acquired by the ſtroke.

When the bodies move towards one another with equal quantities of motion, as in the third general caſe; the motion communicated is equal to the motion of either before the ſtroke. For as in this caſe, both their motions are deſtroyed by the ſtroke; it is plain, that which ever of the bodies is conſidered as giving the ſtroke (and either of them may) it muſt communicate juſt as much motion to the other, as the other has at the time of the ſtroke; for by this means the motion communicated, as it is directly oppoſed to the former motion of the body, will be juſt ſufficient to deſtroy the ſame, and by ſo doing cauſe the body to reſt.

When the quantities of motion in two bodies moving directly towards each other are unequal, which is the fourth general caſe; the motion communicated is determined by the following rule. Let the body which had the leſſer motion before the ſtroke be multiplied into the common velocity after the ſtroke, and to the product let the motion which it had before the ſtroke, be added, and the ſum will be the motion communicated. For as the body, to which the motion is communicated, does after the ſtroke move in a direction contrary to what it did before, it is evident, that beſides the motion, wherewith it is carried in that contrary direction, it muſt have received as much more in the ſame direction, as was ſufficient to withſtand the motion it had before the ſtroke in an oppoſite direction; for till that motion was deſtroyed by an equal motion oppoſed thereto, the body could not change it's direction, and move backward.

LECTURE V.

Of the Collision of Elastick Bodies.

HAVING given you an account of the collision of bodies void of elasticity, I come now to consider the effects thereof in such as are elastick; by which I mean bodies that consist of such parts as yield and give way when pressed, and which restore themselves upon the removal of the pressure: if the force wherewith they restore themselves be exactly equal to the pressure whereby they are bent inward, then are the bodies said to be perfectly elastick; and such are all those bodies supposed to be, wherewith experiments are usually made for confirming the theory relating to the collision of elastick bodies; but as there is not perhaps in nature any body perfectly elastick, if among the experiments that are now to be made, any shall be found to vary a little from the theory, such variation must be looked upon as arising rather from the want of perfect elasticity in the bodies, than from any error in the theory it self, or in the calculations grounded thereon. LECT. II.

The method which I shall observe in treating of the percussion of elastick bodies is this; First, to lay down one general proposition concerning such percussion, and then, Secondly, to deduce the laws relating to the four general cases mentioned in my last lecture, and to confirm each of those laws by experiments.

Before I lay down the proposition, I must observe to you, that wherever I mention the striking body I thereby mean that body which is in motion where one of the two is quiescent, as also that body which moves swiftest when they both move the same way, and lastly, that body which has the greatest quanti-

LECT. V. ty of motion, when they move in oppoſition to one another, or in this caſe, if their motions be equal, then either of them may be taken indifferently for the ſtriking body.

This being premiſed, the PROPOSITION is as follows.

If of two bodies perfectly elaſtick, one be at reſt, and the other in motion; or if they both move either the ſame or contrary ways, ſo as that one ſhall ſtrike the other; let them be conſidered as void of elaſticity, and by the propoſition laid down in my laſt lecture, let the motion of each body after the ſtroke be found, and by one of the four rules laid down in the ſame lecture, let the motion communicated by the ſtriking body to the other be likewiſe found, and let this motion be ſubducted from the motion of the ſtriking body after the ſtroke, and added to that of the body which received the ſtroke, and the reſidue will be the motion of the ſtriking body, and the ſum the motion of the other body after reflection. For, ſince the bodies are ſuppoſed to be perfectly elaſtick, their parts which are bent in by the ſtroke will reſtore themſelves with a force equal to that which bends them in, but the force which bends them in, is meaſured by the quantity of motion communicated by the ſtriking body to the other, and therefore the parts of each body which are bent inward will reſtore themſelves with ſuch a force, as is ſufficient to generate a motion equal to that which is communicated, conſequently, the bodies will by virtue of their elaſticity throw one another contrary ways, each with a quantity of motion equal to that which the ſtriking body communicates to the other; for which reaſon, if that motion be ſubducted from the motion remaining in the ſtriking body after the ſtroke, as being contrary thereto, and added to the motion of the other body after the ſtroke, as conſpiring therewith, the reſidue and ſum will give the true motions of the bodies after reflection.

To

To apply what has been ſaid to the four general caſes, the firſt whereof is where one of the bodies is at reſt at the time of the ſtroke. If a body perfectly elaſtick ſtrikes another of the ſame kind and of equal magnitude at reſt, the ſtriking body will communicate all it's motion to the other and remain at reſt; for by the firſt of the four rules laid down in my laſt lecture, the ſtriking body will, upon the ſtroke, communicate half it's motion, and by the propoſition now laid down, a quantity of motion equal to that which is communicated, muſt be ſubducted from the motion remaining in the ſtriking body, and be added to the motion of the body which receives the ſtroke, by which means the ſtriking body will have no motion left; but the other body will have a quantity of motion equal to what the ſtriking body had before the ſhock. For the confirmation of which, let two equal ivory balls be ſuſpended as were thoſe of clay; and let one of the balls fall from any height, and ſo as to ſtrike the other at reſt, the ball which receives the ſtroke will aſcend to the ſame height from which the other fell, and will leave the other at reſt. LECT. IV. Exp. 1.

If inſtead of one there be two, three, or more quieſcent balls contiguous to one another, that which is fartheſt removed from the ſtriking ball will fly off with the velocity of the ſtriking ball, and all the intermediate balls together with the ſtriking ball will quieſce; for as the ſtriking ball imparts all it's motion to the firſt of the quieſcent balls, ſo does that in like manner to the ball which lies next beyond it, and that again to a third, and ſo on; till at length the laſt ball meeting with none other to reſiſt it, flies off with all the motion of the ſtriking ball, leaving that and the intermediate ones at reſt. Exp. 3.

If two balls be let fall together contiguous to one another, upon the ſtroke the two fartheſt will fly off, leaving the others at reſt; for as the foremoſt Exp. 3.

LECT. V. of the two moving balls is carried equally ſwift with the ſubſequent, it cannot during it's motion receive any impreſſion from the ſubſequent ball; conſequently, when it makes the ſtroke, it will produce the ſame effect in the quieſcent balls as if the ſubſequent ball was away; that is, it will by means of the intermediate balls communicate all it's motion to the laſt, and make that fly off; but no ſooner has it made the ſtroke, and thereby parted with it's own motion, but the ſubſequent ball impells it, and imparts to it all it's motion, and this motion being propagated through the ſeveral intermediate balls as before, makes the laſt but one to fly off, and that in ſuch a manner as to keep pace with, and cloſely purſue the other; becauſe in the ſame inſtant of time that the foremoſt of the two moving balls makes it's ſtroke, it likewiſe receives the ſtroke from the hindmoſt ball, and of conſequence, the flying off of the two laſt balls, which is the effect of the double ſtroke, muſt happen at one and the ſame time.

Exp. 4. For the ſame reaſon that two balls fly off where the number of ſtriking balls is two, three will fly off when there are three ſtriking balls, and four, where there are four, and ſo on whatever be the number of ſtriking balls, an equal number will conſtantly go off.

Exp. 5. If two elaſtick balls be unequal, for inſtance, if one be double the other, and if the greater be let fall from the height of nine inches, and ſtrike the ſmaller at reſt; they will both move forward after the ſtroke, the ſtriking body with one third of the motion which it had before the ſtroke, and the other with two thirds; and the ſtriking body will aſcend to the height of three inches, and the other to the height of twelve. For ſince the ſtriking body is to the quieſcent as two to one, it will by the firſt of the four rules laid down in my laſt lecture, communicate one third of it's motion to it, and on account of

of the elaſticity a quantity of motion equal to what is communicated, muſt be taken from the motion remaining in the ſtriking body, and added to the motion of the other; conſequently, the ſtriking body will retain one third only of it's motion, the other two thirds being communicated to the body which receives the ſtroke; wherefore ſince the ſtriking body is as two, and the height from which it falls as nine, it's motion muſt be as eighteen, one third of which, to wit, ſix, it will retain after the reflection; and the other two thirds, to wit, twelve, will be the motion of the other body, and theſe motions being divided by the bodies, will give three and twelve for the quotients; which quotients are as the velocities of the bodies after reflection, or as the heights to which they aſcend. LECT. V.

Exp. 6. On the other hand, if the larger ball be quieſcent, and the ſmaller be let fall from the height of nine inches, it's motion will be as nine, whereof two thirds will by the firſt of the four rules be communicated by the ſtroke to the greater, and one third only will remain in the ſtriking ball, from which on account of the elaſticity muſt be taken as much as was communicated to the larger ball, that is, two thirds, but upon ſubducting two thirds from one third, there will remain one third negative; which ſhews, that the ſtriking ball will be reflected with one third of the motion it had at the time of the ſtroke, ſo as to aſcend backward to the height of three inches; and the quieſcent ball to which two thirds of the ſtriking ball's motion was communicated by the ſtroke, will likewiſe on account of the elaſticity receive two thirds more, ſo as to be carried forward with a motion equal to what the ſtriking ball had at the time of the ſtroke, and one third more; that is to ſay, with a motion which is as twelve, which being divided by two, the quantity of matter in the ball gives ſix for the velocity, or the height to which that ball muſt aſcend.

LECT. V. From what has been ſaid it follows, that when the quieſcent ball is ſmaller than the ſtriking ball, there can be no reflection, becauſe in that caſe the ſtriking ball will, by virtue of the ſtroke, communicate leſs than half it's motion, and the motion which is to be taken from the ſtriking ball, on account of the elaſticity, being equal to the motion communicated, will upon the ſubduction always leave ſome motion in the ſtriking ball to carry it forward, conſequently, it cannot be reflected. Where the two balls are equal there will be likewiſe no reflection, but the ball which was quieſcent will go forward with all the motion of the ſtriking ball, and the ſtriking ball will become quieſcent; as is evident from what was ſaid concerning that caſe. But where the ſtriking ball is leſs than the quieſcent, it will be reflected, and there will likewiſe be an augmentation of motion in the greater ball; for the ſmaller ball muſt upon the ſtroke communicate more than half it's motion to the greater ball, and there muſt likewiſe, on account of the elaſticity, as much motion be ſubducted from the ſmaller ball,and added to the larger, as is communicated; wherefore, ſince two equal quantities of motion, each of which exceeds half of the ſmaller ball's motion, are to be ſubducted from the ſmaller ball, and given to the larger; it is plain, that the ſmaller muſt loſe all it's motion and ſomething more, that is, it muſt be carried backward or reflected; and the greater ball muſt go forward with more motion than was in the ſmaller at the time of the ſtroke, that is, there will be an augmentation of motion; and the exceſs of motion in the greater ball, above the motion which the ſmaller ball has at the time of the ſtroke, is ever equal to the motion wherewith the ſmaller ball is reflected after the ſtroke, as is evident from what has been ſaid. If therefore motion be communicated from a ſmaller elaſtick body to a larger, by means of ſeveral intermediate bodies each

larger than the other, the motion will be augmented in each of them, and the motion of the laſt will greatly exceed that of the firſt; and this augmentation of motion is greateſt when the bodies are in a geometrical progreſſion; for inſtance, if there be two bodies which are as one and four, and if the ſmaller communicates motion to the larger by means of one intermediate body, the motion will be greater in the larger body, if the middle body be as two, that is, a geometrical mean between the two; than if it be as one and an half, or two and an half, or three, or in ſhort in any other proportion whatever but that of the geometrical mean. For the proof of which, let the leſſer body be expreſſed by unity, and the larger by the ſquare of a, and the geometrical mean will be expreſſed by a; ſo that the three bodies taken in their order from the leaſt, will be expreſſed by the ſymbols in the firſt ſtep, and the motion produced in the ſecond body by the ſtroke of the firſt, will be expreſſed by the ſecond ſtep; and the motion produced in the third by the ſtroke of the ſecond, will be expreſſed by the third ſtep. Again, let another body greater or leſs than a be ſubſtituted in the room thereof, and let the difference between that body and a be called x, in this caſe, the bodies will be expreſſed by the ſymbols in the fourth ſtep, and the motion produced in the ſecond by the ſtroke of the firſt, will be expreſſed by the fifth ſtep; and the motion produced in the third by the ſtroke of the ſecond, will be expreſſed by the ſixth ſtep; but this fraction of the ſixth ſtep is leſs than that of the third ſtep; for if from the product ariſing from the multiplication of the denominator of this fraction into the numerator of that, be ſubſtracted, the product which ariſes from the multiplication of the numerator of this into the denominator of that; that is, if from the ſeventh

LECT. V.

LECT. V. step the eighth be subducted, there will remain the quantity which is expressed in the ninth step.

Whence it appears, that the former product is greater than the latter; and therefore, by the 2d COROL. of the 19th PROP. of the 7th book of the elements, the numerator of the former fraction bears a greater proportion to it's denominator, than that of the latter fraction does to it's denominator, that is, the fraction in the third step which expresses the motion of the greatest body when the intermediate one is a geometrical mean, is greater than the fraction in the sixth step, which expresses the motion of the greatest body when the middle body is not a geometrical mean, consequently, the motion is more augmented when the intermediate body is a geometrical mean, than when it is greater or less in any proportion.

1st. $1, a, a^2$

2d. $\dfrac{2a}{1+a}$

3d. $\dfrac{a^3}{a^3+2a^2+a} \times 4$

4th. $1, a \pm x, a^2$

5th. $\dfrac{2 \times a \pm x}{1 + a \pm x}$

6th. $\dfrac{a^3 \pm a^2x}{a^3+2a^2+a\_a^2x \pm 2ax \pm x +{}^2} \times 4$

7th. $a^6+2a^5+a^4 \pm a^5x + 2a^4x \pm a^3x + a^3x^2$

8th. $a^6+2a^5+a^4 \pm a^5x \pm 2a^4x \pm a^3x$

9th. ——————————— a^3x^2

To give you an instance, how prodigiously motion may be augmented by being successively communicated to several bodies in a geometrical progression

greſſion; if twenty elaſtick bodies be placed one after another, each ſucceeding body exceeding the foregoing in the proportion of twenty to one; and if motion be communicated through the ſeveral intermediate bodies from the firſt to the laſt, it will be ſo far augmented, as to be two hundred thouſand times greater in the laſt body than in the firſt; ſo that if we ſuppoſe the firſt to be a cannon ball, moving with the ſame velocity wherewith it flies from the mouth of a cannon, which from the obſervations of Mr. DERHAM, I ſhall ſuppoſe to be at the rate of 612 feet in a ſecond, though there are pieces of cannon which diſcharge their balls with double that velocity, the motion of the laſt body will be ſo great as if applied to the ball would carry it at the rate of above twenty three thouſand miles in one ſecond of time, which velocity is five thouſand times as great as the velocity of a body revolving about the earth, by the force of gravity at a ſmall diſtance from its ſurface; for a body ſo revolving will not come round in leſs than an hour and twenty four minutes.

From the increaſe of motion in elaſtick bodies, a reaſon may be drawn for the augmentation of ſound in ſpeaking trumpets; for as the ſpeaking trumpet is narroweſt at the mouth-piece, and thence widens and enlarges continually to the extremity, the air within it, which is an elaſtick fluid, as ſhall be ſhewn hereafter, may be conſidered as divided into a great number of cylindrical bodies, of very ſmall but equal altitudes, the baſis of the firſt being equal to the mouth of the trumpet, and the baſes of the reſt increaſing one above another as they are more and more removed from the mouth; upon which account the motion that is impreſſed by the force of the voice on the firſt cylindrical body of air, grows larger in the ſecond, and larger ſtill in the third, and ſo on, till at length at the exit of the tube it becomes ſo large as to magnify the ſound to a great

 degree;

LECT. V. degree; and of the feveral kinds of trumpets, thofe magnify the found moft, that are of fuch a figure as arifes from the revolution of the logarithmick curve about it's axis; that is, let A G be the logarithmick curve, and H O it's axis, the figure arifing from the revolution of A G about H O, is fuch as a fpeaking trumpet ought to have in order to give it the greateft advantage poffible. For from the nature of the curve, if H I, I K, K L, L M, and fo on, be taken equal; the ordinates H A, I B, K C, L D, and fo on, are in geometrical proportion; wherefore if H I, I K, and fo on, be taken very fmall, they will reprefent the equal altitudes of the cylindrical bodies of air in the trumpet; and the ordinates H A, I B, and fo on, will be the radii of their bafes, and the bodies of air being of equal heights will be to one another as their bafes, that is, as the fquares of their radii; but the radii being to one another in a geometrical proportion, their fquares will be fo too; confequently, the little cylindrical bodies of air will be in a geometrical progreffion, the fmalleft whereof lies next the mouth, and the largeft at the exit of the tube; for which reafon the augmentation of found will be greater, *cæteris paribus*, in a trumpet of fuch a form than of any other form whatever.

Pl. 2. Fig. 8.

But to proceed to the fecond general cafe, wherein both the bodies move one and the fame way, but the fubfequent more fwiftly than the preceding.

Exp. 7. If two equal elaftick bodies move in the fame direction, and in fuch a manner as that one may overtake and ftrike the other, upon the ftroke they will change their quantities of motion with each other; for inftance, if the motion of the fubfequent body before the ftroke be double the motion of the preceding body, then will the preceding body after the ftroke, have double the motion of the fubfequent body after the ftroke; and the preceding body after the ftroke, will move with the fame velocity

where-

wherewith the ſubſequent body moved before the ſtroke; and the ſubſequent body will after the ſtroke be carried with the velocity of the preceding body before the ſtroke; ſo that upon the ſtroke the bodies will change their motions and velocities. For ſince by ſuppoſition the ſum of the motions is three, and ſince the bodies are equal, the motion of each after the ſtroke, ſetting aſide the elaſticity, muſt be one and an half; and by the ſecond rule for determining the quantity of motion communicated by the ſtriking body to the other; the motion communicated in this caſe will be as one half, and ſo likewiſe will the motion ariſing from the elaſticity, which being deducted from the motion which remains in the ſtriking body after the ſtroke, and added to that of the preceding body, leaves the motion of the former as one, and of the latter as two; ſo that upon the ſtroke the motions will be changed. Wherefore if two ivory balls of an equal ſize be let fall at the ſame time, one from the height of ſix inches,and the other from the height of three, after the ſtroke, the preceding ball will riſe to the height of ſix inches, and the ſubſequent to the height of three only.

If the bodies be unequal and move the ſame way, their motions and velocities after the ſtroke may in like manner be diſcovered by the help of the propoſition. For inſtance, if the ſubſequent body be as two, and have twelve parts of motion, and the preceding body as one, and it's motion as three; the motion of the ſubſequent body after the ſtroke will be as eight, and that of the preceding body as ſeven, and the velocity of the former will be as four, and that of the latter as ſeven; for the ſum of the two motions before the ſtroke being fifteen, and the bodies being as one and two, the motion of the leſſer body after the ſtroke, ſetting aſide the elaſticity, will be as five, and that of the greater as ten; but the motion of the leſſer body before the ſtroke

LECT. V. was as three, consequently, the communicated motion is as two; wherefore adding so much on account of the elasticity to the motion of the lesser body, and subducting as much from that of the greater body which in this case is the striking body, we shall have eight for the motion of the greater, which being divided by two, the quantity of matter in the greater, gives four for it's velocity; and we shall have seven for the motion of the lesser body, which because the quantity of matter in the lesser is as one, will likewise express the velocity. Wherefore if two

Exp. 8. ivory balls one double of the other be let fall at the same time, the larger from the height of six inches, and the smaller from the height of three, after the stroke the lesser will ascend to the height of seven inches, and the greater to the height of four.

Exp. 9. On the other hand, if the smaller ball be let fall from the height of six inches, and the greater from the height of three; after the stroke the lesser will ascend to the height of two inches, and the greater to the height of five, and the motion of the former will be two, and that of the latter ten; for since the smaller ball is as unity, and falls from the height of six inches, it's motion at the time of the stroke is six; and since the larger ball is as two, and falls from the height of three inches, the motion thereof at the time of the stroke is likewise six; and the sum of those two motions which is twelve, being divided between the bodies in proportion to their quantities of matter gives eight for the motion of the greater, and four for the motion of the lesser, which motions they would have after the stroke, supposing they were not elastick; and since the motion of the greater body before the stroke was six, the motion communicated to it by the stroke is two, which by reason of the elasticity being subducted from four, the motion of the striking body, and added to eight, the motion of the other body, gives two and ten for the motions of

of the two bodies, which motions being divided by the respective bodies, give two and five for the velocities.

If two equal bodies meet one another with equal quantities of motion, which is one branch of the third general caſe, they will rebound with the ſame motions and ſame velocities wherewith they approached; for were they void of elaſticity they would upon the ſtroke ſtand ſtill, becauſe they communicate to one another a quantity of motion equal to that which each of them has at the time of the ſtroke, and that in a contrary direction; but by the propoſition, each of them muſt on account of the elaſticity receive as much motion as was communicated by the ſtroke; and the motions which are thus received by the bodies being equal, and contrary to the motions wherewith the bodies met, and which were deſtroyed by the ſtroke, muſt carry the bodies backward with the ſame velocities wherewith they approached. Wherefore, if two equal ivory balls be let fall at the ſame time from equal heights, ſo as to meet one another, upon the ſtroke they will be reflected back to the heights from which they fell.

If the balls be unequal, for inſtance, if one be double the other; let the larger fall from one half only of the height from which the ſmaller deſcends, by which means when they meet their motions will be equal, and upon the ſtroke they will be reflected each to the height from which it fell.

Where the bodies meet one another with unequal motions, which is the fourth general caſe, if the bodies be equal, they will both be reflected, and each of them will recede with the motion and velocity wherewith the other approached; that is, they will change their motions and velocities; for let us ſuppoſe the motions of the two bodies to be as ſix and three; if they were void of elaſticity the body which has the ſmalleſt quantity of motion would upon the ſtroke be turned back, and the two

LECT. V.

bodies would be carried with the difference of their motions divided equally between them, that is, the motion of each would be as one and an half, and the motion communicated would by the fourth rule be as four and an half; but a quantity of motion equal to what is communicated, muſt be ſubducted from the motion remaining in the ſtriking body, and added to the motion of the other, that is, four and an half muſt be ſubducted from one and an half, and likewiſe added thereto; whereby there will be three negative for the motion of the ſtriking body, which ſhews that it will be carried back with a motion which is as three; and there will be ſix poſitive for the motion of the other body, which ſhews that it will be carried with a motion which is as ſix, in the direction of the ſtriking body before the ſtroke; that is, it will be reflected; ſo that each of them will be carried back with the

Exp. 12. motion wherewith the other approached. Wherefore, if two equal balls of ivory be let fall at the ſame time, one from the height of ſix inches, and the other from the height of three; upon the ſtroke they will return back, but that which fell from the height of ſix inches will riſe only to the height of three, whereas that which fell from three inches will riſe to ſix.

If the balls be unequal, and meet one another with unequal quantities of motion, their motions after the ſtroke may in like manner be determined by the help of the rule laid down in the propoſition;

Exp. 13. for inſtance, if two ivory balls which are as one and two be let fall at the ſame time, the greater from the height of ſix inches, and the ſmaller from the height of three; in this particular caſe, the greater ball will upon the ſtroke loſe all it's motion, and the ſmaller will be reflected with the difference of their motions, ſo as to riſe to the height of nine inches; for ſince the larger ball, which deſcends from the height of ſix inches, is as two, it's motion is as

twelve,

twelve, whilst the motion of the smaller ball, which is as unity, and descends only from the height of three inches, is as three, the difference of which motions is nine; and this being divided between the bodies in proportion to their quantities of matter, gives six for the motion of the larger, and three for that of the smaller; and with these motions the bodies would be carried after the stroke, supposing they were void of elasticity; but because of the elasticity, a quantity of motion equal to what is communicated by the striking body to the other, which in this case is six, must be taken from the motion of the greater body, and added to that of the smaller, which two motions being six and three, the remainder after subduction, which expresses the motion of the greater body, will be nothing; and the sum arising from the addition, which expresses the motion of the smaller ball, will be nine.

LECTURE VI.

OF THE CENTER OF GRAVITY, BALANCE, AND LEVER.

MY design in this lecture is to give you an account of the first and second of the mechanic powers, commonly called the balance and the lever; but I shall first take notice of some things relating to heavy bodies, the knowledge of which is in a great measure necessary to the right understanding of what shall be said concerning the mechanic powers in general. And first, in every body there is a certain point, commonly called by the writers of mechanics, the center of gravity; the nature of which will best appear from it's chief properties, which are these. LECT. VI.

1st, If a body be suspended by it's center of gravity, it will continue in any position whatever wherein

LECT. VI.

Exp. 1.

wherein it is placed; whereas if it be suspended by any other point, it will not rest in any other position but where the center of gravity is either directly above, or directly beneath the point of suspension; thus, if two beams be supported, the one by an axle passing through it's center of gravity; the other by an axle which doth not pass through the center of gravity, but through such a point, as when the beam is parallel to the plane of the horizon, lies directly above the center of gravity; the former will rest in any position, whether it be perpendicular, parallel, or inclined to the horizontal plane; but the latter will rest in the parallel position only; and should it by any force be removed from that position, it will, upon the removal of the force, begin to move in order to recover the parallel position, and after several vibrations will at length settle therein.

A second property of the center of gravity is, that where that is supported the whole body is likewise sustained; for which reason the whole weight of a body may be looked upon as applied to that single point and as centered therein.

A third property of this center is, that it continually endeavours to move downward towards the center of the earth, and where all lets and impediments are removed does actually descend; and therefore if in any case a body seems to move upward by the force of gravity, it will be found that the center of gravity descends notwithstanding any appearance to the contrary. Thus, if two rulers be so placed as to meet in an angle at one of their ends, and there to rest upon an horizontal plane, whilst at their other end they are raised a little above the plane; and if a body consisting of two equal similar cones united at their bases, be laid upon the rulers in such a manner, that the edge of their bases may lie between the rulers, it will when left to it self begin to roll towards the elevated extremities of the rulers, and upon that account appear to ascend, whereas in

Exp. 2.

reality

reality it moves downward; for if a ſtring be ſtretched horizontally beneath the rulers, ſo as that it may touch the edge of the baſes of the cones at the concourſe of the rulers, it will be found that the edge of the baſes deſcends below the ſtring, and that more and more as the body moves nearer to the higher end of the rulers. LECT. VI.

Whilſt the body rolls upon the rulers, the parts of the cones which reſt thereon do, by reaſon of the widening of the rulers, grow continually ſmaller; upon which account, at the ſame time that the body aſcends along the plane of the rulers, it is as it were carried down another plane equal in length to the ſide of the cone, and whoſe perpendicular altitude is equal to the ſemidiameter of the baſes of the cones; and therefore, if the perpendicular altitude of the rulers in that part where their diſtance is equal to the length of the double cone, be leſs than the ſemidiameter of the baſes, the body will move up along the rulers, becauſe, by ſo doing, it will in reality deſcend, and the deſcent thereof will be equal to the difference between the ſemidiameter of their baſes, and the perpendicular altitude of the rulers in that part where their diſtance is equal to the length of the cones; but if that perpendicular altitude be equal to the ſemidiameter, the body will reſt on any part of the rulers, being carried as much upward on one account, as it is downward on the other; and if the altitude of the rulers be a little increaſed, ſo as to exceed the ſemidiameter of the baſes of the cones, the body will roll down the rulers, and thereby deſcend through a ſpace equal to that exceſs.

If a cylinder be ſo contrived as to have it's center of gravity near one of it's ſides, which may be done by making a wooden cylinder hollow towards one ſide, and then filling it with lead; when it is placed on an inclined plane in ſuch a manner as that the ſide Exp. 3.

LECT. VI. ſide which is neareſt to the center of gravity may lean towards the upper part of the plane, it will aſcend provided the inclination of the plane be not too ſmall, but the center of gravity will at the ſame time deſcend; for it will ſuitably to it's nature endeavour to move downward, and thereby cauſe the cylinder to revolve about it's axis; and this revolution will make the cylinder, and conſequently it's center of gravity, to move up the plane; ſo that the center of gravity will have as it were two motions, one upward, ariſing from the progreſſion of the cylinder along the plane, the other downward, occaſioned by the rotation of the cylinder about it's axis; but the deſcent occaſioned by the latter motion, will be greater than the aſcent ariſing from the former; as will appear by ſtretching a line horizontally at the ſame height with the center of gravity before the cylinder begins to rowl, for after the rotation ceaſes the centre of gravity will be beneath the line; ſo that upon the whole, that center will be found to deſcend notwithſtanding the aſcent of the cylinder on the plane.

When the elevation of the plane becomes ſo great that the aſcent ariſing from the progreſſion becomes equal to, or greater than the deſcent ariſing from the rotation, the cylinder will, in the former caſe, continue at reſt, and in the latter rowl down the plane.

A line drawn from the center of gravity of any body, perpendicular to the plane of the horizon, is called the line of direction of the center of gravity, becauſe when the body is carried downward by the force of gravity, if it meets with no let or obſtacle, it's center of gravity will deſcribe that line. The chief property of this line is, that as long as it falls within the baſe of the body, ſo long the body ſtands, whereas no ſooner does it fall beyond the baſe, but the body tumbles; as will appear from the

the following experiment; let a piece of wood be set on a moveable plane with a plummet hanging from it's center of gravity, and let the plane be gradually elevated till at length the plum-line, (which, as it is always perpendicular to the horizon, will represent the line of direction,) falls beyond the base; the wood will not tumble as long as the plummet line falls within the base, whatever be the elevation of the plane whereon it stands, but the moment that line gets beyond the base the body falls. LECT. VI. Exp. 4.

The reason why a body stands during the continuance of the line of direction within it's base is, that no motion can arise in any body from the force of gravity, unless the center of gravity can by such motion be carried downward; but as long as the line of direction of any body falls within the base, it's center of gravity is supported, and therefore cannot descend; and consequently, the body will remain unmoved; whereas upon the removal of the line of direction beyond the base, the center of gravity ceases to be supported, and is therefore at liberty to descend.

From what has been said it appears, why, among bodies descending on inclined planes, some, for instance cubes, only slide, whilst others, as globes or cylinders, rowl; the lines of direction falling beneath the bases of the former, but not the latter.

The center of motion in any body is a fixed point or axis about which the several parts of a body do move, and in moving describe circular arches.

The direction of any power or weight is, that strait line wherein it moves or endeavours to move. And the moment of any power or weight is, that force wherewith it either moves or endeavours to move, and it is always proportional to the product arising from the multiplication of the power or weight into the velocity wherewith it moves or would move if it were not hindred by some opposite

LECT. posite power or weight; and therefore, if the product arising from the multiplication of one weight or power into it's velocity, be equal to the product arising from the like multiplication of any other weight or power into it's velocity, the moments of those two weights or powers must be equal; and this will always be where the weights or powers are to one another reciprocally as their velocities; consequently, two weights or powers may balance, if as much as one exceeds the other in magnitude so much must it be exceeded by the other in velocity; and herein consists the whole force and efficacy of all mechanical engines; for they are so contrived as to diminish the velocity of one weight or power and to increase that of the other, by which means a very small weight or power may become a balance to one exceedingly great, as will appear from what shall be said concerning the mechanick powers, which are commonly reduced to six, namely, the *balance*, the *lever*, the *pulley*, the *axle in the wheel*, the *wedge*, and the *screw*, of each of which in their order.

The BALANCE, strictly speaking, is a beam supported by an axle whereon it turns; which axle therefore is the center of motion; the parts of the beam which lie on each side of the axle are called it's arms, and those parts of the arms to which the weights are applied are called the points of suspension; concerning which it must be observed, that the appending weight, whatever be the length of the cord by which it hangs, acts with the same force and in the same manner as if it's center of gravity was applied to the point of suspension; so that it matters not what the distance is between the weight and point of suspension, as will appear from the following ex-

Exp. 5. periment. Let a weight appended at one arm of a balance be counterpoised by a weight at the other, and let it by means of a cord be hung at different distances below the point of suspension; the position of

of the balance will remain unvaried, and the weights will continue to counterpoiſe each other at all thoſe diſtances. LECT. VI.

The moment of any weight appended at the arm of a balance, is proportional to the product ariſing from the multiplication of the weight into the diſtance of the point of ſuſpenſion from the axis of the balance; for, as was before ſaid, the moment of a weight is proportional to the product of the weight into it's velocity, and in this caſe the velocity of the weight is as the diſtance of the point of ſuſpenſion from the axis; for ſince the weight acts in the ſame manner as if it's center of gravity was applied to the point of ſuſpenſion, whatever be the velocity wherewith that point moves round the axis, the ſame will the velocity of the weight be; but the velocities wherewith the ſeveral points in the arm of a balance move round the axis, are as the ſpaces, that is, as the circular arches, which they deſcribe in the ſame time, which arches from the nature of the circle are to one another as their reſpective radii, that is, as the diſtances of the points from the axis. Thus, if A B repreſents the arm of the balance moving round the axis at A, the velocities of the points B and D, which deſcribe the arches B C and D E, will be as thoſe arches, becauſe they are deſcribed in the ſame time; but from the nature of the circle, thoſe arches are to one another as their radii AB, and A D, that is, as the diſtances of thoſe points from the axis; conſequently, the moment of a weight appended at the arm of a balance, is as the product of the weight into the diſtance of the point of ſuſpenſion from the axis. Whence it follows, that if two weights be appended at the arms of a balance in ſuch a manner, as that the diſtances of the points of ſuſpenſion from the axis ſhall be reciprocally proportional to the weights, thoſe weights will counterpoiſe each other, and the balance will be in *æquilibrio*; for inſtance, if two equal weights Pl. 2. Fig. 9. Exp. 6.

 be

LECT. VI. be applied at equal diſtances from the axis, the balance will not incline to either ſide, but remain parallel to the horizon, the weights in this caſe counterpoiſing one another. Again, if one weight be larger than the other in any proportion, for inſtance, in the proportion of three to one, if the point at which the ſmaller is applied be thrice as far diſtant from the axis as the point at which the larger is applied, the balance will be in *æquilibrio*.

Exp. 7.

On this *æquilibrium* ariſing from the ſuſpenſion of weights at diſtances reciprocally proportional to the weights, is founded the *Statera Romana*, otherwiſe called the ſteel-yard, which conſiſts of two arms very unequal in length, but equally poiſed by means of a weight annexed to the ſhorter, from which likewiſe hangs a ſcale in order to receive ſuch things as are to be weighed; the longer arm is divided into a number of equal parts beginning from the axis, and ſuſtains a weight which ſlides from one end to the other; which weight being aplied to the ſecond diviſion, will counterpoiſe double the weight in the ſcale of the ſhorter arm, that it will when applied to the firſt diviſion; and triple when applied to the third diviſion; and ſo on, whatever be the diviſion to which it is applied, the weight in the ſcale of the ſhorter arm muſt be proportional thereto; otherwiſe the products ariſing from the multiplication of the weights into their reſpective diſtances from the axis would not be equal, and conſequently would not balance each other.

Exp. 8.

Exp. 9.

On the ſame *æquilibrium* is likewiſe founded the deceitful balance, which is ſo contrived as though one arm be longer than the other, yet is the ſhorter made ſo much thicker than the longer, as thereby exactly to poiſe the ſame; upon which account the balance appears to be juſt, and conſequently ſuch weights as counterpoiſe are judged equal, whereas in truth that which is appended at the longeſt arm is leſs than the other, and that in the proportion of the

the length of the ſhorter arm to that of the longer; for inſtance, if the longer arm be to the ſhorter as ten to nine; a weight of nine ounces applied at the longer arm, will counterbalance ten ounces appended at the ſhorter. LECT. VI.

Several weights appended at ſeveral diſtances from the axis in one ſide of a balance, will counterpoiſe ſeveral others appended likewiſe at ſeveral diſtances on the other ſide; provided the ſum of the products which ariſe from the multiplication of the weights on one ſide into their reſpective diſtances from the axis, be equal to the ſum of the products ariſing from the like multiplication of the weights on the other ſide into their reſpective diſtances. Thus, if on one ſide a weight of one ounce be appended at the diſtance of two inches from the axis, and another of two ounces at the diſtance of three inches, and a third of three ounces at the diſtance of four inches; and if on the other ſide be appended one weight of five ounces at the diſtance of an inch from the axis, and another of three ounces at the diſtance of five inches; the two latter will balance the three former; for the product of five into one, being added to the product of three into five, gives the ſum of twenty; as does likewiſe the addition of the three products of one into two, two into three, and three into four. Exp. 10.

The chief uſe of the balance commonly called a pair of ſcales, is to compare the weights of different bodies together; and that this machine may be as exact and perfect as poſſible, it is requiſite, 1ſt, that the center of gravity of the beam be placed a little below the axis, becauſe in this caſe, when there is an *æquilibrium*, the beam will not reſt in any poſition but the parallel; conſequently, the weights which are compared together will appear to be equal, as they really are; whereas if the axis be placed beneath the center of gravity, ſhould the center of gravity be moved out of the perpendicu-

 lar

LECT. VI. lar line, which can ſcarcely be avoided, it will not return, but from it's tendency downward will be carried lower, ſo as to give the beam an inclined poſition; for which reaſon the weights will appear to be unequal, though in reality they are not ſo; and the ſame inconvenience will ariſe if the axis paſſes through the center of gravity, for in that caſe it has been already ſhewn, that the beam, notwithſtanding the *æquilibrium*, will reſt in any poſition.

Secondly, the arms of the beam ought to be exactly equal both as to weight and length, the reaſon of which is evident, from what was ſaid concerning the deceitful balance.

Thirdly, the points from which the ſcales are ſuſpended, ought to be in one right line paſſing through the beam's center of gravity; for by this contrivance the weights will act directly againſt each other, ſo that no part of either will be loſt on account of any oblique direction.

Fourthly, the friction of the beam againſt the axis ought to be as little as poſſible; becauſe, ſhould the friction be great, it will require a conſiderable force to overcome it; upon which account, though one weight ſhould a little exceed the other, it will not preponderate, the exceſs not being ſufficient to overcome the friction, and bear down the beam.

That the friction may be as little as poſſible, the parts of the beam which play upon the axis, as alſo the axis it ſelf, ſhould be well poliſhed, and the axis ſhould be made as ſmall as the uſes of the balance will admit; but as friction cannot be intirely prevented, to remedy the inconveniences ariſing from it as much as poſſible, the arms of the beam ought to be made as long as they conveniently can; becauſe the longer the arms are, the leſs will the weight be that is requiſite to overcome the friction; the moments of weights increaſing in proportion to their diſtances from the center of motion, as has been already ſhewn.

I ſhall

I ſhall cloſe what I had to ſay concerning the balance, by laying before you one property of it, which is ſomewhat ſingular and ſurpriſing; though it has not, that I can find, been taken notice of by any of the mechanick writers, * namely, that if a man ſtanding in one ſcale, and counterpoiſed by a weight in the other, lays his hand to any part of the beam, and preſſes it upward, he will thereby deſtroy the balance, and make the ſcale wherein he ſtands to preponderate.

LECT. V.

Exp. 11.

In order to account for this property, let A B represent the beam of a pair of ſcales playing on the axis at C, and let a man ſtanding in the ſcale D, and counterpoiſed by a weight in the ſcale E, lay his hand to ſome part of the beam, either on the ſame ſide of the axis with himſelf as at H, or on the other ſide as at K, and preſs the ſame upward; inaſmuch as action and reaction are always equal, it is manifeſt that with whatever force the hand preſſes upward againſt the point H or K, with the ſame the hand, and conſequently the man's whole body, is preſſed downward; and therefore the ſcale D wherein he ſtands, bears the ſame preſſure from his feet that the point H or K does from his hand; but the preſſure upon the ſcale D may be looked upon as applied to the beam at the point A from which the ſcale hangs; conſequently, the ſame force which preſſes up the point H or K, preſſes down the point A; wherefore putting F to denote that force, $F \times HC$ will expreſs the moment wherewith the arm AC is preſſed upward when the hand is applied at H, and $F \times KC$ the moment wherewith the arm

Pl. 2. Fig. 10.

* The property here mentioned had not been taken notice of by any of the Mechanick Writers, when the Author compoſed this Lecture; but has been publiſhed ſince, both in the Philoſophical Tranſactions for the Year 1729 and in a courſe of experimental Philoſophy, by Dr. DESAGULIERS, to whom our Author communicated it, as he told me and many others, about thirteen or fourteen Years ago when he was in *London*.

LECT. VI. BC is preſſed upward the hand being applied at K; and in both caſes F×AC will expreſs the moment wherewith the arm AC is preſſed downward by means of the reaction; if therefore the hand be applied at H, it is manifeſt that as the arm AC is at one and the ſame time preſſed upward by a force which is as F×HC, and downward by a force which is as the ſame F×AC, and as HC is ever leſs than AC, the arm AC muſt deſcend with the difference of thoſe forces, that is, with a force equal to F×AH, which is the diſtance of the hand from the point A; if the hand be applied at K, the arm CB is preſſed upward, and conſequently AC downward, with a force equal to F×KC, and upon account of the reaction AC is likewiſe preſſed downward with a force equal to F×AC, and therefore it muſt deſcend with a force equal to the ſum of thoſe two forces, that is, with a force equal to F×AK the diſtance of the hand from the point A; ſo that the ſcale D muſt preponderate whether the hand be applied to that part of the beam which lies on the ſame ſide of the axis with the man, or to that which lies on the other ſide; and if D be put to denote the diſtance of that point to which the hand is applied from the point A, the force wherewith the preponderating ſcale deſcends will be univerſally as F×D, that is, as the force which the hand exerciſes againſt the beam, multiplied into the diſtance of the hand from the point A. And if the force wherewith the hand preſſes the beam be required, it may be diſcovered by throwing in as much weight into the ſcale E as is ſufficient to balance the force of the hand, and to prevent the deſcent of the ſcale D; for putting W to denote that weight, it's moment is as W×BC or AC, which being equal to F×D the moment of F, F will be found equal to $W \times \frac{AC}{D}$, that is, to the weight

multiplied

multiplied into half the length of the beam, and divided by the distance of the hand from A. For instance, if the balancing weight be twenty pounds, and the distance of the hand from A be to half the length of the beam as one to two, the force wherewith the hand presses the beam is equal to twenty pounds multiplied by two and divided by unity, that is, it is equal to forty pounds; from what has been said it follows, that when the hand is applied to that part of the beam which lies on the same side of the axis with the man, the force of the hand upon the beam is greater than the weight which balances it in the scale E, and less than the same when the hand is applied to that part of the beam which lies on the other side of the axis with respect to the man; for in the first case, $W \times \frac{AC}{D}$ is greater than W, and in the latter less, inasmuch as AC is in the former case always greater, and in the latter less than D.

The second, and indeed the most simple of all the mechanick powers is the LEVER; an engine chiefly made use of to raise large weights to small heights. By the writers of mechanicks it is supposed to be an inflexible line void of all gravity; though such as as are in common use are both flexible and weighty. In every lever there is one immoveable point, about which as a center all the parts of the lever turn; and whatever supports that point is called the prop; and with regard to the different situations of the moving power, and the weight to be moved in respect to the prop, the lever is divided into three kinds; the first of which is where the prop is placed between the moving power and the weight to be raised; which kind of lever is represented, wherein C denotes the prop, B the weight, and A the power. In this lever there will be a balance between the power and the weight, provided they be

Pl. 2. Fig. 11.

 to

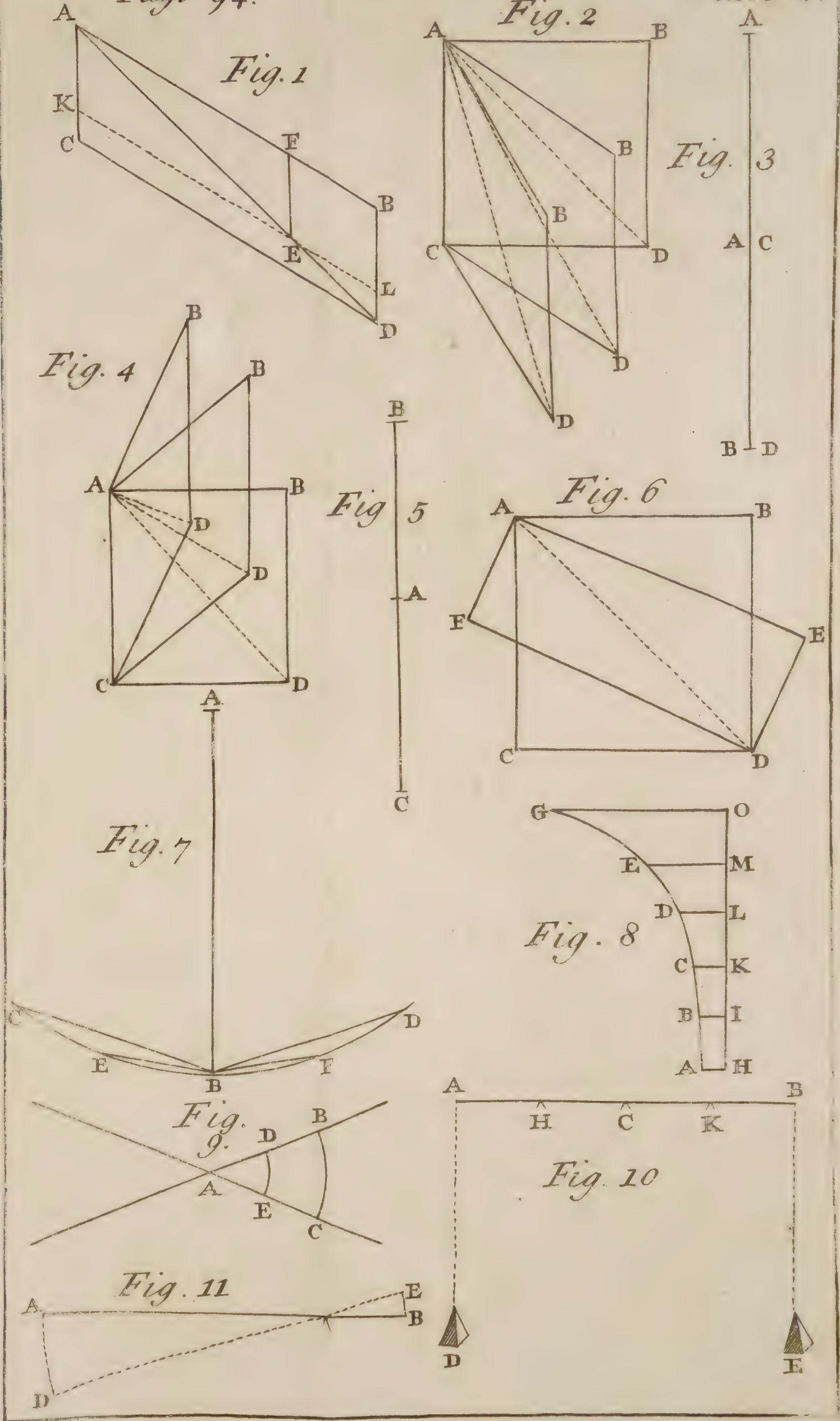
LECT. VI. to one another reciprocally as their diſtances from the prop; that is to ſay, if the power at A be to the weight at B, as CB to CA; for upon the motion of the lever round it's fixed point C, the power at A will deſcribe the arch AD in the ſame time that the weight at B deſcribes the arch BE; conſequently, the velocity of the power will be to the velocity of the weight, as the arch AD to the arch BE; that is, becauſe the arches are ſimilar, as is evident from the manner wherein they are generated, as AC to CB. That therefore the product ariſing from the multiplication of the power into it's velocity may be equal to the product of the weight into it's velocity, or in other words, that their moments may be equal, the power muſt bear the ſame proportion to the weight, that BC the diſtance of the weight from the prop bears to AC the diſtance of the power from the prop. For inſtance, if BC be to AC as one to two, and if a man's ſtrength be
Exp. 12. ſuch as that without the help of a machine he can ſupport an hundred weight, he will by the help of this lever be enabled to ſupport two hundred; becauſe as BC is to AC, which by ſuppoſition is as one to two, ſo muſt the power at A be to the weight at B; but the power at A is ſuppoſed to be equal to one hundred, conſequently the weight muſt be equal to two.

As in this lever the prop may be placed either at the middle diſtance between the moving power and the weight, or nearer to one than the other, it is evident that there may be a balance between the power and the weight, either when they are equal, or when the one exceeds or is exceeded by the other according to the different ſituations of the prop.

To this kind of lever may be reduced ſeveral ſorts of inſtruments, ſuch as ſciſſars, pincers, ſnuffers, each of which may be conſidered as made up of two levers, whoſe prop is the ſame with the pin

which

Page 94. Plate 2.



which rivets them together. Quarry crows are likewise levers of this kind, concerning which it must be observed, that the larger and more ponderous they are, provided they are not so big as to become unmanageable, the more useful they must be, because the weight of that part of a crow which lies on the same side of the prop with the power, and which usually far exeeds the other part in length, acts in conjunction with the power, and thereby facilitates the raising of the stones. LECT. VI.

If the arms of this lever, instead of lying in a right line, meet each other at the prop in a right angle, where A C and B C represent the arms of a lever united at the prop C, in such a manner as to constitute a right angle A C B; if to one arm as CB placed horizontally, a weight be appended at B, and to the other as A C standing perpendicularly a power be applied at A acting in the direction AD: In order to a balance the power must be to the weight as BC to AC, that is, the power and weight must be in the inverse *ratio* of the lengths of the arms to which they are applied. For as the arms turn together upon the prop C, in the same time that the point B describes any arch as B K, the point A must describe a similar arch as A H; consequently, the velocity of A will be to the velocity of B as AC to BC; but as the moment of the power at A is supposed equal to the moment of the weight at B, the power must be to the weight, as the velocity of the latter to the velocity of the former, that is, as B C to A C. Pl. 3. Fig. 1.

To confirm this by experiment, let B C be one fourth of A C, and a weight of twelve ounces be appended at B; to the chord ADF made fast to the point A and passing over a pulley at D, let a weight of three ounces be hung at F so as to pull the arm AC in the direction A D, and there will be a balance. And if B C be one third or one half of A C, then a weight at F, which in the former case is one Exp. 13.

third,

LECT. VI. third, and in the latter one half of P will balance the ſame; and if A C and B C be equal, the balancing weights muſt be ſo too.

From the experiments, and what has been ſaid concerning them, it is evident, that the greater the proportion is which A C bears to B C, the greater is the force of the lever, or the leſs the power at A requiſite to balance a given weight at B. And foraſmuch as the hammer when made uſe of in drawing nails is a lever of this kind, it is manifeſt, that the longer the handle is in proportion to that part of the hammer which lies between the handle and that portion of it which gripes the nail, the leſs will the force be that is requiſite to draw the nail.

The ſecond kind of lever has it's prop at one end, the power at the other, and the weight between, as where C is the prop, A the power, and B the weight; in this lever, in the ſame time that the power at A moves through the arch of a circle whoſe radius is A C, the weight at B moves through a ſimilar arch of a leſſer circle whoſe radius is B C; conſequently, the velocity of the power is to the velocity of the weight as A C to B C; in order therefore to a balance, the power muſt be to the weight as B C to A C; that is, as much as A C, the diſtance of the power from the prop, exceeds B C, the diſtance of the weight from the prop, ſo much muſt the weight exceed the power.

Pl. 3. Fig. 2.

Exp. 14.

As in this lever the diſtance of the weight from the prop is always leſs than the diſtance of the power from the prop, it is evident that there cannot be a balance in any caſe but where the weight exceeds the power.

To this kind of lever may be reduced the oars and rudders of ſhips, cutting-knives fixed at one end, and doors moving upon hinges.

Exp. 15. If in this lever we ſuppoſe the power and the weight to change their places, ſo as that the power

may

may be applied at B between the weight at A and the prop at C, it will become a lever of the third kind; wherein in order to a balance, the power at B muſt ſo far exceed the weight at A, as B C, the diſtance of the power from the prop, is leſs than AC, the diſtance of the weight from the prop. Lect. V. Pl. 3. Fig. 3.

It is evident, that the moving power receives no advantage from this kind of lever, and therefore it is never made uſe of but in caſes of neceſſity, and where the weights to be raiſed cannot be managed in a more convenient manner; as is the caſe of ladders, which being fixed at one end, are by the force of a man's arms reared againſt a wall.

As levers are of ſervice in raiſing weights, ſo are they likewiſe in carrying and ſupporting the ſame; concerning which it is to be obſerved, that when two powers ſupport a weight by help of a lever, the ſum of the powers muſt equal theweight; and the weight being placed between them, their reſpective diſtances therefrom muſt be reciprocally as the powers. Thus, if a weight reſting on the lever at B, be ſupported by two powers, one at A, and the other at C, the diſtance of A from B muſt be to the diſtance of C from B, as the power at C is to the power at A. For in this caſe the lever is of the ſecond kind, where each of the powers is in it's turn to be looked upon as the prop, and then the other power muſt be to the weight as the diſtance of the weight from the prop to the diſtance of the power from the prop; that is, when A is conſidered as the prop, the power at C muſt be to the weight at B, as A B to A C; and when C is conſidered as the prop, the power at A muſt be to the ſame weight at B, as C B to C A. Conſequently, ſince the power at A is to the weight, as B C to A C; and ſince the ſame weight is to the power at C, as A C to A B; the power at A muſt be to the power at C, as B C to B A, that is, the powers muſt be to one another inverſly as their diſtances from the weight; and thus Pl. 3. Fig. 3.

it

LECT. VI. it will appear to be from experiments. For if from the point B of the lever A C a weight as D be suspended, and if two other weights as E and F be suspended from the extreme points A and C by cords passing over pulleys, so as that they may draw the lever directly upward; they will support the weight D provided the sum of those two weights be equal to the weight D, and the weight E be to the weight F as B C to B A.

Exp. 16. Pl. 3. Fig. 4.

Exp. 17. The same thing will happen, if the three weights be made to pull the lever horizontally, which may be done by passing the cords over small wheels or pins placed on a level with the lever.

In shewing what the proportion ought to be between two powers which support a weight placed upon a lever, I have supposed the position of the lever to be parallel to the plane of the horizon; what the proportion ought to be, and in what manner such proportion is determined in inclined positions of the lever, shall be shewn, when I come to treat of powers acting in oblique directions.

If instead of a single lever, several be combined together in such a manner, as that a weight being appended to the first lever, may be supported by a power applied to the last, as in the machine, which consists of three levers of the first kind, and is so contrived as that a power applied at the point L of the lever C, may sustain a weight at the point S of the lever A. The power must be to the weight, in a *ratio* compounded of the several *ratios*, which those powers that can sustain the weight by the help of each lever when used singly and apart from the rest, have to the weight; for instance, if the power which can sustain the weight P by help of the lever A alone, be to the weight as one to five; and if the power whereby the same weight can be sustained by the help of the lever B alone, be to the weight, as one to four; again, if the power which can support the same weight by the help of the letter C alone,

Pl. 3. Fig. 18.

Exp. 18.

alone, be to the weight as one to five; the power which ſupports the weight by means of thoſe three levers joined together will be to the weight in a *ratio* compounded of one to five, one to four, and one to five, that is, it will be as one to an hundred. For ſince in the lever A, a power equal to one fifth of the weight P preſſing down the lever at L, is ſufficient to balance the weight; and ſince it is the ſame thing whether that power be applied to the lever A at L, or the lever B at S, the point S bearing on the point L, a power equal to one fifth of the weight P being applied to the point S of the lever B, and preſſing the ſame downward, will ſupport the weight; but one fourth of the ſame power being applied to the point L of the lever B, and puſhing the ſame upward, will as effectually depreſs the point S of the ſame lever, as if the whole power was applied at S; conſequently, a power equal to one fourth of one fifth, that is, to one twentieth part of the weight P, being applied to the point L of the lever B, and puſhing up the ſame, will ſupport the weight; but it matters not whether that force be applied to the point L of the lever B, or to the point S of the lever C, ſince if S be raiſed, L which reſts thereon muſt be ſo too; but one fifth of the power applied at the point L of the lever C, and preſſing it downward will as effectually raiſe the point S of the ſame lever, as if the whole power was applied at S and puſhed up the ſame; conſequently, a power equal to one fifth of one twentieth, that is, to one hundredth part of the weight P, being applied to the point L of the lever C, will balance the weight at the point S of the lever A; that is, a power which is to the weight, in a *ratio* compounded of the three *ratios*, which the powers have to the weight in each lever taken ſeparately, will be a balance to the weight, when the three levers are uſed jointly. And by the ſame way of reaſoning it will

will be found, that in all machines of this kind, the power requiſite to ſuſtain the weight, is to the weight, in a *ratio* made up of the ſeveral *ratios* of the power to the weight in each lever taken ſeparately, whatever be the number of levers.

In all that has been hitherto ſaid concerning the lever, the power and the weight are ſuppoſed to act in direct oppoſition to each other; and on this ſuppoſition, the power muſt be to the weight in each of the three kinds of levers, in the reciprocal *ratio* of their diſtances from the prop, as has been fully proved with regard to each kind; but where the directions of the power and weight are inclined to each other, the proportion will vary from what has been here determined, as ſhall be ſhewn, when I come to treat of powers acting in oblique directions.

LECTURE VII.

OF THE PULLEY.

LECT. VII. IN this lecture I ſhall give you an account of the *Pulley*, the *Axle in the Wheel*, the *Wedge*, and the *Screw*. The PULLEY is a ſmall wheel that turns about it's axis, and which has a drawing rope paſſing over it. It is made uſe of in raiſing large weights to conſiderable heights; and is of two kinds, fixed and moveable; the ſole uſe of the fixed pulley is to change the direction of the moving power; which in all caſes where weights are to be raiſed to great heights, is exceedingly convenient, and very often of abſolute neceſſity; for inſtance, if the weight P is to be raiſed by the force of a man's hand to any height as A above the reach of the hand, the man muſt quit his place and aſcend in order to carry up the weight, which for the moſt

Exp. 1. Pl. 3. Fig. 6.

3 part

part is found to be inconvenient, and sometimes impracticable; whereas if to a rope as PAF passing over the fixed pulley at A, the weight be made fast at one end as P, and the hand applied to the other end at F, the man by drawing the rope AF downward, will without moving from his place raise the weight as effectually, as if his hand was applied to it and moved upward from P to A; so that in raising weights to great heights the fixed pulley is of singular service, in as much as by changing the direction of the power, it takes off the necessity that a man would otherwise lie under of ascending along with the weight, and by so doing lessens his labour; besides, it has this farther convenience attending it, that by means thereof the joint strength of several persons may be made use of to raise one and the same weight, which in many cases cannot be done, at least not so conveniently, where the weight is raised by the immediate application of the hands; but this pulley does not in the least assist the power, by increasing it's moment; because it neither lessens the velocity wherewith the weight rises, nor arguments that of the power; for whatever be the space through which the power moves by drawing the rope AF, the weight must in the same time be drawn up through an equal space; the rope AP constantly shortning in the same proportion that the rope AF is lengthened; and therefore, wherever any power supports a weight by means of a fixed pulley, that power must be equal to the weight.

When a pulley rises and falls along with the weight, as does this pulley, it is said to be moveable, and with regard to it's use, it is just the reverse of the fixed pulley; for it adds to the moment of the power, but causes no change in it's direction; for if the hand be applied at F to the rope D, in order to raise the weight P appended to the moveable pulley E, it must move directly up- Pl. 3. Fig. 7.

ward

LECT. VII. ward in the very ſame manner, as if it was applied immediately to the weight; conſequently, the direction of the hand which raiſes the weight is no way altered by this pulley, but the moment thereof is doubled, becauſe it is made to riſe twice as faſt as the weight; for in the ſame time that the hand moves upward from F to G, through the ſpace FG equal in length to the two equal ropes D and C, the pulley, and conſequently the weight annexed, will be drawn up through the ſpace E H, whoſe length is equal to one of the ropes only.

In machines conſiſting of ſeveral pulleys, whereof ſome are fixed and ſome moveable, and which have one common rope that goes round them all; if one end of the rope be fixed, as is the caſe in the machines repreſented by theſe figures, in order to a balance, the moving power muſt be to the weight, as one to twice the number of moveable pulleys; becauſe the velocity wherewith the power moves in raiſing weights by the help of ſuch engines, is to the velocity of the riſing weight, as twice the number of moveable pulleys to unity; as I ſhall now ſhew you in the machine, which conſiſts of one fixed pulley as A, and another moveable as E. Since it is one and the ſame rope that is continued from G to F, the part AF which lies beyond the fixed pulley, cannot be drawn down and thereby lengthened, unleſs the two parts D and C, which lie on each ſide of the moveable pulley, be at the ſame time drawn up and ſhortened, and that equally; whence it is evident, that the part AF will be lengthened as faſt again as either D or C is ſhortened, inaſmuch as what each of thoſe parts loſe of their length is added to the length of A F; but the point F to which the power is applied, deſcends as faſt as A F is lengthened, and the point E, to which the weight is faſtened, aſcends as faſt as D or C is ſhortened; conſequently, the velocity of the power is to the velocity of the weight, as two to

Pl. 3. Fig. 8, 9, 10.

Pl. 3. Fig. 8.

to one, that is, as twice the number of moveable pulleys to unity; if therefore a weight appended at F, be to a weight appended at E, as one to two, they will balance each other, as being to one another in the reciprocal *ratio* of their velocities.

Exp. 2.

In the machines, each of which consists of two fixed and as many moveable pulleys, and which differ only in this, that in one the pulleys of the same kind move upon one and the same axis, and in the other upon different axes; I say, in these machines, the velocity wherewith the power moves is to the velocity wherewith the weight rises, as four to one, that is, as twice the number of moveable pulleys to one; for as the part of the rope A F is drawn down and lengthened, the four parts B, C, D, H, which lie on each side of the two moveable pulleys, are drawn up and shortened, and that equally; and what each of them loses of it's length is added to the length of A F; consequently, A F is lengthened four times as fast as each of the other parts shortens; but the power moves as fast as A F lengthens, and the weight rises as fast as the other four shorten; and therefore, the velocity of the power at F is to the velocity of the weight at E, as four to one, or as twice the number of moveable pulleys to unity: for which reason, if a weight be appended at F which is to the weight at E, as one to four, that is, in the reciprocal *ratio* of their velocities, there will be a balance.

Pl. 3. Fig. 9, 10. Exp. 3.

What has been thus proved with regard to the three last machines, namely, that the velocity wherewith a power moves in raising a weight is to the velocity wherewith the weight rises, as twice the number of moveable pulleys to unity, is in the same manner demonstrable with regard to any other machine of the same kind, whatever be the number of pulleys whereof it consists; and therefore, in all machines consisting of several pulleys whereof

LECT. VII. ſome are fixed and others moveable, and round which goes one common rope, fixed at one end, it may be laid down as a general rule, that in order to a balance between the moving power and the weight, the former muſt be to the latter, as one to twice the number of moveable pulleys.

Exp. 4. If the rope which goes round the pulleys, inſtead of being fixed at one end, be faſtened to the weight or to the block which ſupports the moveable pulleys, ſo as to riſe therewith, as in this machine, which conſiſts in five pulleys, whereof three are fixed and two moveable, and in which the end of the rope is joined at G to the block which ſupports the two moveable pulleys; the velocity of the power is to the velocity of the weight, as the ſum of twice the number of moveable pulleys increaſed by unity to one; for in this caſe, the parts of the rope which are equally ſhortened in order to lengthen the part A F, are more in number by one than the ſum of the moveable pulleys when doubled; conſequently, ſince the power at F moves as faſt as A F is lengthened, whilſt the weight at E riſes in proportion only to the ſhortening of the ropes B, C, D, H, K, the velocity of the power bears the ſame proportion to the velocity of the weight, as the ſum of twice the number of moveable pulleys increaſed by unity does to one; and therefore, if the power be to the weight in the inverſe *ratio*, that is, as one to twice the number of moveable pulleys added to unity, there will be a balance. Thus, if in the machine a weight appended at F be to another at E, as one to five, they will balance, and remain unmoved.

Pl. 3. Fig. 11.

Exp. 5. If to any of the forementioned machines be added a runner, that is, a ſingle moveable pulley, which has it's own rope diſtinct from that which is common to the other pulleys, one end whereof is fixed as at L, the other being faſtened to the block at E, and the weight appended at M, the force

Pl. 3. Fig. 12.

force of the former machines will be doubled by this additional pulley; for ſince the point E moves with twice the velocity of the point M, as I ſhewed when ſpeaking of the ſingle moveable pulley, whatever be the proportion which the velocity of the power at F bears to the velocity of the weight when appended at E, it will be doubled if the weight be appended at M; conſequently, the power will by the help of the runner be able to ſuſtain twice the weight that it did before. LECT. VII.

If a machine be combined of one fixed and ſeveral moveable pulleys, put together in ſuch a manner as that each of the moveable pulleys has a ſeparate rope, one end whereof being fixed, the other either paſſes over the fixed pulley, as does that of the firſt moveable pulley E, or is joined to the moveable pulley which lies next above it, as in the caſe of the ropes B,C,D, which belongs to G, H, and I, the ſecond, third and fourth moveable pulleys; B being joined at N to the firſt moveable pulley, C at K to the ſecond, and D at L to the third; the weight being appended to the laſt moveable pulley at H. The velocity wherewith the weight riſes in ſuch a machine is to the velocity of the power, as one to the laſt term of a duple progreſſion, whereof the firſt term is unity, and the number of terms more by one, than the number of moveable pulleys. Pl. 3. Fig. 13. Exp. 6.

For as I proved, when ſpeaking of the ſingle moveable pulley, the velocity of the power at F is to the velocity wherewith the pulley E riſes, as two to one; and ſo likewiſe is the velocity of E, to that of G, and that of G, to that of I, and ſo on, whatever be the number of moveable pulleys, the velocity of each ſucceeding pulley is but one half of the velocity of the preceding; wherefore, if the velocity of the laſt pulley, which is the ſame with the velocity of the weight, be put equal to unity, the velocity of that which immediately precedes it, to wit H, will be as two, and the veloci-

LECT. VII. ty of G, as four, and of E, as eight, and so on, if there be more moveable pulleys, the velocity will be continually doubled, and since the velocity of the last pulley is expressed by unity, that of the first will be expressed by the last term of a duple progression whose first term is unity, and the number of terms equal to the number of moveable pulleys; and consequently, since the velocity of the power is double that of the first moveable pulley, if the duple progression be continued to one term more, that term will express the velocity of the power, the velocity of the weight being as unity; thus, in this machine, the number of moveable pulleys being four, the velocity of the weight at M is to that of the power at F, as one to sixteen; if therefore a weight appended at F be to the weight at M, as one to sixteen, there will be a balance.

Though this engine be of greater force than any other wherein there is the same number of moveable pulleys, yet inasmuch as it does for that very reason raise weights more slowly: men for the sake of dispatch choose rather to make use of such combinations of pulleys as are represented in the 9th and 10th figures, and where they have occasion to raise very large weights, they double the force of those machines by the addition of a runner.

The fourth mechanick power is called the AXLE IN THE WHEEL; which is a simple engine consisting of one wheel fixed to the end of an axle that turns along with the wheel; the manner of raising weights by the help of this machine is thus; the power being applied to some part of the wheel's circumference, turns the wheel, and together with it the axle, by which means a rope that is tied to the weight at one end, and made fast to the axle at the other, is wound about the axle, and thereby the weight drawn up; and for as much as the wheel and it's axle revolve together, in whatever time the power moves through a space equal to the circumference

ference of the wheel, the weight muſt in the ſame time be raiſed up through a ſpace equal to the circumference of the axle, conſequently, the velocity of the power is to the velocity of the weight, as the circumference of the wheel to the circumference of the axle; that is, from the nature of the circle, as the diameter of one to the diameter of the other; if therefore the power be to the weight in the inverſe *ratio* of thoſe diameters; that is to ſay, if the power be to the weight, as the diameter of the axle to the diameter of the wheel, there will be a balance; the power in that caſe being juſt ſufficient to ſupport the weight. For inſtance, if the diameter of the wheel be five inches, and that of the axle one, a weight of one ounce hanging from any point in the circumference of the wheel, will ſupport a weight of five ounces hanging at the axle; and if the diameter of the axle be but half an inch, then will ten ounces at the axle be ſupported by one at the wheel.

Exp. 7.

Where the parts of the axle differ in thickneſs, if weights be hung at the ſeveral parts, they may be ſuſtained by one and the ſame power applied to the circumference of the wheel, provided the product ariſing from the multiplication of the power into the diameter of the wheel be equal to the ſum of the products ariſing from the multiplication of the ſeveral weights into the diameters of thoſe parts of the axle from which they are ſuſpended. Thus a weight of five ounces hanging from the parts of an axle whoſe diameter is one inch, and another of ten ounces from a part whoſe diameter is half an inch, will be balanced by a weight of two ounces hanging from the circumference of the wheel whoſe diameter is five inches; for the ſum of the products of five into one, and of ten into one half, which expreſs the moments of the weights, is equal to ten, as is alſo the product of two into five, which expreſſes the moment of the power.

Exp. 8.

LECT. VII.

If to the axle in the wheel be added one or more wheels with teeth, ſo that motion may be communicated from the firſt wheel to the laſt; the weight being hung from the axle of the laſt wheel, whilſt the moving power is applied to the circumference of the firſt wheel; in order to a balance, the power muſt be to the weight in a *ratio* compounded of the inverſe *ratio* of the diameter of the firſt wheel to the diameter of the laſt axle, and of the inverſe *ratio* of the number of revolutions made by the firſt wheel, to the number of revolutions made by the laſt axle in a given time; for if the firſt wheel and the laſt axle revolved in the ſame time, the *ratio* of the diameter of the wheel to that of the axle, would expreſs the *ratio* of the velocity of the power, to the velocity of the weight; but if the wheel revolves oftener than the laſt axle in a given time, it is evident, that the *ratio* of the velocity of the power to that of the weight, will be greater in that proportion; conſequently, the velocity of the power muſt be to the velocity of the weight in a *ratio* compounded of the *ratio* of the diameter of the firſt wheel to the diameter of the laſt axle, and of the revolutions of the firſt to thoſe of the laſt axle in a given time; and therefore, that there may be a balance between the power and the weight, the former muſt be to the latter inverſly in the ſame compounded *ratio*.

Exp. 9. For inſtance, in a machine conſiſting of two wheels with their axles, wherein the diameter of the firſt wheel is four inches, and that of the ſecond axle a quarter of an inch, and wherein the cogs or teeth of the firſt axle by applying themſelves ſucceſſively to the teeth of the ſecond wheel, turn it about, and therewith it's axle; but the teeth of the firſt axle being in number but one fourth of the teeth of the ſecond wheel, that axle, and conſequently the firſt wheel, muſt revolve four times in order to turn the ſecond wheel and it's axle once; ſo that the revolutions of the

firſt

first wheel in a given time are to the revolutions of the second axle, as four to one: in this machine, in order to a balance, the power must be to the weight inversly in a *ratio* compounded of sixteen to one, and of four to one; that is, it must be to the weight inversly as sixty-four to one; so that a weight of one ounce at the circumference of the first wheel, will support a weight of sixty-four ounces fastened to the second axle. LECT. VII.

Again, in a machine composed of three axles, the two last having wheels with teeth, and the first a perpetual screw which in each revolution of the first axle moves one tooth only of the wheel of the second axle; which wheel having twenty eight teeth, moves round once in the same time that the first axle turns twenty-eight times; and there being a small wheel with fourteen teeth at the other end of the second axle, and these teeth applying themselves continually to the teeth of a wheel fixed on the third axle, which are twenty-eight in number; the wheel of the third axle must revolve but once in the same time that the wheel of the second axle revolves twice, and of consequence the third wheel and it's axle move round but once whilst the first axle performs fifty-six revolutions; and the diameter of the first axle is to that of the last as two to one; in order therefore to a balance between the power which is applied to the first axle, and the weight which is applied to the last; the power must be to the weight inversly in a *ratio* compounded of two to one, and of fifty-six to one; that is, the power must be to the weight as one to an hundred and twelve; so that one ounce hanging from the first axle will support an hundred and twelve ounces hanging from the last axle. Exp. 10.

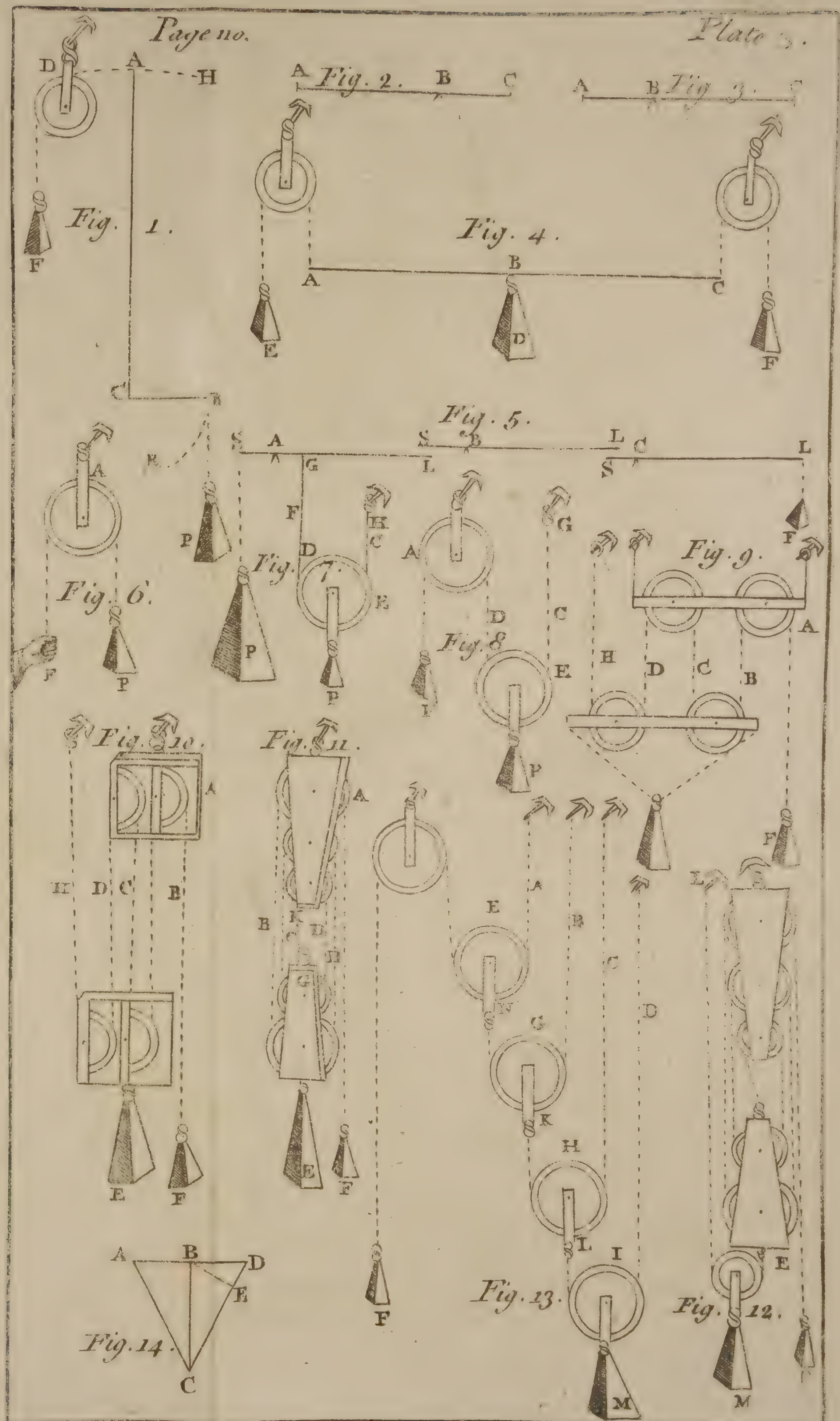
In order to exhibit the force of the WEDGE, which is the fifth mechanick power, let AD represent the base of a wedge, from whose middle point B let the line BE be drawn perpendicular to the Pl. 3. Fig. 14.

 side

LECT. VII. side DC, and the line BC at right angles to AD, and consequently, bissecting the angle ACD made by the concourse of the wedge's sides.

In cleaving timber with a wedge, the force of the mallet which strikes the wedge, is to be looked upon as the moving power, and the cohesion of the parts of the timber, as the resistance or weight to be moved; now, whilst the wedge is driven by the repeated strokes of the mallet from B to C, (for I suppose the edge of the wedge to be placed on the top of a piece of timber at B in order to rend it) the space described by the wood as it yields on each side of the wedge in lines perpendicular to those sides, is equal to BE. Consequently, that the moment of the mallet may be equal to the resistance of the wood, the absolute force of the mallet must be to the force wherewith the parts of the wood cohere, as BE to BC, that is, as the sine of the angle BCD to radius; whence it follows, that all similar wedges are of equal force, for in such the angle BCD is given; it likewise follows, that the powers of dissimilar wedges are inversly as the sines of the angles BCD, or in other words, that the forces requisite to rend timber with such wedges, as directly as the sines BE, which is confirmed by the following experiment.

Exp. 11. Let a machine be so contrived, as to consist of two equal cylinders, rolling upon their axles in an horizontal position along the edges of two rulers, and let them be drawn and kept together by a weight of 2000 grains, hanging freely by a rope, fastened at each end to the cylinders, and let the edge of a wedge be placed between the cylinders; so as that when a sufficient weight is hung to it, it may be drawn down between the cylinders; in this machine the force wherewith the cylinders are drawn together, added to the attrition of their axles in rolling upon the rulers, may be looked upon as the resistance of the timber, and the weight of the wedge,


Page 110.
Plate 5.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.

wedge, together with the appending weight whereby it is pulled down between the cylinders, as the force of the mallet upon the wedge; now, if three wedges be made uſe of, each three inches long, in which the ſines BE are as one, two, and three, their weights likewiſe being in the ſame proportion, the firſt will be drawn down by a weight of 300 grains, the ſecond by one of 600 grains, and the third by one of 900 grains.

To the wedge may be reduced the axe or hatchet, the teeth of ſaws, the chizel, the augur, the ſpade and ſhovel, knives and ſwords of all kinds, as alſo the bodkin and needle, and in a word, all ſorts of inſtruments, which beginning from edges or points grow gradually thicker as they lengthen; and the manner wherein the power is applied to ſuch inſtruments, is different according to their different ſhapes and figures, and the various uſes for which they were contrived.

The next and laſt mechanick power is the SCREW, which conſiſts of two parts, whereof the firſt is called the male or outſide ſcrew, being a cylinder cut in, in ſuch ſort as to have a prominent part going round it in a ſpiral manner, which prominent part is commonly called the thread of the ſcrew; the other part, which is called the female or inſide ſcrew, and by common workmen the nut, is a ſolid body that contains an hollow cylinder, whoſe concave ſurface is cut in the ſame manner as the convex ſurface of the male ſcrew, ſo that the prominent parts of the one may fit the cavities of the other. The chief deſign of this machine is to preſs the parts of bodies cloſely together, and in ſome caſes to break and divide them; when it is made uſe of one part is commonly fixed, whilſt the other is turned round, and in each revolution the moveable part is carried in the direction of the axis of the cylinder through a ſpace equal in length to the interval between two contiguous threads, whereby

LECT. VII. by the parts of the body whereon the preſſure is made are forced to move towards one another through a ſpace equal to that interval; which interval therefore does expreſs the velocity wherewith the ſeveral parts of the body give way to the preſſure, whilſt the circular periphery, which is deſcribed by the power whereby the moveable part of the ſcrew is turned round, expreſſes the velocity of the power; for the moveable part of the ſcrew is uſually turned by means of an handle or handſpike, to ſome part of which the power is applied, and by moving round with that part deſcribes the circumference of a circle; if therefore the moving power be to the reſiſtance of the body which is preſſed, as the diſtance between two contiguous threads of the ſcrew to the circular periphery deſcribed by the power, there will be a balance; and if the power be ever ſo little increaſed beyond that proportion it muſt overcome the reſiſtance, and move the ſcrew; and thus it would conſtantly be, provided there was no reſiſtance from the attrition of the parts of the ſcrew one againſt another; but as that is very conſiderable, there is an addition of power requiſite to overcome it, over and above what is neceſſary to overcome the reſiſtance of the body whereon the preſſure is made: for which reaſon ſuch experiments as are made to ſhew the force of the ſcrew, muſt vary more from the theory, than thoſe which have been made concerning the other mechanick powers, wherein the attrition is far leſs conſiderable; however it will appear from the following experiment, that ſmall powers are ſufficient by the help of the ſcrew to overcome great reſiſtances in the bodies which are preſſed.

Exp. 12. Let a wheel whoſe diameter is four inches, be fixed at it's center to the head of a male ſcrew in a horizontal poſition, and let the end of a rope, which is wound about the groove of the wheel, paſs over a pulley in ſuch a manner as that having

a weight faſtened to it, it may be drawn in a line, that is a tangent to the wheel, by which means the intire gravity of the weight will be employed in turning the wheel; to one end of a lever, ſupported by a prop at the middle, let a weight of ſeven pounds be hung, and let the bottom of the male ſcrew reſt on the other end of the lever; and let the diſtance between the threads of the ſcrew, be equal to one fifth of an inch, and a weight of three ounces and 250 grains being hung to the end of the rope which paſſes over the pulley, will juſt turn the wheel, and thereby thruſt down the ſcrew and with it the end of the lever whereon it reſts, and by ſo doing raiſe up the weight at the other end. LECT. VII.

In this caſe the power which moves the ſcrew, is to the weight raiſed whereby the reſiſtance that is made to the preſſure is meaſured, as one to 24 nearly; whereas it ought not to exceed the proportion of one to 63; for the diameter of the wheel being four inches the circumference is twelve and an half nearly, but 12.5 is to $\frac{1}{5}$, which is the interval between the threads of the ſcrew, as $62\frac{1}{2}$ to one; conſequently, if the power which turns the ſcrew be to the weight that is to be raiſed in the inverſe *ratio* of thoſe numbers, that is, as one to $62\frac{1}{2}$, it ought to balance the weight, and if it be increaſed ever ſo little it ſhould overpower and raiſe the weight: ſince therefore the force that is requiſite to turn the wheel is nearly three times as great as what is neceſſary to overcome the reſiſtance of the weight to be raiſed, it is evident, that almoſt two thirds of that force is imployed in overcoming the reſiſtance ariſing from the attrition of the parts of the ſcrew one againſt another; what the nature of this reſiſtance is, and in what proportion it varies, ſhall be ſhewn hereafter.

LECTURE VIII.

OF COMPOUND ENGINES.

LECT. VIII. THE mechanick powers, which for the moſt part are made uſe of ſeparately, may in many caſes be combined together, and engines thereby formed of ſuch efficacy, as that by the help thereof exceeding great weights may be raiſed by very ſmall powers. In all ſuch compounded machines the proportion which the moving power bears to the weight when they balance each other, is compounded of the ſeveral *ratios* which thoſe powers have to the weight which balance it in each ſimple machine, whereof the compound engine conſiſts. Thus when a machine is compoſed of an axle in the wheel and a pulley, by faſtening the drawing rope of the one to the axle of the other; the power which balances the weight in ſuch a machine, muſt be to the weight, in a *ratio* compounded of the *ratio* which that power has to the weight which balances it by means of the axle in the wheel alone, and of the *ratio* which that power has to the weight, which balances the weight by means of the pulley alone.

Exp. 1. For inſtance, if the nature of the pulley be ſuch, as that a power equal to one tenth part of the weight balances it; and if the axle in the wheel be ſuch, as that a power equal to one fifth part of the weight can ſupport it; the power which balances the weight in the compounded machine, will be to the weight in a *ratio* compounded of one to ten, and of one to five, that is, it will be to the weight as one to fifty; for, ſince the weight is in effect faſtened to the axle of the wheel by means of the rope which goes round the pulleys, it is evident that the axle will be drawn by a force equal to that, which

which when applied to the drawing rope of the pulley is requisite to sustain the weight by means of the pulley, which force is by supposition equal to one tenth part of the weight; but that force at the axle is balanced by a fifth part thereof applied to the wheel; consequently, the power requisite to balance the weight in this machine, is equal to one fifth of one tenth part of the weight, that is, the power is to the weight, as one to fifty, So that one ounce at the wheel will support fifty ounces at the pulley. LECT. VIII.

If a machine be composed of the lever, the axle, and the perpetual screw; the lever being thirteen inches long, and fixed at it's center to an axle, whereon is a perpetual screw, the tooth whereof adapts it self to the teeth of the wheel of an axle, the teeth of that wheel being twenty-four in number, and the diameter of the axle belonging to that wheel equal to six tenths of an inch; in such a machine the power being applied to one end of the lever, and the weight to the axle of the toothed wheel, the former will balance the latter, if it be in proportion thereto, as one to 520; for if the lever to which the power is applied, moved round in the same time with the axle of the toothed wheel whereunto the weight is fastened, the power would be to the weight, as the diameter of the axle to the length of the lever, that is, as six tenths of an inch to thirteen inches, or in whole numbers, as six to an hundred and thirty; but as there are 24 teeth in the wheel of that axle which sustains the weight, and as the endless screw moves but one of those teeth in each revolution of the lever, the lever must go round 24 times in order to turn the axle, which sustains the weight, once; upon which account the power must be to the weight, as one to 24, which *ratio* of one to 24 being combined with the former of six to 130, gives a *ratio* of six to 3120, or of one to 520; so that an ounce weight being made to act with all it's gravity at one end of Exp. 2.

 the

Lect. VIII. the lever in order to turn it round, which may be done by fixing a wheel to the lever, will balance a weight of 520 ounces at the axle of the toothed wheel.

Exp. 3. If to the laſt machine one moveable pulley be added, it will conſtitute a machine of double the force; for the *ratio* of the power to the weight in the foregoing machine, being as one to 520, and in a ſingle moveable pulley, as one to two; the *ratio* compounded of both, will be as one to 1040; ſo that in this machine an ounce will balance 86 pounds 8 ounces; and if the ſtrength of a man's hand be ſuch, as that it can, without the aſſiſtance of an engine, ſupport an hundred pounds, it will by the help of this machine ſuſtain 104000 pounds.

In all that has been hitherto ſaid concerning the mechanick powers, the moving force and the weight or reſiſtance have been ſuppoſed to act in direct oppoſition to one another. I ſhall now conſider the effects of powers acting obliquely, and ſhew in what caſes they balance each other.

And firſt, if three powers acting in oblique directions, be to one another, as the reſpective ſides of a triangle formed by the concourſe of three lines drawn parallel to the directions of the powers; thoſe powers will balance one another. For inſtance, if three powers drawing the point A in the directions AB, AC, and AE, be to one another, as the ſides of the triangle ADE, or ADC, made by the concourſe of the lines AD, AE and ED; or AD, CD, and AC, which lines are parallel to the directions of the powers; they will balance one another, and the point A will remain unmoved.

Pl. 4. Fig. 1. Exp. 4.

For if the line AD be ſuppoſed to denote a power equal to that which acts in the direction AB, but contrary thereto; the power denoted by AD will draw the point A as forcibly towards D, as it is drawn by the oppoſite power towards B; conſequently, there will be a balance between the two powers;

powers; but the power denoted by AD may be resolved into two powers denoted by AE or CD, and AC or ED; which two powers acting together upon the point A in their proper directions AE and AC, will draw it as ſtrongly towards D, as it is drawn by the ſingle power denoted by AD; as is evident from what has been ſaid concerning the reſolution and compoſition of motions and forces; conſequently, two powers which are as AE or CD, and ED or AC, acting in the directions AE and AC, will balance the third power which is as AD acting in the direction AB; that is, two powers, which are as the two ſides of a triangle, acting in directions parallel to thoſe ſides, will balance a third power, which is as the third ſide, and which acts in a direction parallel thereto; and what has been thus proved in particular of two of the powers with regard to the third, is in like manner demonſtrable of any two of the powers with reſpect to the other; conſequently, any three powers which are to one another reſpectively as the ſides of a triangle, and which act in directions parallel to thoſe ſides, will deſtroy each the others effect, and remain in *æquilibrio*. To confirm this by an experiment; let the ſides of a triangle ABC drawn on an horizontal plane, be as two, three, and four; and let CE be parallel to the ſide AB, and the ſide AC continued towards D. Let three ſmall chords be joined together at C, and ſtretched over three pulleys in ſuch a manner, as that one of them may cover the line CD, another the line CE, and the third the line CB; this being done, if a weight of four ounces be hung to the chord which paſſes over CD, and one of three ounces to that which covers CB, and one of two ounces to that which covers CE, there will be a balance; the weights, which in this caſe are the moving forces, being to one another as the ſides of the triangle to which the directions of the weights are parallel.

Pl. 3. Fig. 2. Exp. 5.

If

LECT. VIII.

Pl. 4. Fig. 3, 4. Exp. 6.

If the weight A hangs freely from one end of a balance, ſo as to have it's line of direction DA perpendicular to the arm of the balance; and if another weight as B, be hung at the other end E, in ſuch a manner, as that it's line of direction EC by paſſing over a pulley at C may be oblique to the arm of the balance, the weight B muſt be to the weight A when it counter balances it, as EC to CF, that is, as radius to the ſine of the angle CEF made by the oblique direction of B with the arm of the balance; for if the whole force of gravity in the weight B acting in the direction EC, be denoted by the line EC, it may be reſolved into two forces denoted by EF and FC, acting in the directions of thoſe lines, of which two forces, the latter only which acts in the direction FC perpendicular to the arm of the balance withſtands the force of gravity in the weight A, the other force which acts in the direction EF being intirely imployed in preſſing the balance againſt the axis of it's motion; ſince therefore, that part of the weight B which acts in oppoſition to the weight A, is to the whole weight B, as FC to EC; it is manifeſt, that in order to make the weight B balance the weight A, it muſt exceed the weight A in the ſame proportion that the line EC exceeds the line FC; and thus it is found to be from experiments; for if the pulley be ſo ordered as that EC may be to FC as three to two, then a weight of three ounces appended at E, will balance one of two ounces appended at D.

As a COROLLARY it follows, that the perpendicular diſtances of the lines of direction from the center of motion, are to one another inverſly as the weights; for, if from G the center of motion be let fall GH perpendicular to EC, that line will be the perpendicular diſtance of the direction EC from G; and EG, equal to DG, is the perpendicular diſtance of the direction DA; but the triangles

EFC

EFC and EFG are ſimilar, becauſe their angles at E are equal, and they have each a right angle; conſequently, as EC is to CF, ſo is EG to HG; but the weight B is to the weight A, as EC to FC, that is, as EG or DG to HG; ſo that wherever two powers, which act in oblique directions, are to one another in the inverſe *ratio* of the perpendicular or ſhorteſt diſtances of their lines of direction from the center of motion, they muſt balance one another; whence it follows, that if two weights as A and B, be ſuſpended from two points as D and E in the plane of a wheel placed in a vertical poſition; and if the line DE which is drawn through the two points of ſuſpenſion, paſſes through C the center of motion, the weights will balance, provided they be to one another inverſly as the diſtances of their points of ſuſpenſion from the center of motion, that is, if A be to B, as CE to CD; for ſince the weights hang freely, their lines of direction DA and FB will be perpendicular to the horizon, and of conſequence, parallel to each other; wherefore, if the line HCF be drawn through the center of motion perpendicular to the two lines DA and FB, the triangles DHC and ECF will be ſimilar, conſequently, DC will be to EC, as HC to FC; but by ſuppoſition, the weight A is to the weight B, as CE to CD; that is, as CF to CH; ſo that the weights are to one another inverſly as the perpendicular diſtances of their lines of direction from the center of motion; conſequently, they muſt balance; and though the wheel ſhould be turned upon it's axis, and the diſtances of the lines DA and EB from C be thereby altered, yet will the ſimilarity of the forementioned triangles continue, and of conſequence the balance between the weights will be preſerved; as will appear from the following experiment. Let a weight of one ounce be ſuſpended from the point D, and another of two ounces from the point E; DC be-

LECT. VIII.

Pl. 4. Fig. 5.

Exp. 7.

 ing

Lect. VIII. ing to EC, as two to one, that is, inversly as the weights, there will be a balance, and the wheel will continue at rest. And if by the force of the hand it be turned about it's axis either to the right from I towards K, or to the left towards M, the balance will still continue, and the wheel will remain unmoved when the hand quits it, whatever be it's position.

Pl. 4. Fig. 6. If the points of suspension D and E, be so posited, as that the right line DE which joins them, does not pass through C the center of motion; let that line be divided any where as in G by another line as IL passing through the center C, and there will be a balance, if the appending weights be to one another inversly as the parts of the line DE, that is, if A be to B as EG to DG, provided the position of the wheel be such, as that the line IL may be perpendicular to the horizon; for since the lines EF, GC, and DH are parallel, FC is to HC, as EG to DG; but by supposition, as EG is to DG, so is A to B; wherefore A is to B, as FC to HC, that is, the weights are inversly as the perpendicular distances of their lines of direction from the center of motion, consequently, their moments are equal; but if by turning the wheel about it's axis the line IL be put out of it's perpendicular position, the balance will be destroyed; because, in that case, one of the lines of direction will approach nearer to the center of motion, whilst the other recedes; and of course their perpendicular distances will not continue in the inverse *ratio* of the weights; for if the wheel be moved upon it's axis from I towards K, so as to have the line SCR perpendicular to the plane of the horizon; the line of direction DA will approach towards the center so as to become DP, and it's perpendicular distance from the center of motion will be NC, whilst the other line of direction recedes as far as EQ, and it's perpendicular distance

from

from C becomes equal to OC; for which reason the weight B must preponderate, and move the wheel about it's axis in the direction IKL. And as the wheel continues to move in that direction, the direction of the weight A will approach nearer and nearer to the center of motion, and at length pass beyond it, so as to be on the same side with the direction of the weight B; so that the wheel will then be moved by the joint force of both weights, and continue so to be, till such time as the direction of the weight B, getting on the other side of C, B begins to act in opposition to A; and at length the point I, being brought into the place of L, the weights do again balance each other, the line DE being divided by the perpendicular line IL in the reciprocal *ratio* of the weights. LECT. VIII.

To confirm what has been said by an experiment, let the line DE in the plane of a wheel, be divided in G by the line IL in such a manner, as that DG may be double of EG; then setting the line IL perpendicular, let a weight of one ounce be hung from D, and another of two ounces from E, and the wheel will remain unmoved; let then the wheel be turned a little upon it's axis, either to the right hand or to the left; in the former case, the two ounce weight will prevail, and carry the wheel downward to the right hand, but in the latter the smaller weight will preponderate, and make the wheel to revolve towards the left. Exp. 8.

If the line DE be divided in another point as T, by the line SR, so as that DT may be one third of ET; and if a weight of three ounces be suspended at D, and another of one ounce at E, the same things will happen as in the former experiment; for the line SR being placed vertical there will be a balance; and upon moving it out of that position the balance will be destroyed. Exp. 9.

If the crooked lever FCD be so placed on it's prop at C, as that the arm CF may be parallel to the Pl. 4. Fig. 7.

 the

LECT. VIII. the plane of the horizon, and the arm CD inclined thereto; if two weights as B and A appended at D and F, be in the reciprocal proportion of the perpendicular diſtances of their lines of direction from the prop; that is, if B be to A as FC to EC there will be a balance; for as long as the arm CF continues parallel to the horizon, the weight B hanging from the point D acts in the ſame manner in oppoſition to the weight A, as if it hung from E the extremity of the ſtrait lever FC continued on to E, in which caſe the weight B that balances the weight A muſt bear the ſame proportion to it that FC does to EC; if therefore the arm DC be bent in ſuch a manner, as that EC may be one half or one third of FC in the former caſe a weight of two ounces, and in the latter one of three ounces hanging from D will be counterpoiſed by one ounce hanging from F.

If by moving the lever, the arm FC be put out of it's parallel poſition, the balance will be deſtroyed; for that cannot be preſerved, unleſs the diſtance of B's direction from the prop continues to bear the ſame proportion to the diſtance of A's direction, that EC does to FC; which in this caſe is impoſſible; for firſt, if the point F be moved upward towards H, and of courſe the point D downward towards G, it is manifeſt, that the diſtances of both directions will be leſſened; but the decreaſe of EC in a given time will bear a greater proportion to the decreaſe of FC, than EC does to FC; for by that time the point D has moved from D to G through the arch DG, which meaſures the angle of CD's inclination, EC will vaniſh; whereas FC cannot vaniſh till ſuch time as the point F has moved from F to M through the quadrantal arch FM; but in the ſame time that the point D moves from D to G through the arch DG, the point F can move only from F to H through the arch FH ſimilar to DG; which arch being always leſs than the quadrant, the perpen-

perpendicular diſtance of A's direction from the prop, to wit FC, will not vaniſh upon the arrival of the point F at H, that is, it will not vaniſh ſo ſoon as EC; conſequently, the decreaſe of EC in a given time muſt bear a greater proportion to the decreaſe of FC, than EC does to FC: wherefore EC as diminiſhed in any given time, will be to FC as diminiſhed in the ſame time, in a leſs proportion than that of EC to FC; or in other words, the perpendicular diſtance of B's direction from the prop will bear a leſs proportion to the perpendicular diſtance of A's direction, than EC does to FC; and therefore, the weight A will preponderate. If the point F be moved downward, and conſequently D upward, it is manifeſt from the inſpection of the figure, that the diſtance of A's direction from the prop continually diminiſhes, at the ſame time that the diſtance of B's direction increaſes; and therefore the weight B muſt in that caſe overbalance the weight A.

LECT. VIII.

If FCD be a crooked lever placed as the laſt, and if a weight, inſtead of being hung from the arm DC, be laid thereon at D, and by a vertical plane, as HK, ſet cloſe to it, be hindered from falling off; from the point D whereon the weight reſts, let the line DE be drawn perpendicular to the arm FC continued on towards G; the weight at D will be balanced by the weight A hanging freely from F, provided the weight D be to the weight A, in a *ratio* compounded of EC to CD and of FC to CD; that is, as a rectangle under EC and FC the perpendicular diſtances of the directions of the two weights from the prop, to the ſquare of CD the inclined arm of the lever. For whatever be the moment wherewith the weight A preſſes down the arm FC, the arm DC muſt with an equal moment be preſſed upward, and with it the weight D in the direction DG perpendicular to

Pl. 4. Fig. 8.

 CD:

LECT. VIII. CD; and forasmuch as the same weight presses perpendicularly against HK the vertical plane, it must be pressed backward by the same in an horizontal direction; and at the same time it must have a tendency downward from the force of gravity in the direction ED; so that it is acted upon by three forces in the directions DG, GE and ED; in order therefore to a balance, the forces must be as the sides of the triangle DGE; and the force of gravity which presses it in the direction ED, must be to the force pressing it in the direction DG, as ED to DG, or, because the triangles DGE and CDE are similar, as EC to CD; but as the force which presses it in the direction DG is of equal moment with the weight A, that force must be to the weight A, as FC to CD; consequently, the force of gravity in the weight D must be to the force of gravity in the weight A, that is, the weight D must be to the weight A, in a *ratio* compounded of EC to CD and of CF to CD, or as the rectangle under CE and CF to the square of CD. To confirm

Exp. 10. this by experiment, let a crooked lever as FCD consist of equal arms, and let it be bent in such a manner as that EC may be to CD, as one to two; and let a weight of one ounce be laid on at D, and another of two ounces be hung from F, and they will balance each other; for in this case the product of EC which is as one, into CF which is as two, will be two; and CD being as two, the square thereof will be four; so that the rectangle under EC and CF, is to the square of CD, as two to four, or as one to two; in which proportion therefore the balancing weights must be.

Exp. 11. All things remaining as in the last experiment, excepting that the arm CF is as long again as CD, so that EC, CD, and CF are as one, two, and four; a weight of one ounce at D will be balanced by one ounce hanging freely from F; for CD being

ing as two, it's ſquare is four; and the product of EC, which is as one into FC, which is as four, is likewiſe four. LECT. VIII.

In wheels turned by the force of water falling upon them from an height, and which on that account are commonly called overſhot wheels, the moving power is partly the percuſſive force of the water which falls into the uppermoſt bucket, and partly the gravity of the water contained in the other buckets, which are lodged on the rim of the uppermoſt quarter of the deſcending part of the wheel; and the effects which theſe forces have upon the wheel are greater or leſs in proportion to their abſolute quantities, and the diſtances of their lines of direction from the center of the wheel. Thus, where AIOP repreſents an overſhot wheel, C it's center, K, L, M, N, four buckets fixed on the uppermoſt quarter of the deſcending part of the wheel; AB the direction of the water flowing into the uppermoſt bucket K, CB the perpendicular diſtance of that line from the center C; DE, FG, and HI, the lines of direction of the centers of gravity of the ſeveral portions of water contained in the buckets L, M, N; CE, CG, and CI, the perpendicular diſtances of thoſe lines from the center C. The force of the water flowing into K is proportional to the quantity flowing in in a given time, as alſo to the velocity wherewith it flows, and the diſtance of it's line of direction from the center; and therefore, where the quantity and velocity are given the force will be as BC the perpendicular diſtance of AB the line of direction, from C the center of motion; conſequently, the nearer AB approaches to the tangent in the point A, or the more obliquely the water flows in upon the wheel, the greater will it's force be. The portions of water contained in the buckets L, M, N, have different forces according to their different quantities, and the different diſtances of their lines of direction from the center Exp. 12. Pl. 4. Fig. 9.

LECT. VIII. ter C, their quantities being greateſt, when the diſtances of their directions are leaſt, for the buckets empty as they deſcend; ſo that their force leſſens as they deſcend, by reaſon of the diminution of their quantities, but at the ſame time it likewiſe increaſes on account of the increaſe of the diſtance of their lines of direction from the center of motion; ſo that upon the whole, the force in each bucket may be looked upon as invariable; but whether this be ſo or not, certain it is, that if the wheel be truly centered, and the buckets be equal and alike, and if the water flows in uniformly, the whole moving force muſt continue the ſame as long as the wheel continues to move; and ſince it acts inceſſantly, the motion of the wheel muſt be continually accelerated, and that uniformly; and thus it would be, were it not that when the wheel arrives at a certain degree of velocity, the reſiſtance which is given becomes ſo great as to deſtroy the increments of motion as faſt as they are generated by the moving force; by which means the wheel is made to revolve with one uniform velocity, which is the greateſt that can be given it by that moving power.

Pl. 4. Fig. 10. A plane as AB placed obliquely to BC, which repreſents an horizontal plane, is called an inclined plane; the angle ABC is called the angle of elevation, and it's complement BAC the angle of inclination, the line AC perpendicular to BC is called the height of the plane, and AB it's length. If a weight as P be laid on an inclined plane as AB, and be thereon ſuſtained by a power acting in a direction, as PF, parallel to the inclined plane; in order to a balance, the ſuſtaining power muſt be the weight, as the height of the plane to the length thereof, that is, as AC to AB, or, putting BA for the radius, as the ſine of the angle of elevation to radius; for the weight P is acted upon by three powers in different directions, the firſt of which is the force of

gravity,

gravity, which preſſes it downward in the direction PD perpendicular to BC; the ſecond is the power which draws it in the direction PF parallel to BA; and the third is the plane BA, which does as it were preſs it upward in the direction PH perpendicular to BA; for as the weight P preſſes the plane in a direction perpendicular thereto, it is reacted upon by the plane in a contrary direction. If therefore the line EG be drawn parallel to PD, the ſides of the triangle PEG will be proportional to the three powers, and the force which ſupports the weight on the inclined plane, and which acts in the direction PF, will be to the abſolute weight of the body acting in the direction PD parallel to GE, as PG to GE; but inaſmuch as the triangles PEG and CBA are ſimilar, as PG is to GE, ſo is AC to AB; conſequently, the power neceſſary to ſupport a weight on an inclined plane muſt bear the ſame proportion to the weight ſuſtained, that the height of the plane does to it's length; which is confirmed by experiments; for if a weight of four ounces be laid on a plane whoſe length is to it's perpendicular height, as two to one, it will be counterbalanced by a weight of two ounces, provided the whole gravity thereof be made to act in drawing the other weight in a direction parallel to the inclined plane, which may be done by faſtning one end of a cord to the greater weight, and then ſtretching the cord along the plane, ſo as to keep it parallel thereto, and paſſing it over a pulley at the top of the plane; for the ſmaller weight being tied to the end of the cord which lies beyond the pulley will hang freely, and for that reaſon acts with all it's gravity in a direction parallel to the plane.

LECT. VIII.

Exp. 13.

The ſame weight of four ounces being laid on an inclined plane whoſe length is to it's height as four to one will be ſuſtained by a weight of one ounce hanging freely as before.

Exp. 14.

The

LECT. VIII. The force wherewith a body reſting on an inclined plane preſſes the ſame, is to the weight of the body, as the ſine of the angle of inclination to radius; for in the triangle PEG, PE denotes the force wherewith the body preſſes the plane, and GE the weight of the body; but from the ſimilarity of the triangles, as PE is to GE, ſo is BC to BA; and putting BA for the radius, BC is the ſine of BAC the angle of inclination; wherefore as BC the ſine of the inclination is to the radius AB, ſo is the force wherewith the body preſſes the plane to the abſolute weight of the body. Hence, if upon an inclined lever as AB, reſting on the two props A and B, a weight be laid any where as at P, it will be eaſy to determine what proportion of the weight each prop bears; for drawing the horizontal line AE equal in length to AB, and from the point P whereon the weight reſts letting fall PD perpendicular to AE, if AE be ſuppoſed to denote the whole weight of the body AD will denote that part of it which is ſuſtained by the uppermoſt prop, and DE that part which is ſupported by the lower; for if the lever was horizontal, ſo as that the body might preſs it with all it's gravity, the whole weight of the body would be to that part of it which preſſes the prop B, as BA to PA, as is evident from what has been ſaid concerning the ſecond kind of lever; but as in the inclined poſition of the lever the whole weight of the body does not preſs upon it, that part of the weight which the prop B ſuſtains in the horizontal poſition, muſt be to the part ſuſtained in the inclined poſition, in the ſame proportion with the abſolute weight of the body to the force wherewith it preſſes the inclined plane, that is, as PA to AD; for putting PA for the radius, AD is the ſine of the inclination of the lever; conſequently the whole weight of the body muſt be to that part which preſſes on the prop B in the inclined poſition of the lever, in a *ratio*

Pl. 4. Fig. 11.

compounded

compounded of B A to P A, and of P A to A D, that is, it muſt be as B A to A D, or becauſe A B and A E are equal, as A E to A D; and of conſequence the part ſupported by the other prop A muſt be as D E.

Hence it follows, that if two perſons carry a load fixed upon a lever, the load being placed between them, which is the caſe of chairmen, upon deſcents the foremoſt man will bear the greateſt burthen, and upon aſcents the hindermoſt. It likewiſe follows, that in coaches and all other four-wheel carriages which have the foremoſt wheels ſmaller than thoſe behind, the load muſt be thrown more upon the former than the latter; what effect this has upon the draught, ſhall be ſhewn in my next lecture.

LECTURE IX.

OF FRICTION.

LECT. IX.

IN my laſt lecture I ſhewed you what force is requiſite to ſuſtain a body on an inclined plane. If a body be laid on a plane parallel to the horizon, it does not ſtand in need of any force to ſupport it; for as the direction of gravity is perpendicular to the plane of the horizon, the whole weight of the body muſt be ſuſtained by the horizontal plane whereon it reſts: whence it follows, that if any power endeavours to move a body reſting on an horizontal plane in a direction parallel to the plane, it will meet with no reſiſtance from the weight of the body, that being entirely taken off by the reaction of the plane whereon the body preſſes; but a reſiſtance will ariſe from the attrition of the body againſt the plane; for the ſurfaces of all bodies whatever, even ſuch as are of the fineſt poliſh, being in ſome

meaſure

LECT. IX. meaſure rough and unequal, (as is evident from the obſervations that have been made by the help of microſcopes) when a body is moved upon a plane, the prominent parts both of the body and plane muſt of neceſſity fall into each others cavities, and thereby create a reſiſtance to the motion of the body, inaſmuch as the body cannot be moved unleſs the prominent parts thereof be continually raiſed above the prominent parts of the ſurface whereon it ſlides; and this cannot be done unleſs the whole body be at the ſame time lifted up, and as it were raiſed on an inclined plane equal in height to the forementioned protuberant parts; upon which account the moving power muſt ſuſtain ſome part of the weight of the body, even in moving it along an horizontal plane. But as this is occaſioned by the inequalities in the ſurface, if thoſe were intirely taken off, ſo as to leave the ſurface perfectly ſmooth and even, the reſiſtance ariſing from friction would likewiſe be removed; and ſetting aſide the reſiſtance of the medium, the ſmalleſt force would be ſufficient to move the moſt ponderous body along an horizontal plane. But ſince there are not in nature any bodies, whoſe ſurfaces are perfectly equal; there will ever be ſome reſiſtance ariſing from friction; which reſiſtance will remain unvaried whatever be the magnitude of the ſurfaces that rub one againſt the other, provided the weight which preſſes thoſe ſurfaces together, as alſo the roughneſs of the ſurfaces continue the ſame; for the ſame weight will ever require the ſame force to raiſe it over prominences of a given height, whatever be the magnitude of the ſurface whereon the weight reſts; conſequently, the quantity of reſiſtance will not be varied by varying the magnitude of the ſurface; which may be confirmed by the following experiment.

Exp. 1. Let four pieces of poliſhed box be laid on a poliſhed horizontal plane, and let each piece be ſo loaded as that it's own weight,together with that of

of it's load may be 6685 grains, and let the baſis of one be two inches long, and half an inch broad, and thoſe of the other three, be each four inches in length, but let their breadths be half an inch, an inch, and an inch and an half, ſo that the magnitudes of the baſes may be as one, two, four, and ſix; let then a ſmall cord be faſtened to the end of each piece, and by paſſing over a pulley, be kept in a poſition parallel to the plane, and a weight of 2030 grains hanging from the end of the cord which lies beyond the pulley, will juſt ſuffice to move each piece along the plane; ſo that the reſiſtance ariſing from friction is the ſame in each piece, notwithſtanding the different magnitudes of the ſurfaces whereon they reſt.

LECT. IX.

Exp. 2.

If the roughneſs of the ſurfaces whereon the bodies move be given, the reſiſtance ariſing from friction will vary with the weights of the bodies, and be proportional thereto; for if a certain force be ſufficient to raiſe a certain weight over prominences of a given height, it is manifeſt that a double or triple weight will require a double or triple force to raiſe it to the ſame height. If therefore the pieces of box be ſo loaded, as that each of them with it's load may weigh 13370 grains, that is, as much again as in the laſt experiment, a weight of 4060 grains, that is twice as much as before, will be neceſſary to move them along the ſame plane.

Exp. 3.

If the roughneſs of the ſurface whereon a body moves be increaſed, the reſiſtance will likewiſe increaſe, though the weight of the body remains the ſame; but as the degree of roughneſs in any ſurface cannot otherwiſe be determined than by experiment, ſo neither can the reſiſtance ariſing therefrom; if the plane made uſe of in the laſt experiments be thinly covered with fine ſand, the reſiſtance will thereby become greater in the proportion of about five to four; for the ſame pieces of box which were ſet a going by 2030 grains when the

 plane

LECT. IX. plane was free from ſand, will in this caſe require 2500 grains, that is about one fourth more.

To avoid as much as poſſible the reſiſtance ariſing from friction, which in rough and uneven roads muſt needs be very great, WHEEL CARRIAGES have been contrived; the advantages whereof I ſhall endeavour to explain to you, but I ſhall firſt ſhew you from what cauſe it is that wheels turn round during their progreſſive motion along a plane. If a wheel as ACB playing freely on the axis at A, be lifted off the plane BD by a power applied to the axle, and be carried in any direction whatever, it will not revolve about the axle; for ſince in all wheels that are truly made the axle paſſes through the center of gravity, it is evident, that in this caſe the wheel is ſuſpended by it's center of gravity, and of conſequence will not of it ſelf change it's poſition, but each point thereof will deſcribe a line parallel to the direction of the moving power without any rotation about the axle, in the very ſame manner as if the wheel was fixed to the axle; but if one point of the wheel as B reſts upon the plane BD, and if a power applied to the axle draws the wheel in any direction as AP, ſo as to move it along the plane BD; the motion of the point B will be retarded by the reſiſtance ariſing from friction, whilſt the point C, which meets with no reſiſtance, is carried forward without any retardation of it's motion, and conſequently muſt move forward faſter than the point B; but as all the parts of the wheel cohere, the point C cannot move forward faſter than the point B, unleſs the wheel revolves about it's axis from C towards E; and as the ſeveral points of the wheel's circumference, which are ſucceſſively applied to the plane, ſuffer a retardation in their motion whilſt the oppoſite points move freely, the wheel, during it's progreſſive motion along the plane, muſt continue to revolve about it's axle.

Pl. 4. Fig. 12.

By

By this rotation of wheels about their axles, the resistance arising from friction is very much diminished, and draughts thereby rendered more easy; for in plain roads, where the height of the prominent parts is inconsiderable with respect to the diameter of the wheel; the parts of the revolving wheel which apply themselves successively to the road, may be looked upon in some measure as descending upon the minute prominences, and of course must pass over them without any considerable friction. And so much is the resistance arising from friction diminished in wheel carriages, that if upon the same plane whereon the pieces of box were drawn, a carriage be laid with four equal wheels, each three quarters of an inch in diameter, and loaded in such a manner, as that the weight of the carriage and load may amount to 6685 grains, which was the weight of each piece of box with it's load, it will be set a going by a weight of 420 grains drawing it horizontally, whereas 2030 grains were requisite to move the pieces of box along the same plane.

LECT. IX.

Exp. 4.

From this experiment it appears, that the friction is very much lessened by means of wheels; which diminution is not to be attributed to the wheel's touching the plane in a few points, as may possibly be imagined, but to the rotation of the wheels, for if the wheels of a carriage loaded as before be made fast to the axle, so as not to revolve in their motion, 2030 grains will be necessary to set the carriage a going, that is just as much as was requisite to move the pieces of box.

Exp. 5.

As wheel carriages in general meet with less resistance in their motion than any other, so those of larger wheels, *cæteris paribus*, are less resisted than those of smaller; for the proof whereof, it will be necessary to premise two LEMMAS; the first of which is, *that the secants of angles are to one another inversly as the sines of their complements, that is* AD, *which is the secant of* BAD; *is to* AC, *which is the secant*

Pl. 4. Fig. 13.

LECT. IX. *secant of* BAC, *as* AF, *which is the sine of the complement of* BAC, *to* AH, *which is the sine of the complement of* BAD. For from the nature of similar triangles AD is to AC, as AE to AK, that is, as AE to AG; but AE is to AG, as AF to AH; consequently, AD is to AC, as AF to AH.

The second LEMMA is, *that if two arches of unequal circles have their versed sines equal, the arch of the lesser circle is greater in proportion to the whole periphery, than the arch of the greater circle; or in other words, the angle measured by the arch of the lesser circle, is greater than the angle measured by the arch of the greater circle.* Let HF and DB be two arches of unequal circles, whose versed sines FG and BC are equal; I say, the angle HEF is greater than the angle BAD; for since EF is less than AB, and GF and CB are equal, EF is to GF in a less proportion than AB to CB; consequently, EG is to EF in a less proportion than AC to AB, that is the sine of the angle GHE, is less than the sine of the angle CDA, and of course the angle GHE is less than the angle CDA; consequently, the angle HEG, which is the complement of the lesser angle GHE, is greater than the angle DAC, which is the complement of the greater angle CDA.

Pl. 4. Fig. 14.

Pl. 4. Fig. 15.

These two LEMMAS being premised, let HM represent a plane whereon move the two wheels ABH and KLR, which are of different magnitudes, but equal in weight, and let BC and LM be two obstacles of equal heights, and of such a nature as that the wheels cannot otherwise pass than by surmounting those obstacles; the forces requisite to draw the larger wheel over the obstacle BC, is less than what is requisite to draw the lesser wheel over the obstacle LM equal in height to the former; for since the wheels revolve in passing over the points B and L, their centers of gravity A and K may be looked upon as revolving about the fixed points B and L, and describing the arches AF and KP; consequently,

consequently, the forces which move the wheels may be looked upon as drawing them upon inclined planes, whose directions coincide with the directions of the curves in the points A and K, that is, they coincide with the tangents AE and KO; which tangents being parallel to the tangents of the wheels in the points B and L, that is to say, to DB and NL; the centers of gravity of the two wheels, and consequently, the wheels themselves may be looked upon as drawn up the inclined planes DB and NL; but since the wheels are supposed to be equal in weight, the forces which support them on the inclined planes DB and NL, the height whereof is given, must be to one another inversly as the lengths of the planes; that is, the force which supports the larger wheel on the plane DB, must be to the force supporting the smaller wheel on the plane NL, as NL to DB; that is, putting BC or LM for the radius, as the secant of the angle NLM to the secant of the angle DBC; or, because KS and LM, as also AI and BC are parallel, as the secant of the angle KSL to the secant of the angle AIB; but, from the nature of similar triangles, the angle KSL is equal to the angle KLQ, as is also the angle AIB to the angle ABG; and therefore the force which sustains the greater wheel on the inclined plane DB, is to the force sustaining the lesser wheel on the inclined plane NL, as the secant of the angle KLQ to the secant of the angle ABG; but, by the first *Lemma*, the secant of KLQ is to the secant of ABG, as the sine of BAG to the sine of LKQ; and, by the second *Lemma*, the sine of BAG is less than the sine of LKQ; consequently, the force which raises the greater wheel over the obstacle BC, is less than the force which raises the lesser wheel over the obstacle LM equal in height to the former; but the forces requisite to make the wheels surmount the obstacles are the measures of the resistances, and therefore,

K *cæteris*

LECT. IX. *cæteris paribus*, the greater wheel muſt meet with leſs reſiſtance from the ſame obſtacle than the ſmaller. To confirm this by an experiment; let an obſtacle one tenth of an inch in height be fixed on an horizontal plane, and cloſe behind it let there be placed the carriage with four equal wheels, each three quarters of an inch in diameter, and if it be loaded in ſuch a manner as that the weight of the carriage and load may amount to 6685 grains, it will not be raiſed above the obſtacle by leſs than 2850 grains drawing it in a direction parallel to the plane; whereas if four wheels, each an inch and an half in diameter, be fitted to the ſame carriage, the weight of the whole being the ſame as before, it will be raiſed above the obſtacle by 2050 grains, that is, by 800 grains leſs than were requiſite to raiſe it with the ſmaller wheels.

Exp. 6.

From this experiment it appears, that the reſiſtance which larger wheels meet with in ſurmounting obſtacles, is leſs than the reſiſtance given to ſmaller wheels by the ſame obſtacles; and from what has been demonſtrated it is evident, that the reſiſtance given to the greater wheel is to the reſiſtance given to the ſmaller, as the ſine of an angle meaſured by an arch of the greater wheel, to the ſine of an angle meaſured by an arch of the ſmaller wheel, the verſed ſine of each angle being equal to the height of the obſtacle; ſo that putting R and r for the radii of the two wheels, and x for the verſed ſine or the height of the obſtacle, it follows from the nature of the circle, that as $\frac{\sqrt{2Rx-xx}}{R}$ is to $\frac{\sqrt{2rx-xx}}{r}$, ſo is the reſiſtance given to the larger wheel to the reſiſtance given to the ſmaller, or dividing by $x^{\frac{1}{2}}$, as $\frac{\sqrt{2R-x}}{R}$ to $\frac{\sqrt{2r-x}}{r}$; but as the proportion

proportion of these sines is not fixed, but varies with the height of the obstacle, so likewise must the proportion, which the resistance given to the greater wheel bears to the resistance given to the smaller; and all that can be determined in this case is, that larger wheels ever meet with less resistance in surmounting obstacles than smaller; and that the disproportion between the resistances suffered by each wheel, increases with the height of the obstacle. Indeed where the obstacle vanishes, which is the case when wheels move upon planes, the expressions for the resistances, and consequently the resistances themselves, are as $\frac{1}{\sqrt{R}}$ and $\frac{1}{\sqrt{r}}$, that is, the resistances are inversly as the square roots of the semidiameters of the wheels; so that where the heights of the wheels are as one and two, the forces requisite to draw them along the same horizontal plane, are as fourteen and ten, that is, inversly as the square roots of one and two, which is confirmed by experiments; for whereas the carriage whose wheels are three quarters of an inch in diameter, required 420 grains to move it along the horizontal plane, the weight of the carriage and load being 6685 grains; the carriage whose wheels are $1\frac{1}{2}$ inch in diameter, when loaded in the same manner, will be set a going by 300 grains; but 420 is to 300, as 14 to 10, that is, the forces requisite to move the two carriages along the same plane, are inversly as the square roots of the heights of the weights.

LECT. IX.

If the nature of the obstacle be such, as to be bore down by the pressure of the wheel, the larger wheel will in this respect likewise have the advantage over the smaller, and depress the obstacle with greater force. For let LK be continued to T, so that TL may be equal to AB; and since the wheels are supposed to be equally weighty, let AB and TL ex-

Pl. 4. Fig. 13. 15

LECT. IX. T L expreſs the abſolute forces of the two wheels acting againſt the obſtacles in the directions AB and KL; it is evident from what has been ſaid concerning the reſolution of forces, that the force denoted by A B may be reſolved into two forces; one whereof may be denoted by AG, and the other by G B, whereof A G alone acts in depreſſing the obſtacle B C, inaſmuch as it bears directly down upon it; whereas the other force denoted by GB, inaſmuch as it's direction is perpendicular to the obſtacle, may thruſt it forward, but can contribute nothing towards preſſing it downward from B towards C. In like manner the force denoted by T L, is reſolvable into two forces, which may be denoted by T V and V L, whereof T V alone acts in depreſſing the obſtacle E M; conſequently, the force wherewith the greater wheel depreſſes the obſtacle, is to the force wherewith it is depreſſed by the leſſer, as A G to T V, or as the ſine of the angle A B G to the ſine of the angle T L V or K L Q; but by the ſecond *Lemma*, the angle BAG, which is the complement of G B A, is leſs than the angle L K Q the complement of KLQ; conſequently the angle ABG is greater than TLV, and AG the ſine of the former greater than TV the ſine of the latter; but as A G is to TV, ſo is the depreſſing force of the greater wheel to the depreſſing force of the leſſer; conſequently, the ſame obſtacle is more eaſily depreſſed by the larger wheel than the ſmaller, and of courſe muſt give leſs reſiſtance to the former than to the latter.

If the obſtacle be ſuch, as that it can neither be ſurmounted nor depreſſed, but muſt be driven forward, then indeed the ſmaller wheel has the advantage of the larger; for the forces of the wheels being reſolved as before, the lines GB and VL will expreſs the forces which act in driving the obſtacle forward; but it has been demonſtrated, that GB the ſine of the angle G A B, is leſs than V L the ſine of the angle

angle V T L equal to Q K L; and therefore the force wherewith the greater wheel propels the obſtacle, is leſs than the force wherewith the ſmaller wheel propels the ſame; beſides, as the greater wheel preſſes the obſtacle directly downward with a greater force than the ſmaller, the reſiſtance made by the ſame obſtacle to the propelling force of the larger wheel, will be greater than what is made to the propelling force of the ſmaller; ſo that where the obſtacle is to be propelled, the ſmaller wheel is preferable to the larger; but as in draughts this is rarely if at all the caſe, the obſtacles which are commonly met with in roads being ſuch as muſt either be ſurmounted or depreſſed by the wheels, ſuch wheels are to be preferred as beſt ſerve both thoſe purpoſes, and thoſe I have ſhewn to be the larger wheels; which likewiſe are attended with other advantages beſides what have been already mentioned; for firſt, it frequently happens in rough and uneven roads, that two obſtacles are placed ſo near each other, that before the wheel has quitted one it meets with the other, and reſting upon each, hangs between them; in which caſe the ſmaller the wheel is, the lower it deſcends between the obſtacles, and thereby renders the draught more difficult; inaſmuch as it muſt be raiſed to a greater height in order to paſs over the foremoſt obſtacle, than when the wheel is larger: For the illuſtration of which, let F E and H G repreſent two obſtacles placed at ſo ſmall a diſtance, that the wheel having ſurmounted the firſt but not quitted it, may meet with the ſecond, ſo as to hang between them; it is manifeſt, that as the arch FDH of the leſſer wheel, which lies between the obſtacles, has a greater curvature than F B H the arch of the greater wheel, which lies between the ſame obſtacles, the point D muſt deſcend lower than the point B; conſequently, the ſmaller wheel muſt be raiſed to a greater height than the larger, in order to paſs over the ſame obſtacle;

LECT. IX.

Pl. 5. Fig. 1.

LECT. IX. ftacle; and therefore a greater force will be neceffary to pull up the fmaller wheel, than what is requifite to raife the larger; which is confirmed by the

Exp. 7. following experiment. Let a carriage with four wheels each an inch and an half in diameter, and fo loaded as that the weight of the carriage and load may amount to 6685 grains, be fo placed on the plane before made ufe of, as that the two foremoft wheels may hang between two obftacles whofe diftance is half an inch, and their height likewife half an inch, and a weight of 1150 grains drawing the carriage horizontally, will move the wheels from between the obftacles; whereas if four fmaller wheels be made ufe of each three quarters of an inch in diameter, a weight of 2700 grains will be requifite to draw them from between the obftacles.

As wheels cannot always run upon the nail, but muft frequently meet with heavy roads, they will fink down, and thereby render the draught more difficult; but the larger the wheels are, the lefs *cæteris paribus* will the depth be to which they fink. For if ABC denotes the plane of the road, and if it be of fuch a nature as to fuffer the fmaller wheel to fink down as far as E; it is manifeft that the gravity of the wheel muft overcome the refiftance of as much of the earth whereon it preffes, as is equal to the fegment HED; for it cannot otherwife fink, than by forcing fuch a quantity of the earth out of it's place; and fhould the larger wheel fink to the fame depth, the gravity thereof muft overcome the refiftance of as much earth as is equal to the fegment AEC, that is, it muft overcome a greater refiftance in order to fink to the fame depth with the fmaller; but it cannot poffibly overcome a greater refiftance, becaufe it is fuppofed to have the fame gravity with the fmaller; confequently, it will not fink as deep as the fmaller, and for that reafon will make the draught lefs troublefom.

As

As large wheels have the advantage of ſmall ones with regard to the reſiſtance ariſing from the obſtacles and impediments in the roads, ſo have they likewiſe in relation to the reſiſtance occaſioned by the friction of the box againſt the arm of the axle; not that this reſiſtance is leſs in greater wheels than in ſmaller; for ſince it is not varied by varying the magnitude of the ſurface, as has been ſhewn, if the boxes and arms are truly fitted and of an equal ſmoothneſs, and the weights whereby the arms and boxes are preſſed together be equal, the quantity of reſiſtance will be given, whatever be the magnitude of the wheels, as alſo of the arms of the axle whereon they play; but where the arms of the axles are of equal diameters, (which is commonly the caſe in one and the ſame carriage, though the wheels be unequal) a leſs force is requiſite to overcome the given reſiſtance in a larger wheel than in a ſmaller; for in this caſe the ſemidiameter of the wheel may be looked upon as a lever, whoſe prop or fixed point is at the center of the arm, and the impediment ariſing from the friction of the box againſt the arm may be looked upon as a weight placed upon the lever at the diſtance of the arm's ſemidiameter from the prop, whilſt the moving power is applied to the extremity of the wheels ſemidiameter; and therefore in order to a balance, the power muſt be to the reſiſtance, as the ſemidiameter of the arm to the ſemidiameter of the wheel; ſince then the impediment is given, as alſo the diſtance thereof from the prop, it is evident, that the larger the lever is, and conſequently, the larger the wheel, the leſs is the force requiſite to overcome the reſiſtance. Thus, if B E F repreſents the circumference of the arm of an axle, whereon the wheels AGH and DIK revolve, C the center of the arm, BC it's ſemidiameter, DC the ſemidiameter of the ſmaller wheel, and AC that of the larger; in the

LECT. IX.

Pl. 5. Fig. 2.

LECT. IX.

bigger wheel the length of the lever is AC, and in the fmaller DC; fince therefore the fame impediment is in both levers placed at the fame diftance from the prop C, to wit at B, it will be balanced by a lefs force at A than at D; and the force at A is to the force at D, as DC to AC, that is, inverfly as the femidiameters of the wheels; for the force at A is as BC applied to AC, and the force at D is as the fame BC applied to DC; that is, the force at A which balances the refiftance at B, is to the force at D which balances the fame refiftance, as BC divided by AC, to BC divided by DC, that is, multiplying croffwife, and throwing out BC, as DC to AC. Whence it follows, that when the femidiameter of the arm is given, the more the wheel is enlarged, the lefs will the force be that is requifite to overcome the refiftance arifing from the friction of the wheel againft the arm; fo that upon this account as well as the former, large wheels are to be preferred to fmall ones.

In order to leffen the refiftance arifing from the friction of the box againft the arm of the axle, there has been a late contrivance, whereby the axle, contrary to what is ufual in moft carriages, is made to revolve, and it's arms, inftead of preffing againft the boxes, are made to bear on the circumferences of moveable wheels, which wheels from their ufe in diminifhing the friction, are by the author of this contrivance called *friction wheels*. Now that fuch wheels, where they can be made ufe of, do take off much of the refiftance occafioned by friction, will

Exp. 8.

appear from the following experiments; from the axle of the machine called the axle in the wheel, in which the diameter of the wheel is to the diameter of the axle, as nine to one, let a weight of 23163 grains be hung, and a weight of 2770 grains hanging at the circumference of the wheel, will turn the machine, provided the axle turns on the circumferences

rences of two moveable wheels; whereas if it turns in the pevets it will be neceſſary to add 600 grains more, ſo as to make the whole 3370 grains; conſequently, the reſiſtance occaſioned by friction in the latter caſe, is more than four-fold what it is in the former; for ſince the diameter of the wheel is nine times as great as that of the axle, a weight of 2574 grains at the wheel is requiſite to balance the weight of 23163 at the axle, which balancing weight being deducted from 2770, and likewiſe from 3370 grains, leaves 196 grains for overcoming the reſiſtance in one caſe, and 796 in the other; but 796 is to 196, as four and a little more to one. LECT. IX.

Again, let a ſmall cart with friction wheels be ſo loaded, as that it's own weight added to that of the load, may amount to 20000 grains; and a weight of 54 grains drawing horizontally, will move it along a ſmooth level table, whereas, if the friction wheels be taken off, 322 grains will be neceſſary to ſet it a going. If the cart be ſo loaded as that the weight of the whole may amount to 40000 grains, then in each caſe, a double force will be requiſite to move it, that is to ſay, 108 grains with the friction wheels, and 644 without them; ſo that in this cart the friction wheels take off five parts in ſix of the reſiſtance; for 54 is but a ſixth part of 322, as is likewiſe 108 of 644. And from theſe experiments it does again appear, that under like circumſtances the reſiſtance ariſing from friction, is proportional to the weight, whereby the ſurfaces which rub one againſt the other are preſſed together.

Seeing then that great wheels have in ſo many reſpects the advantage over ſmall ones, it will not be improper in this place to ſhew you, on what account it is, that the wheels of common carrs, as alſo the foremoſt wheels of coaches, chariots, and moſt other four-wheel carriages, are commonly made ſo ſmall as ſeldom to exceed two feet and an half in diameter; and the firſt reaſon of this contrivance

is

LECT. IX. is for the convenience of turning; for as in moſt roads, but more eſpecially ſuch as are narrow, there are windings of ſuch a nature as to allow but a ſmall ſpace for carriages to turn in, it is neceſſary to make uſe of ſuch wheels as can turn in the narroweſt compaſs, and ſuch are ſmall ones; for it is a thing well known to carters, and all others who are uſed to drive wheel carriages, that the larger the wheels are, the greater compaſs do they require in order to turn with eaſe and ſafety; and ſhould they at any time attempt to turn carriages with large wheels as ſhort as thoſe which have ſmaller, the wheels will drag, and thereby render the draught very difficult, and ſometimes endanger the overſetting of the carriage.

But the ſecond, and indeed the principal reaſon for the uſe of ſmall wheels is, that upon aſcents, and in paſſing over obſtacles in rough and hilly roads, as little of the horſe's force may be loſt as poſſible; if roads were level and ſmooth without riſings or impediments, the moſt convenient ſize for wheels, ſeting aſide the neceſſity of turning, would be where the axle is upon a level with the breaſt of the horſe; for ſince the whole force of the horſe in drawing is applied to that part of the tackle which lies upon the breaſt, and to which the traces are joined; and ſince the traces are faſtened to the carriage in ſuch a manner, as that being continued they muſt paſs through the axle of the foremoſt wheels, it is manifeſt, that if that axle be of an equal height with the cheſt of the horſe, the traces, in whoſe plane the line of direction lies, will be parallel to the road whereon the carriage is drawn; conſequently, the whole force of the horſe will be employed in drawing the carriage directly forward, without any loſs or diminution; whereas if the wheels be of ſuch a ſize as that the height of the axle is either greater or leſs than that of the horſe's cheſt, the whole force of the horſe will not be employed in the direct draught; but in the

former

former cafe, fome part of the force will be fpent in preffing the carriage directly downward, and in the latter, in lifting the fame directly upward. For the proof and illuftration whereof, let the firft of the three wheels be of fuch a fize, as that it's axle A may be of an equal height with the horfe's breaft at B; and let the fecond wheel be fo large as that it's axle A may ftand higher than the horfe's cheft at B, and in the third, let the axle be lower than the breaft of the horfe; and in each wheel let the lines of direction of the horfe's draught, to wit a AB be taken equal, and let each of thofe lines exprefs the force of the horfe; it is manifeft, that in the firft wheel, the whole force denoted by AB, is imployed without any lofs in drawing the wheel forward, becaufe the line of direction AB, wherein the force draws, is parallel to EF, the road whereon the wheel moves; whereas, in the fecond and third wheels the lines AB, wherein the forces draw, being inclined to EF, whereon the wheels move, fome part of each force muft be loft; for if each force denoted by AB be refolved into two, to wit CB and AC, whereof CB is parallel to EF, and AC perpendicular thereto; it is evident, that that force alone which is denoted by CB, acts in moving the wheel forward along EF, whilft the force denoted by AC does in the fecond wheel prefs it directly downward againft the road, and in the third lifts it directly upward; whence it follows, that if the force of a horfe be juft fufficient to move the firft wheel, it will not fuffice to ftir the fecond or third. It likewife follows, that if the wheel be fo far inlarged, as that the angle which the line of direction AB makes with the plane EF, approaches nearly to a right one, the line CB will bear a very fmall proportion to AB, whilft AC becomes nearly equal thereto; fo that almoft the whole of the horfe's force will be fpent in preffing down, and thereby increafing the load; whence it appears, that

LECT. IX.

Pl. 5. Fig. 4.

LECT. IX. that notwithstanding the several advantages arising from the largeness of wheels, yet may they be so far increased, as even upon account of their magnitude to render the draught impossible. By the use of small wheels whose axles lie below the level of the horse's chest, provision has been made against the inconvenience last mentioned, and the loss of force (which by reason of the roughness and inequalities of roads cannot wholly be avoided) has been rendered as little as possible, and made to obtain chiefly in level smooth roads, where there is least occasion for the whole force; whereas upon ascents, and in passing over obstacles in rough roads, where the stress is greatest, there little of the force is *lost*; for the proof of which, let the wheel be of such a size, that it's axle A may be below the horse's breast at B, and let AB, as before, denote the force of the horse; if the wheel be drawn along a smooth level road as EF, CB will express that part of the force which draws the wheel along the road, and AC that part of the force which is imployed in lifting up the wheel, which part is lost as to the draught, but however, is not intirely useless; because, by pulling the wheel directly upward, it eases the load, and thereby renders the draught less difficult; though at the same time the draught is by no means as easy as it would be, if the force of the horse was applied at G, so as to draw in the direction AG parallel to EF. If the wheel instead of moving along a smooth road, be to pass over the obstacle DH, or which is the same thing, if it be to be drawn up the ascent EHL; and if the force of the horse be applied at G, so as that the direction of the draught AG may be parallel to EF, and consequently, inclined to EHL; it is manifest upon resolving the force AG into two forces, to wit AK and KG, whereof AK is parallel, and KG perpendicular to EHL; that force alone which is expressed by AK, acts in drawing the wheel up EHL; whereas the force expressed by

Pl. 5. Fig. 5.

KG acts in preffing the wheel directly againft EHL, and thereby adds to the weight of the wheel; fo that in this cafe, fome part of the horfe's force is loft, and the load at the fame time increafed, both which inconveniencies are avoided where the breaft of the horfe is fo far elevated above the axle of the wheel, as that the line of direction A B may be parallel to E H L; for then no part of the horfe's force will be loft, but the whole will be imployed in drawing the wheel directly over the obftacle, or up the afcent; fo that a lefs force will be requifite to draw the wheel over the obftacle D H in the direction AB, than in the direction AG; and this is fully confirmed by experiments. For whereas the little carriage with four wheels, each three quarters of an inch in diameter, being fo loaded as that the weight of the carriage and load amounted to 6685 grains, was not drawn over the little obftacle one tenth of an inch in height, by lefs than 2850 grains acting in an horizontal direction, it will be drawn over by 2450 grains provided the direction be made parallel to the tangents of the wheels in thofe points which touch the obftacle; and 1950 grains will be fufficient to draw the carriage with the larger wheels over the fame obftacle, if the direction of the draught be made parallel to the forementioned tangents, whereas 2050 grains were neceffary when the direction was parallel to the horizontal plane. And if the direction be ftill farther removed from the parallelifm of the tangents, which may be done by depreffing it below the horizontal plane, the force of 2350 grains will be but juft fufficient to furmount the obftacle, and draw the carriage over.

LECT. IX.

Exp. 9.

Though in four wheel carriages, the contrivance of fmall wheels before has it's advantages, yet is it not intirely free from inconveniencies; for by this means the load muft of neceffity be thrown forward, and [illegible] a greater

a greater ſtreſs laid on the foremoſt wheels; whereby the reſiſtance that ariſes from the friction of the axle againſt the wheels will become greater in the foremoſt than in the hindmoſt wheels, in proportion to the greater weight which they ſuſtain. Beſides as the ſpaces deſcribed by wheels in each revolution are nearly equal to the peripheries of the wheels, it is manifeſt that the foremoſt wheels muſt revolve oftener than the hindmoſt, in order to rid the ſame ground. And this frequency of turning requiſite in the foremoſt wheels joined to the greater ſtreſs upon them from the load, as alſo to the greater reſiſtance which they meet with from obſtacles in the road, is the true reaſon why they are more frequently out of order, and ſtand in need of repair much oftener than thoſe behind.

LECTURE X.

MOTION OF BODIES DOWN INCLINED PLANES.

LECT. X. MY deſign in this lecture is to explain the chief properties of the PENDULUM; and in order thereto, I ſhall lay down the following PROPOSITIONS concerning the motion of bodies down inclined planes and curve ſurfaces.

Pl. 5. Fig. 6. PROP. I. *The force wherewith a body deſcends upon an inclined plane, as* AC, *is to the abſolute force of gravity wherewith the ſame body falls freely and perpendicularly, as the height of the plane to the length thereof, that is, as* AB *to* AC.

For it has been proved, that the force requiſite to ſuſtain a body upon an inclined plane, is to the abſolute weight of the body, as the height of the plane

plane to it's length; but the force wherewith a body endeavours to defcend upon an inclined plane, muft be equal to the force which is neceffary to fupport it upon that plane; confequently, the propofition is true.

COROL. I. Hence it follows, that the motion of a body defcending on an inclined plane is uniformly accelerated; for fince the force which carries a body down an inclined plane, has every where, and in all parts of the plane the fame proportion to the abfolute weight of the body, and fince the abfolute weight remains unvaried, the other force muft do fo too; confequently, as it acts inceffantly in equal times, it makes equal impreffions on the defcending body, fo as to generate equal degrees of velocity in the motion thereof; that is, in other words, the motion of a body defcending on an inclined plane is uniformly accelerated.

COROL. II. On account of this uniform acceleration of the motion, the times of defcending, as alfo the velocities acquired at the end of the defcent, are as the fquare roots of the fpaces defcribed, as in the cafe of bodies falling freely; that is to fay, the time wherein a body defcends upon the inclined plane from A to D, is to the time of the defcent, from A to C, as the fquare root of AD, to the fquare root of AC; and the velocity of the body when it has defcended as far as D, is to the velocity thereof when it arrives at C in the fame proportion of the root of AD to the root of AC.

PROP. II. *The velocity acquired in any given time by a body defcending on an inclined plane, is to the velocity acquired in the fame time by a body falling freely and perpendicularly, as the height of the plane to it's length, that is, as* AB *to* AC.

For, by the firft *corollary* of the foregoing *propofition*, the motion of a body down an inclined plane is

LECT. X. is uniformly accelerated, in the ſame manner as the motion of a body falling freely; conſequently, at the end of any given time, the velocities acquired muſt be as the accelerating forces; but by the foregoing *propoſition*, the accelerating force of a body moving down an inclined plane as AC, is to the accelerating force of a body falling freely and perpendicularly, as the height of the plane to it's length; and therefore the velocities acquired in any given time muſt be in the ſame proportion.

PROP. III. *The ſpaces deſcribed in a given time by two bodies moving from a ſtate of reſt, whereof one deſcends on an inclined plane, and the other falls freely, are in the ſame* ratio *of the height of the plane to it's length; that is, the ſpace deſcribed by a body moving along* A C, *is to the ſpace deſcribed by a body falling down the perpendicular* A B, *as* A B *to* A C.

For where the motions are equable, the ſpaces deſcribed in a given time, are as the velocities wherewith they are deſcribed; if therefore the velocities be increaſed in a conſtant uniform manner, the ſpaces deſcribed will likewiſe increaſe in the ſame manner; but by the ſecond *propoſition*, the velocities are augmented in ſuch a manner as in a given time to bear the ſame proportion to one another, as the height of the plane does to it's length; conſequently, the ſpaces deſcribed in a given time muſt be in that proportion.

COROL. I. If from B, the line BD be drawn perpendicular to AC, AD will be the ſpace deſcribed by a body moving down the plane AC, in the ſame time that a body falls freely down the height of the plane from A to B.

For, from the nature of ſimilar triangles, AC is to AB, as AB to AD; but by the *propoſition*, as AC is to AB, ſo is the ſpace deſcribed in a given time by a body falling freely, to the ſpace deſcribed

by

by a body defcending upon the inclined plane AC; confequently, fince AB is the fpace defcribed by the body falling freely, AD muft be the fpace defcribed in the fame time by a body defcending along AC. LECT. X.

COROL. II. All the chords of a circle are defcribed in the fame time by bodies running down them. For if a circle be defcribed with the diameter AB, which is the height of the inclined plane AC, the point D, which determines the fpace AD through which a body defcends upon the inclined plane, whilft another falls freely from A to B, will be in the periphery of the circle, becaufe the angle ADB in the femicircle is always a right one; and for the fame reafon, if the height of the plane continuing the fame, the inclination thereof be varied, fo as that it may become AG, the point E which determines the fpace AE, through which a body moves along the plane AG, during the time of a body's fall from A to B, will likewife be in the periphery of the circle; confequently, in the femicircle ADB all the chords as AD and AE will be defcribed in the fame time; and as in the femicircle AFB, whatever chords as BF and BH are drawn through the point B, other chords as AD and AE may be drawn in the other femicircle parallel thereto and equal; it follows, that whether a body falls freely down the diameter AB, or whether it defcends along a chord as HB or FB, it will in the fame time arrive at the loweft point of the circle; or in other words, all the chords of a circle will be defcribed in equal times by bodies running along them. Pl. 5. Fig. 7.

PROP. IV. *The time wherein a body moves down an inclined plane as* AC, *is to the time wherein a body falls freely down* AB *the height of the plane, as the length of the plane to it's height, that is, the times are as the fpaces defcribed.* Pl. 5. Fig. 6.

 For

LECT. X. For by the second *Corol.* of the first *Prop.* the time of a body's motion along the inclined plane from A to C, is to the time of it's motion from A to D, as the square root of AC to the square root of AD; but by the second *Corol.* of the third *Prop.* the time of a body's motion along the inclined plane from A to D, is equal to the time of the fall from A to B; and therefore the time of the motion along the plane from A to C, is to the time of the perpendicular fall from A to B, as the square root of AC, to the square root of AD, that is, because from the similarity of triangles AC, AB, and AD are in continued proportion, as AC to AB, or as the length of the plane to it's height.

COROL. Hence it follows, that if several inclined planes have equal altitudes, the times wherein those planes are described by bodies running down them, are to one another as the lengths of the planes.

Pl. 5. Fig. 7. For the time of the descent along AC, is to the time of the fall down AB, as AC to AB, and the time of the fall down AB, is to the time of the descent along AG, as AB to AG; consequently, the time of the descent from A to C, is to the time of the descent from A to G, as AC to AG, that is, the times are as the lengths of the planes.

Pl. 5. Fig. 6. PROP. V. *The velocity acquired at the end of the fall, by a body falling down the perpendicular height of an inclined plane as* AB, *is equal to the velocity acquired at the end of the descent by a body moving down the inclined plane, from* A *to* C.

For by the first *Prop.* the accelerating force of a body falling freely from A to B, is to the accelerating force of a body moving along the plane AC, as AC to AB; and by the fourth *Prop.* as AB is to

to AC, ſo is the time of the fall from A to B, to the time of the deſcent from A to C; ſo that the forces which accelerate the bodies during their motions, are to one another, reciprocally as the times that they continue to act; conſequently, at the end of thoſe times, the velocities generated muſt be equal. For inſtance, if AB be one half of AC, the force which accelerates the body in it's fall from A to B, is to the force which accelerates the body in it's deſcent from A to C, as two to one; but the time that a body takes to fall from A to B, is but one half of the time that a body takes to deſcend from A to C; ſo that the accelerating force which acts upon the body during it's motion from A to C, though it be but one half of the accelerating force which acts upon the body during it's fall from A to B, yet does it continue to act twice as long; and therefore muſt in the end produce the ſame velocity. LECT. X.

Corol. Hence it follows, that the velocities acquired by bodies in falling down inclined planes, are equal where the heights of the planes are equal.

For, the velocity acquired in falling from A to C, is equal to the velocity acquired in falling from A to B, as is alſo the velocity acquired in falling from A to G; conſequently, the velocities acquired in falling from A to C, and from A to G, are equal. Pl. 5. Fig. 7.

Prop. VI. *If a body deſcends along ſeveral contiguous planes as* AB, BC, *and* CD, *the velocity which it acquires in it's deſcent from* A *to* D, *is equal to the velocity acquired by the perpendicular fall from* H *to* D, *on ſuppoſition that the body is not retarded by the ſhocks it ſuffers in the angles* B *and* C. Pl. 5. Fig. 8.

For drawing the horizontal lines HE and DF through the points A and D, and producing the planes CB and DC as far as G and E; by the

 Corol.

LECT. X. *Corol.* of the laſt *Propoſition*, the ſame velocity is acquired in the point B, by a body in deſcending from A to B, as in deſcending from G to B; conſequently, the ſame velocity is acquired in the point C, by a body deſcending from A through B to C, as in deſcending from G to C; but by the ſame *Corollary*, the velocity acquired in deſcending from G to C, is equal to the velocity acquired in deſcending from E to C; wherefore, the velocity in the point D, acquired by the deſcent along the three planes AB, BC, and CD, is equal to the velocity acquired by the deſcent from E to D, which velocity by the foregoing *Propoſition*, is equal to the velocity acquired by the perpendicular fall from H to D.

Pl. 5. Fig. 9. COROL. Hence it follows, that if a body deſcends along the arch of a circle as AB or of any other curve, the velocity acquired at the end of the deſcent, is equal to the velocity acquired by falling down CB, the perpendicular height of the arch.

For curves may be looked upon as compoſed of an infinite number of right lines inclined one to another.

Pl. 5. Fig. 10. PROP. VII. *If two planes as* AB *and* BD *joined together at* B, *have equal degrees of elevation with two other planes as* EF *and* FH *joined together at* F, *and if* AB *be to* EF *as* BD *to* FH; *the time of a body's fall down the planes* ABD, *will be to the time of the fall down* EFH, *as the ſquare root of* AB *and* BD *taken together, to the ſquare root of* EF *and* FH *taken together.*

Let AB and EF be produced till BC becomes equal to BD, and FG equal to FH. Since AB is to BC, as EF to FG, AB is to AC, as EF to EG; and ſince thoſe four quantities AB, AC, EF, and EG are proportional, their ſquare roots will be ſo too. Again, ſince the planes AC and EG are equally elevated,

elevated, they may be looked upon as parts of one and the ſame plane, and therefore, by the ſecond *Corol.* of the firſt *Prop.* the time of a body's fall from A to C, is to the time of the fall from E to G, as the ſquare root of AC to the ſquare root of EG, or as the ſquare root of AB to the ſquare root of EF; but the time of a body's fall from A to B, is to the time of the fall from E to F, as the ſquare root of AB to the ſquare root of EF; ſo that the time of the fall from A to C, is to the time of the fall from E to G, in the ſame proportion of the time of the fall from A to B, to the time of the fall from E to F; conſequently, the time of the fall from B to C, ſuppoſing the motion to begin from A, muſt be to the time of the fall from F to G, ſuppoſing the motion to begin from E, in the ſame proportion of the root of AB to the root of EF; if the bodies after their fall from A to B and from E to F, inſtead of moving along BC and FG continue their motions along BD and FH, ſince thoſe two planes are equally inclined to AB and EF, and ſince BD is equal to BC, and FH equal to FG, whatever proportion the time of the body's motion along BD bears to the time of it's motion along BC, the ſame will the time of the motion along FH bear to the time of the motion along FG; but it has been already proved, that the time of the motion along BC, is to the time of the motion along FG, as the ſquare root of AB to the ſquare root of EF; wherefore, the time of the motion along BD, is to the time along FH, as the ſquare root of AB to the ſquare root of EF, that is, in the ſame proportion with the time along AB to the time along EF; and therefore, the ſums of thoſe times will be in the ſame proportion; that is to ſay, the time of the motion along AB, added to the time of the motion along BD, is to the time of the motion along EF, added to the time of the motion along FH, as the ſquare root of AB to the ſquare root of EF; but

LECT. X. it has been proved, that the ſquare root of AB, is to the ſquare root of EF, as the ſquare root of AB and BD taken together, to the ſquare root of EF and FH taken together; and therefore, the time of a body's fall from A through B to D, is to the time of the fall from E through F to H, as the ſquare root of ABD to the ſquare root of EFH, which was to be proved. And what has been thus proved with regard to two planes on each ſide, is in like manner demonſtrable with regard to any number of planes, provided thoſe on one ſide be proportional to thoſe on the other, and that the correſponding planes have equal degrees of elevation.

Pl. 5. Fig. 11. COROL. Hence it follows, that if bodies deſcend through the arches of circles, the times of deſcribing ſimilar arches ſimilarly poſited, are as the ſquare roots of the arches. For inſtance, if bodies move down the ſimilar arches AB and CD, which are ſimilarly poſited with regard to the horizontal plane ED, the time of deſcribing AB, is to the time of deſcribing CD, as the ſquare root of AB to the ſquare root of CD.

For all circles whatever may be conſidered as ſimilar polygons, conſiſting of an indefinite number of ſides indefinitely ſmall; and therefore, ſimilar arches muſt conſiſt of an equal number of ſides proportional the one to the other; and foraſmuch as the angles which thoſe ſides contain are equal, if the arches be ſimilarly poſited, the correſponding ſides in each arch muſt have equal degrees of elevation; and conſequently, the times of deſcribing the arches will be as their ſquare roots.

In my lecture upon gravity, I ſhewed you, that if a body be thrown directly upward, it will riſe to the ſame height whence, if it fell from a ſtate of reſt, it would by the end of the fall acquire the ſame velocity wherewith it is thrown up; I likewiſe ſhewed you, that the time of the riſe is equal to that of the fall. I now ſay,

PROP.

PROP. VIII. *That the ſame things do likewiſe obtain with regard to bodies thrown up obliquely, whether they aſcend upon inclined planes or along the archs of curves.*

Becauſe the ſame forces which accelerate the motions of bodies deſcending on ſuch planes or curves, do in the very ſame manner retard the motions of ſuch bodies as aſcend thereon; and therefore, whatever be the time requiſite for a body to deſcend upon an inclined plane or through the arch of a curve, in order to acquire any velocity, the ſame muſt the time be, wherein that velocity is deſtroyed in a body aſcending upon the ſame plane or curve; and whatever be the length of the plane or curve, through which a body deſcends in order to acquire any velocity, the ſame muſt the length of the plane or curve be, through which it muſt aſcend in order to have that velocity deſtroyed.

COROL. Hence it follows, that if by any contrivance a body be made to deſcend through the arch of a circle as from C to A, and with the velocity acquired by the deſcent to aſcend along the arch AD of the ſame circle, the arch AD which it deſcribes in it's aſcent, will be equal to the arch CA deſcribed in the deſcent; and the times in which thoſe arches are deſcribed will be equal. Pl. 5. Fig. 12.

And this is the caſe of the PENDULUM; which is a heavy body as A, hanging by a ſmall cord as BA, and moveable therewith about the point B, to which the cord is fixed. If when the cord is ſtretched the weight be raiſed as high as C, and thence let fall, it will by it's own gravity deſcend through the circular arch CA; and by the *Corol.* of the ſixth *Prop.* it will have the ſame velocity in the point A, that a body would acquire in falling perpendicularly from E to A; and, by the firſt LAW OF NATURE, it will endeavour to go off with that velocity in the tangent AF; but being by the force of the Exp. 1.

 the

LECT. X. the chord made to move in the periphery CAD, it will riſe through the arch AD as high as D, where loſing all it's velocity, it will be turned back by it's gravity, and deſcending through the arch DA, will, upon it's arrival at A, have the ſame velocity as before, with which it will aſcend to C; and thus it will continue it's motion forward and backward along the curve CAD, which motion is called an *oſcillatory* or *vibratory* motion; and each ſwing from C to D, as alſo from D to C, is called a *vibration*; and if the pendulum ſuffered no retardation in it's motion from the reſiſtance of the air, nor from the friction of the chord againſt the center about which it moves, the arches deſcribed in each vibration would be exactly equal, and the motion of the pendulum would continue for ever; but whereas the motion of the pendulum is continually retarded by the forementioned cauſes, the arches deſcribed in each vibration muſt grow leſs and leſs continually, and at laſt vaniſh together with the motion of the pendulum.

The vibrations of one and the ſame pendulum vibrating in unequal circular arches, are performed very nearly in equal times, provided the arches are but ſmall. Thus, in the pendulum AB, the vibration through the arch CAD, is performed very nearly in the ſame time wherein the pendulum vibrates through the arch EAF, on ſuppoſition that the arches CA and EA are but ſmall.

Pl. 5, Fig. 13.

For, drawing the chords CA and AD, as alſo EA and AF, inaſmuch as the arches are ſuppoſed to be ſmall, they will not differ much either as to length or declivity from their reſpective chords; conſequently, the times of deſcribing the arches CA and EA, by a heavy body running along them, will be nearly equal to the times of deſcribing the chords; but by the ſecond *Corol.* of the third *Prop.* the times of deſcribing the chords are equal; wherefore the times of deſcribing the arches

arches CA and EA, muſt be nearly equal; and ſo likewiſe muſt the double of thoſe times, or the times wherein the pendulum vibrates through the unequal arches CAD and EAF. And this is confirmed by experiment. For if two pendulums of an equal length, be ſet going at the ſame inſtant of time, ſo as to vibrate through ſmall but unequal arches, they will for a long time keep pace together; and continue to begin and end their ſwings without any ſenſible difference as to point of time, during a great number of vibrations. LECT. X. Exp. 2.

If a pendulum as BA, vibrates through the circular arches CAD and EAF, the velocity which it acquires by that time it arrives at the loweſt point A, is as the chord of the arch which it deſcribes in it's deſcent; that is, the velocity which it acquires in deſcending from C to A, is to the velocity acquired in it's deſcent from E to A, as the chord CA to the chord EA. Pl. 5. Fig. 14.

For, drawing the horizontal lines EK and CH, the velocity acquired in falling from H to A, is to the velocity acquired in falling from G to A, in the ſubduplicate *ratio* of HA to GA, as I proved in my lecture upon gravity; that is, becauſe, from the nature of the circle HA, CA, and GA, are in continued proportion, as CA to GA; for the ſame reaſon, the velocity acquired in falling from G to A, is to the velocity acquired in falling from K to A, as GA to EA; conſequently, the velocity acquired in falling from H to A, is to the velocity acquired in falling from K to A, as CA to EA; but by the *Corol.* of the ſixth *Prop.* the velocity acquired in falling from H to A, is equal to the velocity acquired in the deſcent from C to A, and the velocity acquired in falling from K to A, is equal to the velocity acquired in the deſcent from E to A; wherefore, the velocity acquired in deſcending through the arch CA, is to the velocity acquired in deſcending through the arch EA, as the chord CA to the chord EA.

Hence

LECT. X. Hence it appears, that if the arch of a circle wherein a pendulum vibrates, be so divided in the points 1, 2, 3, 4, and so on, beginning from the lowest point A, as that the chords drawn from A to the several points of division, may be to one another, as those numbers, the velocities acquired by a pendulum in the lowest point A, when let fall successively from the several points of division, will be as the numbers affixed to the respective points; and it was upon this account, that in the experiments relating to the collision of bodies, the balls were constantly let fall from such heights, as that the chords of the arches which they described in their descent, might be to one another in the same proportion with the velocities wherewith the balls were supposed to meet at the lowest point.

The times wherein pendulums of unequal lengths vibrating in similar arches, perform their vibrations, are to one another, as the square roots of their lengths; for instance, the time wherein the pendulum BA vibrates through the arch FG, is to the time wherein the pendulum BC vibrates through the arch DE similar to FG, as the square root of BA to the square root of BC.

For, by the *Corol.* of the seventh *Prop.* since the arches FA and DC are similar and similarly posited, the time of the descent through FA, is to the time of the descent through DC, as the square root of FA, to the square root of DC; but by the *Corol.* of the eighth *Prop.* the time of the descent through FA, is one half of the time of the vibration from F to G, and the time of the descent through DC, is one half of the time of the vibration from D to E; consequently, the time of the vibration through FG, is to the time of the vibration through DE, as the square root of FA, to the square root of DC; that is, because the arches FA and DC are similar, as the square root of BA to the square root of BC, that is, the times of the vibra-

tions

tions are as the ſquare roots of the lengths of the pendulums. And foraſmuch as the times wherein pendulums perform their vibrations, are to one another inverſly as the number of vibrations performed in a given time; the numbers of vibrations performed by pendulums in a given time, are to one another inverſly as the ſquare roots of the lengths of the pendulums. For inſtance, if the length of the pendulum BA, be to the length of the pendulum BC, as one to 4, the number of vibrations performed in any given time by the ſhorter pendulum, is to the number of vibrations performed in the ſame time by the longer, as the ſquare root of 4 to the ſquare root of one, that is, as two to one; which caſe is experimentally confirmed by two pendulums, whereof the longer being 39.125 inches, Exp. 3. vibrates in one ſecond of time; and the ſhorter being 9.781 inches, vibrates in half a ſecond, and performs two vibrations in the ſame time that the longer performs one.

		Inches.	
Length of a pendulum vibrating	in a 2d.	39.125	*Halley*
		39.207	*Newton*
	in $\frac{1}{2}$ a 2d.	9.781	*Halley*
		9.801	*Newton*

The time of a pendulum's vibration is no way altered by varying the weight thereof; for ſince the gravity of every body is proportional to it's quantity of matter, as I proved in my lecture upon gravity, all bodies in the ſame circumſtances are moved by the force of gravity with the ſame velocity; and therefore, if the length of a pendulum continues the ſame, it will perform it's vibrations in the ſame time, whatever be the magnitude of the appending weight; which may be confirmed by the following experiment. Let two unequal weights be hung by Exp. 4. two threads ſo as to conſtitute two pendulums equal in length, and let them at the ſame inſtant of time

LECT. X. fall from equal heights, they will keep pace together so as to perform their vibrations in equal times.

In the foregoing part of this lecture I shewed you, that the vibrations of one and the same pendulum, vibrating through unequal but small circular arches, are performed in times that are very nearly, but not precisely, equal. Whence it follows, that however useful such a pendulum may be in measuring time where great exactness is not requisite, yet can it by no means be admitted as an accurate measure of time, unless by some contrivance it be made to perform all it's vibrations in equal arches, which, considering the unavoidable imperfections of all machines, is extremely difficult, if not impossible; for it has been found by experience, that the best regulated pendulum clocks, wherein the greatest care has been taken to make the pendulums vibrate in equal arches, have notwithstanding varied in a course of time, so as to stand in need of a new regulation, which they could not possibly do in case the pendulums, whereon the regularity of all the other movements depends, continued constantly to vibrate in equal arches.

In order therefore to obtain an exact unerring measure of time, it is necessary to make a pendulum vibrate in such a manner, as that all it's swings, whether they be through larger or smaller arches, may be performed in times exactly equal; and this may be done by making a pendulum vibrate in the curve of a cycloid, as I shall now demonstrate; but I shall first shew you the manner wherein that curve is generated, and what it's chief properties are, as also by what contrivance a pendulum is made to vibrate in such a curve.

Pl. 5. Fig. 16. If a circle as CEF, which touches the right line AB in the point C, be moved along that line in the manner of a wheel from C to D, so as to perform an intire revolution; the point C will by virtue of it's double motion describe the curve line

CID, which curve line is called a *cycloid*; and the right line CD is called the *base*, the line IK perpendicular to the base at it's middle point is called the *axis of the cycloid*, and the point I the *vertex*, and the circle CEF or KLI is called the *generating circle*. LECT. X.

From any point in the cycloid as H, let a right line as HL, be drawn parallel to the base CD, and continued till it meets the generating circle KLI, described about the axis IK; and let the line HM touch the cycloid in the point H; this being done, the chief properties of the cycloid are these three.

First, the arch IPL of the generating circle, intercepted between the vertex of the cycloid and the point L, wherein the right line HL meets the generating circle, is equal in length to the right line HL.

Secondly, The chord IL of the circular arch IPL, is parallel to the right line MH, which touches the cycloid in the point H.

Thirdly, the cycloidal arch IH intercepted between the vertex and the point H, is double the chord IL.

The demonstrations of these properties may be seen in HUYGENS, WALLIS, COTES, and others who have wrote of the cycloid.

The contrivance whereby a pendulum is made to vibrate in the curve of a cycloid, is thus. A cycloid as AVB, being described on the base AB, let the axis VD be produced towards C, till DC becomes equal to VD; through the points C and A, and C and B, let two semi-cycloids CA and CB be drawn, each equal to half of AVB, their vertices being at A and B; if then we suppose CA and CB to be two plates of some breadth, and an heavy body to hang from the point C by a string equal in length to CV, and to vibrate between the plates CA and CB, the upper part of the string will constantly Pl. 5. Fig. 17.

stantly

LECT. X. ſtantly apply it ſelf to that plate towards which the body moves, and by ſo doing cauſe it to move in the cycloid AVB, as has been proved by HUYGENS the author of this contrivance; and likewiſe by COTES in his treatiſe *de motu pendulorum*, where he has delivered the whole doctrine of pendulums in four THEOREMS, which I ſhall here lay down and explain.

Pl. 5. Fig. 17. THEOREM I. *If a pendulum vibrating in a cycloid as* BVA, *begins it's motion downward towards* V, *from any point taken at pleaſure as* L, *and if upon a* radius *as* VL, *equal in length to the cycloidal arch* VL, *a circle be deſcribed; the velocities of the pendulum in the ſeveral points of the cycloidal arch, will be as the right ſines in the circle which are raiſed from the correſponding points in the* radius; *for inſtance, if in the* radius LM *be taken equal to* LM *in the cycloid, and from the point* M *in the* radius *correſponding to the point* M *in the cycloid, be raiſed the right ſine* MX, *the velocity of the pendulum in the point* M, *after it has deſcended from* L, *will be as the ſine* MX.

For the proof of which, from the points L and M in the cycloid, let the right lines LOR and MQS be drawn perpendicular to the axis, cutting the generating circle in O and Q, from whence to the vertex, let the right lines OV and QV be drawn. By the *Corol.* of the ſixth *Prop.* the velocity which the pendulum acquires in deſcending along the cycloid from L to M, is equal to the velocity acquired by a body in falling perpendicularly from R to S; but the velocity which a body acquires in falling perpendicularly, is in the ſubduplicate *ratio* of the ſpace deſcribed, as I proved in my lecture upon gravity; conſequently, the velocity acquired by the pendulum in it's deſcent from L to M, may be expreſſed by the ſquare root of RS;

RS; but RS, being equal to the difference between RV and SV, the velocity in the point M may be expreſſed by the ſquare root of the difference between RV and SV; or, becauſe RV multiplied into the axis DV, is to SV multiplied into the ſame DV, as RV to SV, the velocity may be expreſſed by the ſquare root of the difference between the product of RV×DV and SV×DV; but from the nature of the circle, the product of RV×DV is equal to the ſquare of VO; and the product of SV×DV is equal to the ſquare of VQ; wherefore, the velocity at M may be expreſſed by the ſquare root of the difference between the ſquare of VO and the ſquare of VQ; but, by the third property of the cycloid, VO is equal to one half of the cycloidal arch V L, and V Q to one half of the arch V M; wherefore, as V O ſquare, is to V Q ſquare, ſo is VL ſquare, to VM ſquare; conſequently, the velocity of the pendulum at M may be expreſſed by the ſquare root of the difference between the ſquare of V L and the ſquare of V M; but the cycloidal arches V L and V M are by ſuppoſition equal to V L and V M in the *radius* of the circle; and, from the nature of a right-angled triangle, the difference between the ſquare of VX, which is equal to VL, and the ſquare of V M, is equal to the ſquare of MX; wherefore, the velocity of the pendulum at the point M, is as the ſquare root of MX ſquare, that is, as MX, as was aſſerted in the *Theorem*. And what has been thus proved with regard to the velocity at the point M, is in like manner demonſtrable with regard to the velocity at any other point as N; namely, that it is as the right ſine NY raiſed from the point N in the *radius* correſponding to the point N in the cycloid; ſo that the velocities of a pendulum deſcending in a cycloid, are in the ſeveral points of the cycloidal arch, as the right ſines in a circle which are raiſed from the correſponding points of the *radius*, the *radius* being equal

LECT. X.

LECT. X. equal in length to the cycloidal arch intercepted between the vertex and that point from which the pendulum begins it's motion. Thus VM and VN in the *radius*, of the circle, being taken equal to VM and VN in the cycloid, so as that the points M, N, V, in the *radius*, may correspond to the points M, N, V, in the cycloid, the velocities of the pendulum in those points are to one another, as the sines MX, NY, and VZ, the *radius* VZ expressing the greatest velocity at the vertex V.

Pl. 5. Fig. 17. THEOREM II. *If a body be supposed to move uniformly in the curve of the circle, with a velocity equal to the velocity acquired by the pendulum in it's descent from* L *to* V, *which velocity is, as was just now shewn, expressed by the* radius VZ; *any arch of the circle as* XY *taken at pleasure, will be described by the body moving along it in the forementioned manner, in the same time that the pendulum, which begins it's motion from the point* L *in the cycloid, describes the cycloidal arch* MN, *corresponding to and equal in length to* MN, *that part of the* radius, *which lies between the sines* MX *and* NY, *which terminate at the extremities of the circular arch* XY.

Let the sine FGH, be drawn indefinitely near to the sine MX, and let XG be drawn parallel to MF; and let MF in the cycloid be equal to MF in the *radius* of the circle. By the foregoing *Theorem*, the velocity of the pendulum in the point M, is as MX; and therefore, since F is supposed to be indefinitely near to M, the little cycloidal arch MF, equal to MF in the *radius*, is to be looked upon as described by the pendulum with a velocity which is as MX; and the little circular arch XH, is by supposition described with a velocity which is as VZ, equal to VX; and the triangles MXV and GXH being similar, inasmuch as the angles at M and G are right ones, and the angle MXV equal to GXH,

GXH, becauſe GXV is the complement of each of them to a right one; XH is to XG equal to MF, as VX or VZ to MX; that is, XH and MF are to one another, as the velocities wherewith they are deſcribed; conſequently, they muſt be deſcribed in the ſame time. And what has been thus demonſtrated of MF and XH, is in like manner demonſtrable of the ſeveral correſponding parts in the cycloidal arch MN, and circular arch XY; conſequently, the whole cycloidal arch MN, will be deſcribed by the pendulum in the ſame time, that the circular arch XY is deſcribed by a body moving along it uniformly with the velocity expreſſed by VZ; and by the ſame way of reaſoning, the time of deſcribing any other cycloidal arch as LV, is equal to the time of deſcribing the correſponding circular arch LZ.

Corol. As a *Corollary* it follows, that the time wherein a pendulum deſcribes any arch of a cycloid as MN, may be expreſſed by the correſponding circular arch XY.

For, as the motion along the curve of the circle is ſuppoſed to be uniform, the time of deſcribing any arch as XY, muſt be as the length of the arch; but by the *Theorem*, the times of deſcribing the circular arch XY, and the cycloidal arch MN, are equal; conſequently, the time in which the pendulum deſcribes the cycloidal arch MN, is as the circular arch XY.

Theorem III. *The time of one intire vibration of a pendulum moving in a cycloid, is to the time wherein a body falls perpendicularly through a ſpace equal in length to the axis of the cycloid, as the periphery of a circle to it's diameter.*

All things being ſuppoſed as before, the time of deſcribing the ſemicircular periphery LZP with the velocity Pl. 5. Fig. 17.

M

LECT. IX. velocity expreſſed by VZ, is to the time of deſcribing the ſemidiameter LV with the ſame velocity, as the ſemicircular periphery to the ſemidiameter, or as the whole periphery to the diameter; but the time of deſcribing the ſemicircular periphery LZV with the velocity VZ, is equal to the time of an intire vibration; for, by the eighth *Prop.* the time wherein the pendulum deſcribes the cycloidal arch LV, is one half of the time wherein it performs an intire vibration; and by the ſecond *Theorem*, the time wherein a pendulum deſcribes the cycloidal arch LV, is equal to the time wherein the quadrantal arch of the circle, to wit LZ, is deſcribed with the velocity expreſſed by VZ; conſequently, the time of an intire vibration is equal to the time of deſcribing the ſemicircular periphery LZP; and the time of deſcribing the ſemidiameter LV with the velocity VZ, is equal to the time of a body's fall down the height of the axis DV; for, by the ſecond *Corol.* of the third *Prop.* the fall down the axis DV, is performed in the ſame time with the deſcent along the chord OV; and by the eighth *Prop.* the velocity acquired at the end of the deſcent along the chord OV, will in the ſame time with that of the deſcent deſcribe a ſpace equal to twice OV; but by the third property of the cycloid, twice VO is equal to the cycloidal arch LV, which by ſuppoſition is equal to the ſemidiameter VL; and conſequently, the velocity acquired at the end of the deſcent along the chord OV, is ſuch, as will in a time equal to that of the fall down the axis DV, deſcribe the ſemidiameter LV; but, by the *Corol.* of the ſixth *Prop.* the velocity acquired at the end of the deſcent along the chord OV, is equal to the velocity acquired by the pendulum in it's deſcent along the cycloidal arch from L to V, which by the firſt *Theorem* is as VZ; wherefore, the time of deſcribing the ſemidiameter LV with the velocity VZ, is equal

equal to the time of the fall down the axis DV; but it has been already proved, that the time of describing the ſemicircular arch LZP with the velocity VZ, is to the time of deſcribing the ſemidiameter LV with the ſame velocity, as the periphery of the circle to it's diameter; and it has been likewiſe proved, that the time of deſcribing the ſemicircular arch with the velocity VZ, is equal to the time of an intire vibration of the pendulum; conſequently, the time of ſuch a vibration is, to the time of the fall down the axis, as the periphery of the circle to it's diameter. LECT. X.

COROL. From what has been proved it follows, that the time of a vibration of a pendulum moving in a given cycloid is given; or in other words, that all the vibrations of ſuch a pendulum, whether they be in larger or ſmaller arches, are performed in times exactly equal.

For, as it has been proved, that the time of the vibration which begins from the point L is, to the time of the fall down the axis, as the periphery of the circle deſcribed on a *radius* equal to the cycloidal arch VL, to it's diameter; it may in like manner be demonſtrated, that if the vibration begins from any other point as M, the time thereof will bear the ſame proportion to the time of the fall down the axis, that the periphery of a circle deſcribed on a *radius* equal in length to the cycloidal arch VM, does to it's diameter; but the *ratio* of the periphery to the diameter in any one circle, is the ſame with that in any other; wherefore, the times of the vibrations through unequal arches, have all the ſame *ratio*, to the time of the fall down the axis, and of conſequence muſt be equal.

From this equality in the times of the ſwings it is, that this kind of pendulum is preferable to ſuch as vibrate in circular arches, as being a more exact and juſt meaſure of time; a minute of mean or equal time being preciſely meaſured by ſixty ſwings

LECT. X. of a pendulum of this kind, whofe length is equal to three horary feet, which anfwers to 39 inches and one eighth of our meafure, according to Doctor HALLEY; or to 39 inches and one fifth, according to Sir ISAAC NEWTON; and now that I have mentioned *mean* or *equal*, otherwife called *true time*, it will not be improper in this place, to fhew you wherein it differs from that time, which by aftronomers is called *unequal* and *apparent time*.

As time in it felf does not fall under the notice of our fenfes, and as the parts thereof go on in a continued fucceffion one after another, no two exifting together, it is impoffible to difcover the equality or inequality of any two portions of time, by an immediate comparifon of one with the other; and therefore, it was neceffary for thofe who firft thought of diftinguifhing the parts of time, to have recourfe to fomething fenfible, and of a different nature from time, as a meafure thereof. And as nothing feems better fitted to ferve this purpofe, than fuch natural appearances as fall under every man's notice, and at the fame time have frequent returns, it is highly probable, that in the firft ages of the world, men obferving the frequent rifings and fettings of the fun, took the one or the other for their firft meafure of time, calling that portion of time which paffed between two rifings or fettings, which immediately fucceeded each other, by the name of a *day*; in like manner it is rational to fuppofe, that upon obferving the frequent returns of the full and new moons, they made the one or the other their fecond meafure of time, calling that fpace which paffed between two fucceffive new or full moons by the name of a *moon* or *month*. And it is likely, that for fome time they contented themfelves with thefe meafures, without knowing or confidering whether they were exact or not: but in procefs of time, as men became better acquainted with the motions of the heavenly bodies, they dif-

covered

covered ſome irregularities in the apparent motion of the ſun, and of conſequence, an inequality in the natural days which depend on that motion; inaſmuch as the portion of time, which paſſes between the ſun's departure from the plane of any meridian and it's next return thereunto, is not always the ſame. By conſidering the cauſes of this inequality, they were led into a method of making ſuch corrections in the natural days, by adding to ſome, and taking from others, as reduced them all to a mean equal length; each day being made to conſiſt of 24 equal *hours*, each of which is divided into ſixty equal parts called *minutes*, and each of theſe into ſixty others called *ſeconds*, and theſe again into *thirds*, and ſo on in a *ſexageſimal* progreſſion, the parts of each denomination being conſtantly equal among themſelves. And theſe parts of time thus reduced to an equality conſtitute the mean or equal time, as it ſtands diſtinguiſhed by aſtronomers, from the unequal or apparent time, which is meaſured by the apparent motion of the ſun.

In order to have a conſtant meaſure of equal time, HUYGENS contrived a method of adapting pendulums to clocks, whereby their motions are ſo exactly regulated, as that in a clock whoſe movements are rightly adjuſted, the ſeconds, minutes, and hours, are for ſome time pointed out with the greateſt exactneſs; I ſay, for ſome time only, becauſe it is not poſſible that any clock whatever ſhould continue exactly true for a long courſe of time; for as the pendulums of clocks according to HUYGENS's firſt contrivance, and by the general practice of clock-makers at this day, are made to vibrate in circular arches, where the times of the vibrations are not preciſely equal, unleſs the arches through which the pendulum moves be ſo too, if the wheels on account of the thickening of the oil by froſty weather, or from any other cauſe grow more

LECT. X. sluggish, so as to give the weight, which in clocks is the moving power, greater resistance than according to the first adjustment, the force of the crown-wheel upon the palates of the pendulum will likewise be diminished, and of consequence, the pendulum being thrown less forcibly will move through smaller arches than before, and by so doing, will measure out smaller portions of time, the time of sixty swings not amounting to a minute, upon which account the clock must gain, and go too fast. On the other hand, whenever the parts of the movement which rub one against another do, by reason of the thinning of the oil by the heat of the weather, grow more slippery, or from their constant friction become more smooth, so as to give less resistance to the moving power than according to the first adjustment, the crown-wheel acts more forcibly on the pendulum, and causes it to vibrate in larger arches, by which means the time of each swing is inlarged, and of course the clock loses and goes too slow. To remedy these inconveniences, HUYGENS thought of a second method of adapting pendulums to clocks, so as to make them perform their vibrations in cycloidal arches; by which means, though the force of the crown-wheel upon the pendulum should vary, so as to cause it to vibrate sometimes in larger and sometimes in smaller arches, yet will not any variation arise from thence in the times of the vibrations; as is evident from the *Corollary* of the third *Theorem*; so that in clocks whose motions are governed by pendulums vibrating in cycloidal arches, the irregularities arising from the variation of the force of the crown-wheel upon the pendulum are wholly avoided; and yet a clock of this kind will not always go true; for as the pendulum cannot vibrate in the curve of a cycloid, unless the uppermost part of the string does as often as it moves from the perpendicular

pendicular towards either ſide, form itſelf into a cycloidal arch; and as this cannot be done unleſs that part of the ſtring be made of ſilk, or ſome other ſoft and pliable ſubſtance, which as ſuch is apt to imbibe the moiſture of the air; whenever the weather becomes remarkably moiſt, the watery particles, which float in the air, will inſinuate themſelves into the pores of the ſtring, and by ſo doing cauſe it to contract and ſhorten; upon which account, the vibrations of the pendulum will be quickened, as will appear from the next *Theorem*, and the clock will gain. So that neither a clock of this, nor of any other kind can go exactly true for any long courſe of time, which is a thing well known to clock-makers, who have frequently experienced the beſt regulated clocks to vary in the compaſs of a few months, ſome ſeconds from the equation table, ſo as to ſtand in need of a new regulation.

LECT. X.

THEOREM IV. *The times wherein pendulums of different lengths as* CV *and* AB *perform their vibrations, are to one another in the ſame proportion with the ſquare roots of the lengths of the pendulums.*

Pl. 5. Fig. 17. 18.

For, by the third *Theorem*, the time wherein the pendulum CV performs it's vibrations, is to the time wherein a body falls down the axis DV, as the circumference of a circle to it's diameter; and by the ſame *Theorem*, as the circumference of a circle is to the diameter, ſo is the time wherein the pendulum AB performs it's vibrations, to the time wherein a body falls down the axis EB; conſequently, the time wherein the pendulum CV performs it's vibrations, is to the time wherein the pendulum AB performs it's vibrations, as the time of the fall down DV, is to the time of the fall down EB; but as I proved in my lecture upon gravity, the time of the fall down DV, is to the time of the

Pl. 5. Fig. 17.

Pl. 5. Fig. 18.

 fall

LECT. X. fall down EB, as the ſquare root of DV, to the ſquare root of EB; or, becauſe DV is one half of CV, and EB one half of AB, as the ſquare root of CV, to the ſquare root of AB; wherefore, the time wherein the pendulum CV performs it's vibrations, is to the time wherein the pendulum AB performs it's vibrations, as the ſquare root of CV, to the ſquare root of AB, that is, the times are as the ſquare roots of the lengths of the pendulums; ſo that if one pendulum be four times as long as another, the ſhorter will vibrate in half the time, ſo as to perform two vibrations in the ſame time that the longer performs one.

In this *Theorem*, as alſo in every thing elſe that has been hitherto ſaid concerning the pendulum, the force of gravity is ſuppoſed to be given; whence it follows, that if pendulums of different lengths, as Pl. 5. Fig. 17, 18. CV and AB, perform their vibrations in equal times, the force of gravity in ſuch pendulums muſt vary, and that in proportion to the lengths of the pendulums, that is to ſay, the force of gravity in the pendulum CV, muſt be to the force of gravity in the pendulum AB, as CV to AB. For, as the times of the vibrations are ſuppoſed to be equal, the times of the perpendicular falls down the axes DV and EB muſt likewiſe be equal, inaſmuch as they have been proved to be proportional to the times of the vibrations; ſince therefore, forces which act conſtantly and uniformly are to one another as the v locities which they generate in any given time, the force of gravity which carries a body down DV, muſt be to the force of gravity which in the ſame time carries a body down EB, as the velocity acquired at the end of the fall down DV, to the velocity acquired at the end of the fall down EB; but I proved in my lecture upon gravity, that the velocity acquired in falling down DV, is ſuch as will in a ſpace of time equal to that of the fall, carry a body thro' a ſpace equal to twice DV, that is, thro' a ſpace equal to the

the length of the pendulum CV; and therefore, the time being given, the velocity may be expreſſed by the length of the pendulum; and for the ſame reaſon, the velocity acquired in falling down EB, may be expreſſed by the length of the pendulum AB; conſequently, the force of gravity which moves the pendulum CV, is to the force of gravity which acts upon the pendulum AB, as the length of the former, to the length of the latter. Since therefore, it has been found by experience, that a pendulum which vibrates in a ſecond of time under the line, muſt be lengthened as it is removed from the line, and that more and more as it's diſtance therefrom increaſes; it is manifeſt, that the force of gravity is leſs in the æquatorial parts of the earth, than in any other, and that it increaſes continually as the diſtance from the line increaſes, ſo as to be greateſt under the poles; in what proportion this increaſe of gravity is made, and from what cauſe it proceeds, I ſhewed in my lecture upon gravity. LECT. X.

As the ſeveral parts of the cycloidal arch LV, have different inclinations to the plane of the horizon, it is evident, from what has been ſaid concerning the motion of bodies upon inclined planes, that the force which accelerates the motion of a pendulum in it's deſcent from L to V, muſt continually vary; it being greateſt in the point L, and thence continually leſſening as the cycloidal arch ſhortens, till at length in the point V it intirely vaniſhes; and what is particularly remarkable in this caſe is, that the accelerating forces in the ſeveral points of the cycloid, are to one another in the ſame proportion with the cycloidal arches intercepted between the vertex and the reſpective points; for inſtance, the force which accelerates the pendulum in the point L, is to the force which accelerates the ſame in the point M, as the arch LV, to the arch MV. Pl. 5. Fig. 17.

For,

LECT. X.

For, as the points L and M have the ſame directions with their tangents, the accelerating forces in thoſe points muſt be the ſame with the forces which accelerate the motions of bodies deſcending along the tangents; or becauſe the chords OV and QV in the generating circle, are by the ſecond property of the cycloid, parallel to the tangents at L and M, as the forces which accelerate bodies in their deſcent upon the chords OV and QV; but foraſmuch as thoſe accelarating forces act conſtantly and uniformly, they muſt be to one another, as the velocities which they generate in a given time; and therefore, ſince it has been proved, that the chords OV and QV are deſcribed in the ſame time, the accelerating forces are as the velocities acquired at the end of the deſcent along thoſe chords; but it has likewiſe been proved, that thoſe velocities are as the lengths of the chords; conſequently, the force which accelerates a body deſcending along the chord OV, is to the force which accelerates a body deſcending along the chord QV, as OV to QV; but foraſmuch as by the third property of the cycloid, OV is one half of LV, and QV one half of MV, as OV is to QV, ſo is LV to MV; and therefore, the accelerating force along OV, is to the accelerating force along QV, as the cycloidal arch LV, to the arch MV; but it has been proved that the accelerating force along OV, is the ſame with the accelerating force in the point L, and that the accelerating force along QV, is the ſame with the accelerating force in the point M; conſequently, the force at L, is to the force at M, as LV to MV; and what has been thus demonſtrated of the forces at the points L and M, is in like manner demonſtrable of the forces at any other points, ſo that in a pendulum deſcending in the arch of a cycloid, the accelerating force is in every point as the length of the cycloidal arch intercepted between the point and the

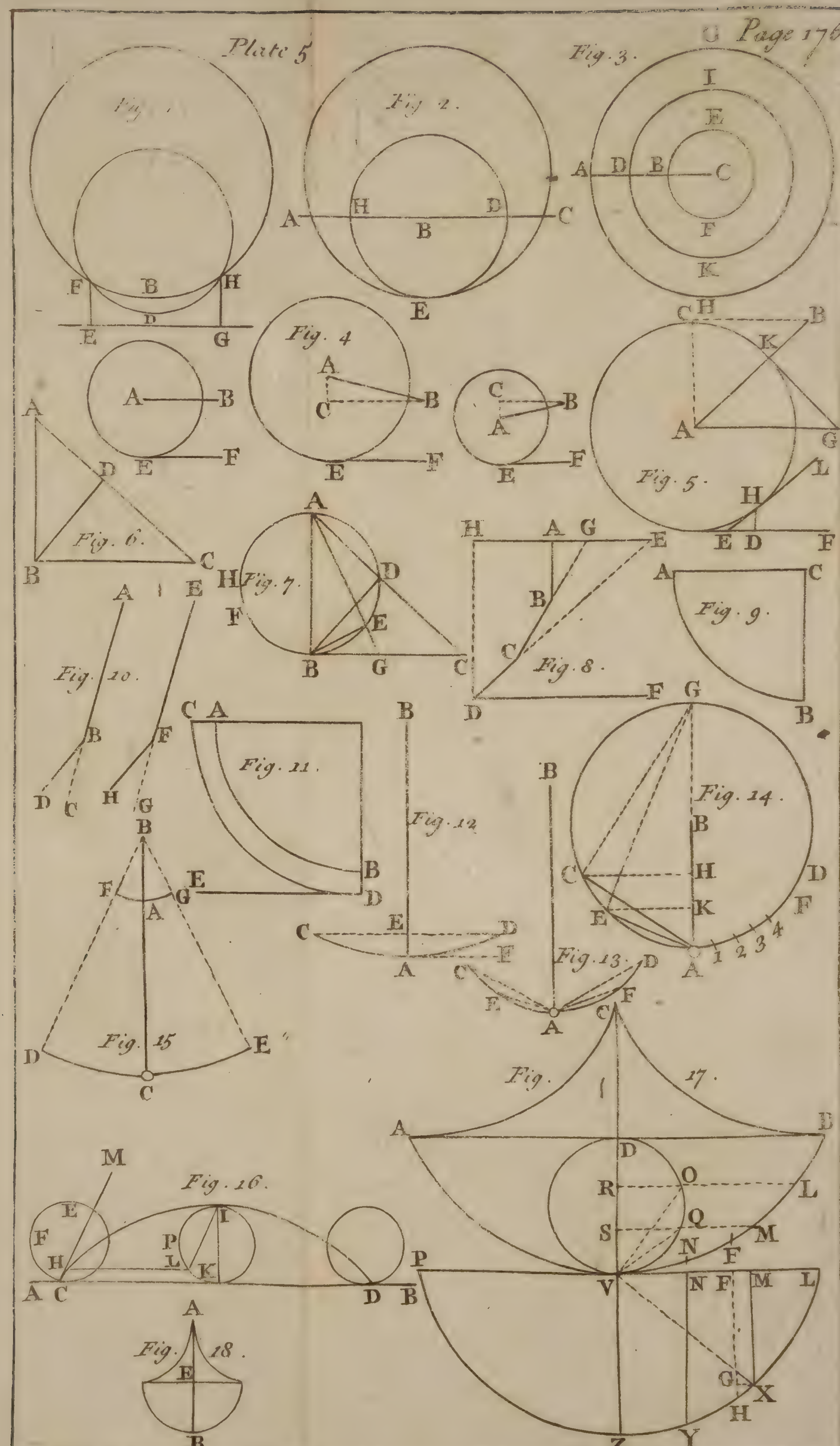


Plate 5
Page 176
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12
Fig. 13.
Fig. 14.
Fig. 15
Fig. 16.
Fig. 17.
Fig. 18.

the vertex; or in other words, the force is every where proportional to the ſpace to be deſcribed.

This then being the law of the accelerating force, and it having been proved, that the pendulum, whether it begins it's motion from L or M, or any other point in the cycloid, will arrive in the ſame time at the loweſt point V; it follows, that if ſeveral bodies, placed at different diſtances from any point or center, begin to move towards it at the ſame inſtant of time, with forces that are every where proportional to the diſtances from the center, they will all arrive at the center at the ſame inſtant of time; which I thought fit to mention in this place, in order to avoid the trouble of demonſtrating the ſame, when I come to treat of the motions of muſical ſtrings, towards the explaining of which this property will be of uſe.

LECTURE XI.

OF THE MOTION OF PROJECTS.

AS the DOCTRINE OF PROJECTS, whereof I intend to treat in this lecture, cannot be rightly apprehended without ſome knowledge of the *parabola*; I ſhall by way of introduction ſhew the manner wherein that curve is generated, and point out ſuch of it's properties as I ſhall have occaſion to make uſe of in explaining the motion of projects, referring you for their demonſtrations to thoſe authors who have wrote of the *conick ſections*. LECT. XI.

If a cone as ABC, be touched by a plane in the right line AB, and be cut by another plane parallel to the former, the curve which ariſes from the interſection of the plane with the ſurface of the cone is called a *parabola*; being ſuch as is repreſented in *Fig.* 2, in which the higheſt point P is called the *principal vertex*; Pl. 6, Fig. 1. Pl. 6. Fig. 2.

LECT. XI. *tex*; the right line CAP paſſing through the point P, and perpendicular to the tangent at that point, is called the *axis*; a right line as DA, drawn from any point in the curve perpendicular to the axis, is called an *ordinate to the axis*; PA the part of the axis intercepted between the vertex and the ordinate, is called the *abſciſſe to that ordinate*; a right line, being a third proportional to the abſciſſe and it's reſpective ordinate, is called the *principal parameter*, or the *parameter to the axis*; a right line as DEH, drawn from any point in the curve parallel to the axis, is called a *diameter*; a right line as PE, intercepted between any point in the curve and the diameter, and parallel to BD which touches the curve in the point D, is called an *ordinate to that diameter*; DE the part of the diameter lying between the vertex D and the point E, is called the *abſciſſe to the ordinate* PE; and a right line, being a third proportional to the abſciſſe DE and the reſpective ordinate EP, is called the *parameter to the diameter* DH, or *to the vertex* D.

The ſquare of any ordinate divided by the reſpective abſciſſe, is equal to the reſpective parameter; thus the ſquare of DA divided by PA, or the ſquare of OQ divided by PQ, is equal to the principal parameter; and the ſquare of EP divided by DE, as alſo the ſquare of LM divided by DL, is equal to the parameter belonging to the vertex D. The ſquares of the ordinates to the axis, or to one and the ſame diameter, are to one another in the ſame proportion with their reſpective abſciſſa's. Thus the ſquare of DA is to the ſquare of OQ, as PA to PQ; and the ſquare of PE is to the ſquare of ML, as DE to DL.

In one and the ſame *parabola*, the principal parameter is the leaſt of all the parameters; and the other parameters increaſe, as the diſtance of their vertices from the principal vertex increaſes, though not in the ſame proportion.

If

If from any point in a *parabola* as D, an ordinate be drawn to the axis, and if from the ſame point a tangent be drawn upward, it will meet the axis when produced; and AB, the part of the axis intercepted between the ordinate DA and the tangent DB, will be biſected by P the principal vertex.

Theſe things being premiſed; if a body be thrown in any direction whatever that is not perpendicular to the plane of the horizon, it will in it's motion deſcribe a *parabola*.

For the proof of which, let AE be the direction of the projection, which in the 3d Fig. is parallel to the horizon, and in the 4th and 5th inclined thereto; and let AE be the ſpace which the project would deſcribe in any given time by means of the force impreſſed, ſuppoſing it had no motion downward from the force of gravity; likewiſe let AB be the ſpace through which it would deſcend in the given time by virtue of it's own gravity, ſuppoſing it had no other motion; then compleating the parallelogram ABCE, it is manifeſt from what was formerly ſaid concerning the compoſition of motion, that at the end of the given time, the project muſt by virtue of it's double motion, be found in the point C; but, foraſmuch as the motion impreſſed in the direction AE is uniform, the ſpace deſcribed, that is AE, muſt be as the time in which it is deſcribed; conſequently, AE ſquare, or BC ſquare, is as the ſquare of the time; but AB or EC, which is the ſpace deſcribed in the ſame time by the force of gravity is likewiſe as the ſquare of the time, as I proved in my lecture upon gravity; conſequently, AB is as the ſquare of BC; and therefore, from the nature of the *parabola*, the point C through which the project moves, muſt be in the curve of a *parabola*, whoſe diameter is AB, the vertex A, the point from whence the project begins it's motion, and the

Pl. 6. Fig. 3. 4. 5.

 parameter

LECT. XI. parameter belonging to that vertex, BC ſquare divided by AB, or AE ſquare divided by EC; and what has been thus demonſtrated of the point C, is in like manner demonſtrable of all the other points through which the project moves; conſequently, the line which it deſcribes is a *parabola*.

The velocity of a project in any point of the *parabola* as A, is ſuch as a body acquires in falling down the fourth part of the parameter belonging to that point. For the velocity of the project in the point A is ſuch, as would carry it from A to E in the ſame time that a body deſcends from E to C; and the velocity acquired in the deſcent from E to C is ſuch as, in the ſame ſpace of time with that of the fall, would carry a body through a ſpace equal to double EC; conſequently, that velocity is to the velocity of the project in the point A, as twice EC to AE, or as EC to $\frac{1}{2}$ AE; but as EC is to $\frac{1}{2}$ AE, ſo is the velocity acquired in falling from E to C, to the velocity acquired in falling down the fourth part of the parameter belonging to the vertex A; for, by the nature of the *parabola*, the parameter belonging to the vertex A, is equal to $\frac{AEq}{EC}$; wherefore, the velocity acquired in falling from E to C, is to the velocity acquired in falling down the fourth part of the parameter, as the ſquare root of EC to the ſquare root of $\frac{\frac{1}{4}AEq}{EC}$, which ſquare roots are to one another, as EC to $\frac{1}{2}$AE, as may appear by multiplying each into the ſquare root of EC; ſo that the velocity acquired in falling through a fourth part of the parameter belonging to the vertex A, and the velocity of the project in the point A, have one and the ſame proportion to the velocity acquired in falling from E to C; conſequently, from the nature of proportionals, thoſe two velocities muſt be equal.

 Hence

Hence it follows, that if projects move through the fame or different *parabolas*, the fquares of their velocities in the feveral points of the *parabolas* are to one another, as the parameters belonging to the refpective points; for, fince the velocities in the feveral points are equal to the velocities acquired in falling down the fourth part of the parameters belonging to thofe points, and fince the fquares of the velocities acquired in falling down the fourth part of the parameters, are to one another as the fpaces defcribed, as I proved in my lecture upon gravity, it is evident that the fquares of the velocities wherewith projects move through the feveral points of the *parabolas* which they defcribe, are to one another in the fame proportion with the quarter parts of the parameters belonging to thofe points; but the quarter parts of the parameters being to one another as the whole parameters, the fquares of the velocities in the feveral points of the *parabolas* muft bear the fame proportion to one another, that the parameters do which belong to thofe points.

Since this is the cafe, and fince by the nature of the *parabola* the principal parameter is lefs than any other, and that the other parameters grow larger as the points to which they belong are more diftant from the principal vertex; if a project be caft obliquely upward, as in *Fig*. 4. from A towards E, it's velocity muft continually decreafe as it rifes and approaches the uppermoft point P, wherein the velocity being leaft muft thence increafe continually as the project defcends and recedes from the point P; and as in one and the fame *parabola*, where the diftances of any two points, as A and C, from the principal vertex P, are equal, the parameters belonging to thofe points are likewife equal; it is manifeft, that a project muft have equal velocities in thofe points; and of confequence, fetting afide any difference which may arife from the refiftance of the air (Pl. 6. Fig. 4.)

the

LECT. XI. the project will, *cæteris paribus*, ſtrike a mark as forcibly in the point K as it does at it's firſt ſetting out in the point A.

The velocity wherewith a project is thrown being given, the velocity thereof in any point of the curve may be thus determined. In the *parabola* of *Fig.* 3. let the axis BA be continued upward to D, ſo as that DB may equal the height from which a body muſt fall, in order to acquire the ſame velocity wherewith the project ſets out from G; then from any point in the curve taken at pleaſure as K, let the horizontal line KL be drawn, and the velocity of the project in the point K, will be to the velocity wherewith it began it's motion from G, as the ſquare root of DL, to the ſquare root of DB. For, in my lecture upon gravity, I proved, that if a body be thrown directly upward from B towards D, with the ſame velocity that it acquires in falling from D to B, it will in any point of it's aſcent as L, have the ſame velocity that it would acquire in falling from D to that point; but the velocity acquired in the deſcent from D to L, is to the velocity acquired in the deſcent from D to B (which velocity is by ſuppoſition equal to the velocity wherewith the body is thrown up) as the ſquare root of DL, to the ſquare root of DB; and by the eighth *Prop.* of my laſt lecture, the velocity of the project at K, is the ſame with the velocity at L; conſequently, the velocity thereof at K, is to the velocity wherewith it ſet out from G, as the ſquare root of DL, to the ſquare root of DB. Whence it follows, that if DB be equal to 1600 feet, and DL to 400, the velocity of the project at K, is but one half of the velocity which it had at it's ſetting out from G; and if DL be equal to 900 feet, then is the velocity at K, three fourths of the velocity at G; ſo that a project being thrown obliquely upward with ſuch a velocity as would carry it to the height of 1600 feet if thrown directly upward,

will

will lose a fourth part of it's velocity by the time it has risen to the perpendicular height of 700 feet, and one half of it's velocity when it has risen 500 feet more. LECT. XI.

The velocity wherewith a project is thrown from any given place being given, as also the position of a mark, the directions wherein the project must be thrown, in order to hit the mark, may be determined in the following manner.

Let A be the place from whence the project is thrown, C the mark situated in the line AC whose length is given, as also the angle CAB, which it makes with the horizontal line AB; at A erect the perpendicular AP, equal to the parameter belonging to the point A, which parameter is given, inasmuch as the velocity wherewith the project is cast from the point A is given; for it is equal to four times the height from which a body must fall in order to acquire that velocity. Let AP be bissected by the line KH, cutting it perpendicularly in G; at A erect AK perpendicular to AC, and let it be continued till it meets KH. From the point of concourse K, with the *radius* KA, let the circle AHP be described. This being done, let a right line as BCEI, be erected perpendicular to the horizontal line AB, so as to pass through the mark C, and if possible to cut the circle in two points as E and I; AE and AI are the two directions, in either of which, the project being cast with the given velocity, will hit the mark. Pl. 6. Fig. 6.

For, drawing the lines PE and PI, the angles CAE and APE are equal, from the nature of the circle; and from the nature of parallel lines, the angles CEA and EAP are equal; consequently, the triangle AEC is similar to the triangle PAE; and therefore, PA is to AE, as AE to EC; wherefore, multiplying the extremes and means, and dividing by EC, PA is equal to $\frac{AEq}{EC}$. In like man-

LECT. XI. ner, the triangles PAI and CAI being similar, PA is equal to $\frac{AIq}{IC}$. Wherefore, since PA is equal to the parameter at the point A, it follows, from the nature of the *parabola*, that those *parabolas* which the project describes when thrown in the directions AE and AI, must pass through the point C; consequently, the mark will be hit by a project thrown in either of those directions.

Whenever the mark is placed at such a distance from A on the line ACM, suppose at M, as that the perpendicular NMH which passes through the mark, becomes a tangent to the circle at H, the mark is then at the utmost limit on the line AM, to which a project thrown with the given velocity can reach, and there is but one direction, to wit AH, wherewith the mark can be hit; for it is evident, that any other direction must terminate in some point of the circumference above or below the point H; whence if a perpendicular be let fall to the horizontal line AN, it must of necessity fall on this side of HN with respect to A, and of consequence, cut the line AM in a point less distant from A than is the point M.

The line AH, which denotes the direction of the project, when thrown to the greatest distance possible on the line AM, bissects the angle PAM, which measures the visible distance between the zenith or vertical point P and the mark M. For, by the nature of the circle, the angle MAH is equal to the angle HPA; and forasmuch as in the triangles HPG and HAG, the sides PG and AG are equal by construction, and GH common to both, and the angles at G right ones, the angle HPG is equal to HAG, consequently HAG or HAP is equal to MAH, that is, the angle PAC is bissected by the line AH.

If

If the line ACM be ſituated below the horizontal line AN, which is the caſe when the mark is ſeated on a deſcent, let all things be conſtructed as before, and the ſame things will obtain; to wit AI and AE, will be the directions neceſſary to hit the mark at C; and the line AH will biſſect the angle PAM, which meaſures the apparent diſtance between the zenith and the mark; and the point M will be the utmoſt limit on the line AM, of a project thrown with the given velocity; the demonſtrations of which are exactly the ſame as in the foregoing caſe. LECT. XI. Pl. 6. Fig. 7.

If the mark be placed on a level, the line ACM will coincide with the horizontal line ABN, and the parameter AP, will paſs through the centre of the circle and become a diameter, the points K and G coinciding. Pl. 6. Fig. 8.

In this caſe, the horizontal diſtance of the mark, to wit AC or AB, is as the ſine of the doubled angle of elevation; or in other words, the horizontal range, or the diſtance to which a project is thrown on the plane of the horizon with a given velocity, is as the ſine of the doubled angle of elevation.

For AC, the horizontal range of a project thrown with a given velocity in the direction AE, is equal to DE, the ſine of the angle AKE; but, from the nature of the circle, the angle AKE is double the angle APE, which is equal to CAE, the angle of elevation; conſequently AC, the horizontal diſtance of the mark, or the diſtance to which a project is thrown on the plane of the horizon with a given velocity, is as the ſine of the doubled angle of elevation.

Hence it follows, that in order to throw a project with a given velocity, to the greateſt diſtance poſſible on the plane of the horizon, the direction of the projection muſt be elevated in an angle of 45 degrees; for, ſince the ſine of twice 45 or 90 de-

LECT. XI. grees is equal to the *radius*, and of consequence, the greatest of all the sines; and since the horizontal ranges at the several angles of elevation are to one another, as the sines of the doubled angles of elevation, it is manifest, that the greatest range, or as it is usually called by gunners, the greatest random, must be when the project is cast in a direction whose elevation is 45 degrees; moreover, the greatest random is ever equal to one half the parameter at the point from which the projection is made; for the line AM, which expresses the greatest random, is equal to the *radius* KH, or half the diameter AP, which by the construction, is equal to the parameter belonging to the point A; so that where the velocity with which a project is thrown is given, the utmost distance which that project can reach on the horizontal plane, is likewise given; for it is equal to twice the height, from which a heavy body must fall in order to acquire the velocity wherewith the project is thrown; the parameter belonging to the point A, having been already proved equal to four times that height.

A second consequence of the horizontal ranges being as the sines of the doubled angles of elevation is, that if two projects be thrown with equal velocities, in directions whose elevations are equally distant from 45 degrees above and below, for instance, if the elevation of one be 60 degrees, and that of the other 30, whereof the former exceeds 45 degrees, and the latter falls short thereof by 15 degrees, the horizontal ranges will be equal; or, in other words, the two projects will fall on the plane of the horizon, at the same distance from the place of projection; for as the sum of any two arches of a quadrant, whereof one exceeds 45 degrees, as much as the other is exceeded thereby, is equal to a quadrant, it is manifest, that two such arches are complements to each other; wherefore, since by the nature of the circle, the sine of a doubled arch is equal

to

to the ſine of it's doubled complement, the ſines of LECT. XI.
the doubled angles of two elevations equally diſtant from 45 degrees above and below, muſt be equal; and ſo of conſequence muſt the horizontal ranges which are proportional to thoſe ſines. And thus it would conſtantly be, were it not for two cauſes which do in ſome meaſure diſturb this law of projects, ſo as to make the horizontal ranges of the higher elevations to fall ſhort of thoſe of the lower.

The firſt of theſe diſturbing cauſes is the *air*, which as it reſiſts, and thereby retards the motions of projects, muſt, *cæteris paribus*, cauſe a greater retardation in thoſe motions which are of longeſt continuance; conſequently, ſince the higher the elevation of the direction is, the longer is the time of the project's motion, as ſhall be ſhewn hereafter; if the directions wherein two projects are caſt with equal velocities, be equally diſtant from 45 degrees, the one above and the other below, the project which is thrown in the higher direction, will be more retarded than that which is thrown in the lower; and of courſe, will fall on the plane of the horizon at a leſs diſtance from the place of projection.

The ſecond diſturbing cauſe obtains with regard to ſuch projects only as are thrown by the force of *gun-powder*. As the force of the powder acts upon the ball during it's continuance in the barrel, ſo does it likewiſe to ſome diſtance beyond the muzzle; and by ſo doing makes the ball to move forward in a right line, which line is commonly called the *line of impulſe of fire*; at the end of which, the ball quitting the blaſt of the powder, begins to move in the curve of a *parabola*.

Now, though the air gave no reſiſtance to projects, yet muſt the horizontal ranges of a ball ſhot out of the ſame piece with equal charges, in two directions equally diſtant above and below 45 degrees, be different on account of the different directions of the

 line

LECT. XI. line of impulſe of fire; for, let us ſuppoſe a gun at A, to diſcharge two equal balls with equal quantities of powder, one in the direction AB, and the other in the direction AC, AB being as far above 45 degrees, as AC is below it; and let AB and AC denote the lines of impulſe of fire, ſo that at B and C the balls will begin to move in *parabolic* curves; from which points let fall the perpendiculars BD and CE; it is manifeſt, that AD, which is the ſine of the complement of the higher elevation, will denote that part of the horizontal range which is owing to the line of fire, when the project is thrown according to the higher elevation; and AE, the ſine of the complement of the lower elevation, will be that part of the horizontal diſtance, which is owing to the line of impulſe when the project is thrown according to the lower elevation; conſequently, ſince the ſine of the complement of a leſſer angle is ever greater than that of a larger angle, the horizontal range of the lower elevation muſt exceed that of the higher, ſo that where projects are thrown with the ſame velocity by the force of powder, in directions equally diſtant above and below 45 degrees, thoſe muſt range fartheſt which are thrown according to the lower elevations, as well on account of the line of fire, as of the reſiſtance of the air.

Pl. 6, Fig. 9.

The altitude to which a project riſes, is as the verſed ſine of the doubled angle of elevation; for the proof of which, let AE be the direction of the projection, and let AC or AB be biſſected in T, and from the point of biſſection erect the perpendicular TR; ſince the point T is equally diſtant from A, where the project begins it's motion, and from B, where the motion of the project ceaſes, TR will be the axis of the *parabola* which the project deſcribes; and, from the nature of the *parabola*, will be biſſected in V by the principal vertex wherefore, TV will be the height to which the pro

Pl. 6. Fig. 8.

ject

ject riſes; but, from the nature of ſimilar triangles, ſince AT is one half of AC, TR is likewiſe one half of CE, and conſequently, TV one fourth of CE; wherefore, from the nature of proportionals, TV is as CE; but CE is equal to AD the verſed ſine of the angle AKE, which by the nature of the circle, is double APE; and APE is likewiſe, by the nature of the circle, equal to BAE the angle of elevation; wherefore, TV, or the height to which the project riſes, is as CE the verſed ſine of the doubled angle of elevation; hence it follows, that the greater the elevation is, the higher the project will riſe, inaſmuch as the verſed ſines of the doubled angles of elevation increaſe continually with the elevation, till at length the elevation becoming perpendicular, the verſed ſine of the doubled elevation becomes equal to the diameter, which being the greateſt of all the verſed ſines, the altitude of the perpendicular projection muſt likewiſe be greateſt; and it is equal to one fourth of the parameter; for, I ſhewed you in my lecture upon gravity, that if a body be thrown up with any velocity, it will riſe to the ſame height, from whence if it fell from a ſtate of reſt, it would by the end of the fall acquire the ſame velocity wherewith it is thrown up; and in this lecture I proved, that the velocity wherewith a project moves in any point of the *parabola*, is equal to the velocity acquired by a heavy body in falling down the fourth part of the parameter belonging to that point; conſequently, a project thrown up with a given velocity from the point A, will riſe to a height equal to the fourth part of the parameter belonging to that point. Hence it appears, that the greateſt height of the perpendicular projection, is equal to half the greateſt random, inaſmuch as the greateſt random has been proved equal to half the parameter belonging to the point A.

The time of the flight of a project thrown with a given velocity, is as the ſine of the angle of eleva-

LECT. XI. tion: for inſtance, the time of the flight of a project thrown with a given velocity in the direction AE, is as the ſine of CAE, the angle of elevation; for ſince the project moves through the curve of a *parabola* from A to C, by virtue of its uniform motion in the direction AE, and of it's accelerated motion in the direction EC, it is evident, that the time of it's flight through the *parabola*, muſt be equal to the time of it's uniform motion from A to E; but as the velocity is given, the time of the motion from A to E, muſt be as the ſpace deſcribed, that is, as AE; or by the nature of proportionals, as one half of AE; but AE being the chord of the arch AE, which meaſures AKE, the doubled angle of elevation, one half of AE is equal to the ſine of half the arch AE, that is, to the ſine of the arch which meaſures CAE, the angle of elevation; conſequently, the time of the flight is as the ſine of the angle of elevation. Hence it follows, that the greater the elevation is, the longer the time of the flight will be; as alſo that the time of the perpendicular flight is greateſt of all, the ſine of the perpendicular elevation being equal to *radius*.

If the velocity wherewith a project is thrown be required, it may be determined from experiments in the following manner; by the help of a *pendulum* or any other exact *chronometer*, let the time of the perpendicular flight be taken; then, foraſmuch as the times of the aſcent and deſcent are equal, the time of the deſcent muſt be equal to one half of the time of the flight, conſequently, that time will be known; and, foraſmuch as a heavy body deſcends from a ſtate of reſt at the rate of 16 feet in the firſt ſecond of time, and that the ſpaces through which bodies deſcend are as the ſquares of the times; if we ſay, as one ſecond is to ſixteen feet, ſo is the ſquare of the number of ſeconds which expreſs the time of the deſcent of the project, to a fourth proportional, we ſhall have the number of feet through which the

project

project fell, which being doubled, will give us the number of feet which the project would defcribe in the fame time with that of the fall, fuppofing it moved with an uniform velocity, equal to that which it acquired by the end of the fall; which laft found number of feet, being divided by the number of feconds which exprefs the time of the project's defcent, will give a quotient, expreffing the number of feet through which the project would move in one fecond of time with a velocity equal to that which it acquired in it's defcent, which velocity is equal to the velocity wherewith the project was thrown up; confequently, the velocity wherewith the project was thrown up is difcovered. To illuftrate this by an inftance, let us fuppofe half the time of the perpendicular flight to be 8 feconds; then, as one is to 16, fo is 64, the fquare of 8 feconds, to 1024; which being doubled, and then divided by 8, gives 256 in the quotient; which fhews that the project was thrown upward with fuch a velocity as would carry it, fuppofing it moved uniformly, at the rate of 256 feet in one fecond of time. LECT. XI.

Perhaps it may be objected, that the method here laid down for difcovering the velocities of projects, is founded on experiments in which projects are fuppofed to move freely without any let or impediment, whereas the air refifts and retards all projects in their motions, fo as not to fuffer them to rife to the fame height, or to return with the fame velocity, that they would in cafe they moved *in vacuo*; in anfwer to which, it muft be confeffed, that in the experiments here made ufe of, the air does refift and impede the motions of projects, fo as to fhorten their afcent, and to leffen the velocity of their return; but then this does very little affect the truth of the conclufions which are gathered from thefe experiments concerning the velocities, wherewith projects begin their motions; for, as in the method laid

LECT. XI. laid down, the only thing neceſſary to be known from experiment, is the true time of the flight of a project, ſuppoſing it to move *in vacuo*; if that time can be had from theſe experiments, the velocity wherewith the project ſets out may be rightly determined, notwithſtanding the reſiſtance of the air; but the time of the flight of a project thrown directly upward, is very nearly the ſame *in vacuo*, as in the air; for, as much as the time of a project's aſcent is ſhortned by the reſiſtance of the air, ſo much very nearly is the time of its deſcent lengthned by the ſame reſiſtance, conſequently, the whole time of the flight in air muſt be very nearly equal to the time of the flight *in vacuo*; and therefore, the time of the flight *in vacuo* is got, by taking the time of the flight in air.

Degrees.	*Sines.*	*Verſed ſines.*
30	50	13
45	70	29
60	86	51
90	100	100
120		148

Experiments

Experiments concerning projects made with a small mortar, the length of whose chase was 5½ inches; the diameter of the ball and chase 3½ inches; weight of the hollow ball 23000 grains; length of the chamber 2 inches, and its diameter ¾ inch.

Quantity of powder in grains.	Degrees of elevation.	Horizontal ranges in feet.	Times of the flights in half seconds.
60	30	135	5
60	45	150	6
60	60	120	8
60	90		11
90	30	200	6^{2}
90	45	220	8
90	60	220	9½
90	90		11
120	30	420	9
120	45	450	11
120	60	300	12½
120	90		16
140	45	660	15
140	90		18
180	30	1000	13
180	45	1100	17
180	60	900	21
180	90		26
240	45	1750	20
240	60	1390	25
240	90		32

LECTURE XII.

OF HYDROSTATICKS.

LECT. XII.

IN this lecture I shall give you an account of the gravitation and pressure of WATER, and such other FLUIDS, as are commonly called LIQUIDS.

A *fluid* in general is a body, whose parts yield to any force impressed, and in yielding are easily moved one among another.

The minute particles of fluids do not seem to differ from those of solid bodies; inasmuch as fluids and solids are frequently converted into one another. Thus water and watery fluids are by cold changed into ice; which by heat is again reduced to it's fluid state. Metals of all kinds being melted become fluid, and upon cooling grow solid again. The most solid and ponderous woods, as also the hardest stones may, by the force of fire, in a great measure be converted into water, as is well known to the chemists. And there are not wanting instances in nature of the grossest bodies being turned into the subtile fluids of air and light, and these again into gross bodies. Which changes can scarcely be accounted for, unless we suppose the minute particles of fluids to be of the same nature with those of solid bodies. But be this as it will, most certain it is, that fluids as well as solids consist of heavy particles, whose gravity is ever proportional to the quantity of matter which they contain. This having been found as far as experience reaches to be the universal property of matter, whatever be the form under which it appears. Most indeed of the ancient naturalists, not being sensible of any weight or pressure from the air about them, or from the incumbent water when immersed therein, were of opinion, that the parts of one and the same element did

did not gravitate one upon another; which opinion has been exploded by the moderns as erroneous; and that it is ſo, will appear from the following experiment.

Lect. XII.

Exp. 1.

Let an empty phial cloſe ſtopped and immerſed in water, be ſuſpended from one end of a balance and poiſed; then let the ſtopple be taken out, that the water may run in, the phial upon receiving the water will preponderate, and bear down the arm of the beam from which it hangs; which evidently proves, that the parts of water retain their gravity in water, ſo as to preſs and bear down upon the parts beneath them; otherwiſe the phial would not become heavier upon the admiſſion of the water.

From the gravity of the parts it follows, that ſetting aſide all external impediments, the ſurface of a liquid contained in a veſſel muſt be ſmooth and level; for ſhould any part ſtand higher than the reſt, it muſt deſcend by the force of it's gravity, and in ſo doing, ſpread and diffuſe itſelf till it comes to be on a level with the other parts. As the gravity of the parts reduces the upper ſurface to a level, ſo does it likewiſe occaſion a preſſure on the lower parts, greater or leſs in proportion to their depth below the ſurface, each part ſuſtaining a preſſure equal to the weight of all thoſe which lie above it; whence it follows, that the parts which are at equal depths below the ſurface, are equally preſſed, and of conſequence muſt be at reſt, contrary to the opinion of thoſe, who make the nature of fluidity to conſiſt in the conſtant actual motion of the parts one among another. Should this equality of preſſure at any time be deſtroyed, then indeed a motion will ariſe in the parts of the fluid, and continue till the preſſure becomes equal again, as may appear from the following experiment; whereby the truth of what has been ſaid concerning the preſſure of the ſuperior parts of fluids on thoſe beneath them, will likewiſe be confirmed.

LECT. XII. Take a glaſs tube open at both ends, and ſtopping one end with a finger, immerge the other in water to any depth whatever; upon the immerſion, the water will riſe in the tube, but the height to which it riſes, whilſt the upper orifice continues ſtopped, will be but ſmall; but upon removing the finger, it will riſe to the ſame height with the water without *.

Exp. 2.

When the tube is immerſed, that portion of water which lies beneath the orifice ceaſes to be equally preſſed with the other portions that are at the ſame depth; for that portion bears no other preſſure than what ariſes from the ſpring of air included in the tube, (which preſſure is equal, as ſhall be ſhewn hereafter, to the preſſure ariſing from the weight of the external air) whereas, the other portions do not only bear the preſſure of the air, but likewiſe the weight of the incumbent water; foraſmuch therefore, as the portion of water which lies beneath the orifice, is preſſed down leſs forcibly than the adjacent portions, it muſt give way and riſe in the tube, but the height to which it riſes, whilſt the upper orifice of the tube continues ſtopped, can be but ſmall; becauſe, as the water riſes it compreſſes the air in the tube, and thereby ſtrengthens its ſpring, ſo as to make it preſs with greater force; and when the air is ſo far compreſſed by the riſing water, as that the force of it's ſpring, added to the weight of the elevated water, makes the ſame preſſure on that portion of water which lies beneath the orifice, as the joint weight of the atmoſphere and external water does on the other portions, which are at the ſame depth with the former, then the water ceaſes to riſe. Upon opening the upper orifice of the tube, by the removal of the finger, the compreſſed air finding a paſſage through that orifice, expands and dilates itſelf till it becomes of an equal denſity with

* The water is tinged of a fine blue purple colour with a few grains of Sal Armoniack and Copper.

the external air; by which means, the preſſure ariſing from the condenſation of the air is taken off, and of conſequence, the water which lies beneath the orifice is leſs preſſed than the adjacent portions, and for that reaſon muſt riſe, and continue ſo to do, till the elevated water in the tube gravitates as forcibly on the water beneath the orifice, as the external water does on the neighbouring portions; but this it cannot poſſibly do, till it comes to be of an equal height with the external water. LECT. XII.

Should a lighter liquid be poured on the external water, the water within the tube will riſe yet higher than before; and the height to which it riſes above the ſurface of the external water, will be ſo much leſs than the height of the lighter liquor above the ſame ſurface, by how much the ſpecifick gravity of the water exceeds that of the lighter liquor; for inſtance, if the ſpecifick gravity of the water be to the ſpecifick gravity of the lighter liquor, as two to one, the height of the water in the tube above the level of the external water, will be to the height of the lighter liquid, as one to two; becauſe in that caſe, one part of water makes an equal preſſure with two parts of the lighter liquid. To illuſtrate this by an experiment.

Exp. 3. Let oil of turpentine, whoſe ſpecifick gravity is to the ſpecifick gravity of water, as 83 to 100, be poured on the external water to the height of 8 inches and an half, and the water will riſe in the tube to the height of 7 inches and $\frac{3}{10}$ above the level of the external water; that is, the heights of the water and oil will be in the reciprocal proportion of their ſpecifick gravities; for 7 and $\frac{3}{10}$ is to 8 and $\frac{1}{2}$, or, which is the ſame thing, 73 is to 85, very nearly, as 83 to 100.

The ſame thing is in like manner confirmed by the following experiment.

Exp. 4. Let one end of a ſmall tube open at both ends, be immerſed in mercury contained in a larger tube, and let water be poured upon the mercury in the larger

Lect. XII. larger tube to the height of 34 inches; the mercury will riſe in the ſmaller tube to the height of 2 inches and an half above the level of the mercury in the larger tube; ſo that the height of the mercury in the ſmaller tube above the level of the mercury in the larger, will be to the height of the water above the ſame level in the reciprocal proportion of their ſpecifick gravities; for $2\frac{1}{2}$ is to 34, as 1 to $13\frac{3}{5}$, which numbers expreſs the proportion of the ſpecifick gravity of water to that of mercury.

The preſſure which the lower parts of a liquid ſuſtain from the weight of thoſe which lie above them, exerts itſelf every way in all manner of directions, and that equally; or in other words, whatever be the force wherewith a drop of any liquid is preſſed downward by the weight of the incumbent liquid, with the very ſame force is that drop preſſed upward, as alſo laterally and obliquely, and in a word, in all kind of directions whatever; otherwiſe the drop, which from the nature of fluidity, readily yields and gives way to any impreſſion, muſt by reaſon of the preſſure from above move out of it's place; but this it cannot poſſibly do, becauſe the drops all around it being equally preſſed from above, do on all ſides reſiſt the motion of that drop, with the ſame force that it endeavours to move; conſequently, the drop muſt continue at reſt, and be preſſed on all ſides with the ſame force that it is from above; and what has been thus proved of one drop, is in like manner demonſtrable of all the reſt; and therefore, the preſſure on the lower parts of a liquid exerts itſelf equally every way, as will appear from the following experiment.

Exp. 5. Let four tubes open at both ends be immerſed in water to the ſame depth, their upper orifices being firſt ſtopped, and let the lower orifices be ſo ſituated, as that the water in entring may move directly upward in one, and directly downward in another,

obliquely

obliquely in a third, and horizontally in the fourth; upon opening the upper orifices, the water will riſe in all of them to the ſame height with the external water, as being preſſed in the ſeveral directions with a force equal to the weight of the incumbent water. LECT. XII.

From the preſſure of liquids upwards it is, that ſolid bodies ſpecifically lighter than liquids, are made to aſcend when immerſed therein. For when a ſolid body is immerſed in a liquid, it preſſes that part of the liquid whereon it reſts, with a force equal to the weight of a column compoſed of the body it ſelf, and that portion of liquid which lies upon it; and the water preſſes upward againſt the body, with a force equal to the weight of a like column of the liquid alone; which force, inaſmuch as the liquid is heavier than the ſolid, muſt overcome the force wherewith the body preſſes downward, and of conſequence, the body muſt riſe with the difference of thoſe forces; as ſhall be ſhewn more fully in my next lecture. If by any contrivance the preſſure of the liquid from beneath can be taken off, a body though ſpecifically lighter will not riſe in a liquid but remain immerſed, as in the following experiment.

A braſs plate being joined to one end of a cylindrical piece of wood, and another plate of the ſame ſize and ſhape being fixed in water; let the cylinder be totally immerſed,and let it's plate be laid upon the other in ſuch a manner, as that no water may get between; the cylinder though ſpecifically lighter will remain beneath the water, being preſſed down by it's own weight and that of the incumbent water, whilſt the contrary preſſure of the water from beneath, is kept off by means of the plate whereon the cylinder reſts. Exp. 6.

As bodies ſpecifically lighter than liquids, are forced up, on account of the preſſure from below being greater than the force wherewith the bodies

 preſs

LECT. XII. prefs downward; fo on the other hand, bodies fpecifically heavier muft fink, becaufe the force wherewith they prefs downward exceeds the preffure from beneath which oppofes their defcent; and the force wherewith they defcend is equal to the difference of thofe forces; as fhall likewife be fhewn in my next lecture. If by any contrivance thofe two forces can be reduced to an equality, then the bodies will not defcend but remain fufpended in the liquid; as in the following experiment.

Exp. 7. Let a brafs plate, whofe fpecifick gravity is to that of water, as 9 to 1, be adapted to the neck of a glafs veffel in fuch a manner, as that being immerfed in water no part of the water may get upon it's upper furface; let it then be immerfed to the depth of nine times it's own thicknefs, (that is, to the depth of 2 inches and $\frac{7}{10}$, the thicknefs of the plate being $\frac{3}{10}$ of an inch) and it will remain fufpended; but upon pouring ever fo little water upon it's upper furface, it will immediately defcend and fall to the bottom.

The plate being immerfed to the depth of nine times it's own thicknefs, is preffed upward by a force equal to the weight of a column of water, whofe height is nine times as great as the thicknefs of the plate; which weight, inafmuch as the fpecifick gravity of the water is to that of the plate, as 1 to 9, is equal to the weight of the plate, that is, to the force wherewith the plate preffes downward; for as none of the water lies on it's upper furface, it can prefs downward with no other force than what arifes from it's own gravity; confequently, in this cafe, the force which refifts the defcent, is equal to the force which promotes it, and of courfe, the plate muft remain in it's place. When a little water is poured on the plate, the weight of that added to the weight of the plate, overcomes the refifting force of the water, and caufes the plate to defcend. Should

Exp. 8. the plate be immerfed to twice the former depth, it

it will not descend though loaded with water to the height of nine times it's own thickness; for as in this case, the depth to which it is immersed is double the former, so likewise is the force wherewith the water presses upward; consequently, that force is sufficient to support twice the weight of the plate, and therefore will sustain the plate when loaded with water, to the height of nine times it's own thickness, such a quantity of water being just equal in weight to the plate.

If by pouring on more water, the force wherewith the plate presses downward be increased; or by raising the plate nearer to the surface of the water, the force wherewith the water presses upward be diminished, the plate will fall to the bottom; and on the other hand, if by immersing the plate to a greater depth, the pressure of the water upward be increased, the plate will be thrust upward against the glass, and would actually ascend were it not hindered by the glass.

From what has been said it follows, that if S be put to denote the number expressing the specifick gravity of the plate, that of water being unity; D be the depth to which the plate is immersed expressed in the thickness of the plate, and H the height of the water upon the plate expressed likewise in the thickness of the plate; D must be equal to the sum of S and H in all cases where the plate remains suspended; and if there be no water upon the plate, then D and S must be equal; wherefore, if in the former case, D be greater than S and H taken together, or in the latter, than S alone, the plate will ascend if not hindered by the glass; and on the other hand, if D be less, the plate will descend and fall to the bottom.

The pressure which the bottom of a vessel sustains from a liquid contained in it, whatever be the shape of the vessel, is equal to the weight of a pillar of the liquid, whose base is equal to the *area* of the

LECT. XII. the bottom, and whofe height is the fame with the perpendicular height of the liquor.

That this is the cafe in veffels that are equally wide from top to bottom, is plain and obvious; inafmuch as the bottom of every fuch veffel does actually fuftain fuch a pillar of liquor. But that the cafe fhould be the fame in irregular veffels, is not fo eafy to conceive; for inftance, that in a veffel which from a large bottom grows narrower as it rifes, fo as perhaps at length to be contracted into a tube, the bottom fhall bear the fame preffure when the veffel is filled, as it would were the veffel equally wide throughout from bottom to top, feems ftrange and furprizing, and yet it is what neceffarily follows from the nature of fluidity; for that part of the bottom which lies directly beneath the tube, fuftains the weight of a pillar of liquor which reaches to the top of the tube, the veffel being fuppofed to be full, and being preffed with the weight of that pillar, re-acts with an equal preffure on that portion of the liquor which touches it; and that preffure, inafmuch as it exerts it felf equally in the liquor every way, is propagated literally through the feveral portions of liquor which are contiguous to the bottom of the veffel; and forafmuch as this lateral preffure does in like manner exert it felf equally every way, the bottom of the veffel muft be equally preffed in every point; confequently, fince that portion of it which lies beneath the tube, bears a preffure equal to the weight of a pillar of liquor, whofe height reaches to the top of the veffel, every other equal portion muft bear a preffure equal to the fame weight; and of courfe, the whole bottom muft be preffed as forcibly, as if the veffel continued of the fame widenefs to the top, and was filled with the liquor.

Exp 9. Pl. 6. Fig. 10, 11. To confirm this by an experiment. Let there be two glaffes open at both ends, and of fuch fhapes as are exhibited in the two figures, whofe lower parts

MN

MN are cylindrical and equal, and of a capacity just sufficient to admit the brass plate made use of in the last experiment; which must be fitted to each of them successively, in order to constitute two vessels of equal bottoms, but of different capacities; and being so fitted, let it be immersed in water, as in the last experiment, to such a depth, as that it will be necessary to load it with water in order to make it sink; that is, let the depth be more than nine times the thickness of the plate, which depth must be the same in both cases; let then water be poured on the plate, and let it be observed what height of water is requisite to force down the plate when the wider vessel is made use of, and it will be found, that the same height will suffice in the narrower vessel; consequently, the small pillar of water in the narrower vessel, must press the plate with a force equal to the weight of a pillar of water of the same height, and of a base equal to the *area* of the plate; for such a pillar does actually press the plate in the larger vessel, as is evident from the bare inspection of the figure, and the pressures made on the plate in both vessels are equal, inasmuch as they overcome equal resistances.

From what has been said it appears, that where the base of a vessel is given, the pressures upon it are as the perpendicular heights of the liquid, whatever be the shape of the vessel. And universally the pressure on any base is measured by the product of the *area* of that base into the perpendicular height of the liquor above it, without any regard to the quantity of liquor contained in the vessel; so that if we suppose a hogshead set on one end, and filled with a liquor, and a small pipe to issue perpendicularly upward from the other end to any height whatever, and to be filled with the same liquor, the bottom will be as strongly pressed, and be in as much danger of bursting out, as if the hogshead

was continued to the ſame height with the pipe, and filled with the liquor.

As the bottom of a veſſel bears a preſſure proportional to the height of the liquor, ſo likewiſe do thoſe parts of the ſides which are contiguous to the bottom; becauſe the preſſure of fluids is equal every way. And as the preſſure which the lower parts of a fluid ſuſtain from the weight of thoſe above them exerts it ſelf equally every way, and is likewiſe proportional to the height of the incumbent fluid, the ſides of a veſſel muſt every where ſuſtain a preſſure proportional to their diſtance from the upper ſurface of the liquor. Whence it follows, that in a veſſel full of liquor, the ſides bear the greateſt ſtreſs in thoſe parts next the bottom; and that the ſtreſs upon the ſides decreaſes with the increaſe of the diſtance from the bottom, and in the ſame proportion; ſo that in veſſels of a conſiderable height, the lower parts ought to be much ſtronger than the upper, that they may be able to withſtand the greater preſſure.

LECTURE XIII.

OF HYDROSTATICKS.

LECT. XIII.

IN this lecture I ſhall explain to you, that part of HYDROSTATICKS which is of uſe in diſcovering the *denſities* and *ſpecifick gravities* of *bodies*.

The DENSITY of any body is meaſured by the proportion which it's quantity of matter bears to it's bulk. For the more numerous the particles of matter are in proportion to the ſpace which they poſſeſs, the greater is the denſity of the body; and the fewer the particles, the leſs the denſity; wherefore, putting D to denote the denſity of a body, Q it's

Q it's quantity of matter, and M it's magnitude, $D=\frac{Q}{M}$; and forasmuch as the quantity of matter in any body is ever proportional to, and measured by, the weight, as I shewed in my lecture upon gravity; if instead of the quantity of matter the weight of the body be substituted, and if that weight be denoted by W, then $D=\frac{W}{M}$; that is, the density is as the weight of the body directly, and the magnitude inversly.

By the specifick gravity of a body is meant the gravity peculiar to that species of matter, whereof the body is a part; and it is measured by the proportion of the absolute weight to the bulk; which proportion in one and the same kind of matter, remains unvaried; and in different kinds, as this proportion is greater or less, so is the specifick gravity which is measured by it. Let S denote the specifick gravity of a body, it's weight and magnitude being denoted by W and M as before; then, from what has been said, $S=\frac{W}{M}$; and by consequence since D is likewise $=\frac{W}{M}$, $S=D$; that is, the specifick gravity of a body is as it's density. And therefore, by finding out the proportion which the specifick gravities of bodies bear one to another, the proportion of their densities is likewise discovered; for which reason I shall take no farther notice of the densities of bodies, but confine my self to the consideration of their specifick gravities alone.

When a solid body is immersed in a liquid, it presses downward, and endeavours to descend by the force of it's gravity; but forasmuch as it cannot descend without moving as much of the liquid out of it's place, as is equal to it in bulk, it is manifest that

LECT. XIII. that it is resisted, and, as I may say, pressed upward by a force equal to the weight of such a portion of the liquid as is equal to it in bulk; consequently, if the specifick gravity of the solid be greater than that of the liquid, that is, if the solid weighs more than an equal bulk of the liquid, the body will descend with a force equal to the excess of it's gravity above the gravity of the liquid: on the other hand, if the gravity of the liquid exceeds that of the solid, the body being as it were pressed upward by a force greater than that whereby it endeavours to go down, will ascend with the difference of those forces, that is, with a force equal to the excess of the specifick gravity of the liquid above that of the solid. When the specifick gravities are equal, the body will neither rise nor fall, but remain suspended at any depth; being pressed as strongly upward by the resisting force of the liquid, as it is downward by it's own weight. Hence it follows, that if by any contrivance the specifick gravity of a solid can be varied, so as to be one while greater, another less, and then equal to the specifick gravity of a liquid wherein it is immersed, the body will sink, or rise, or remain suspended according to the variation of it's specifick gravity. And this is the case in that ludicrous experiment of the little glass images in water, which are made to descend, or rise, or remain suspended at pleasure; the reason of which I shall explain to you, after you have seen the experiment.

Exp. 1. The images being set to float on the water, the top of the vessel must be covered with a bladder closely bound about the neck of the vessel, to the end that the air, which lies upon the surface of the water, may not force it's way out when it is condensed by the hand pressing on the bladder. The images themselves, though lighter, are yet nearly of the same specifick gravity with the water, and being hollow, are full of air, which by means of small holes

holes in their heels communicates with the air without. When the air which lies beneath the bladder is preſſed by the hand, it preſſes on the ſurface of the water; and foraſmuch as the preſſure is propagated through all the water, thoſe portions which are contiguous to the heels of the images, are thereby forced into the holes, by which means the air within is condenſed, and at the ſame time, the weight of the images is increaſed by the additional weight of the influent water. And when ſo much water is forced in, as to render the ſpecifick gravity of the images greater than that of the water, the images deſcend and fall to the bottom; where they remain as long as the preſſure above continues, but when that is taken off by the removal of the hand, the condenſed air in the images dilates and expands it ſelf, and in ſo doing drives out the water; upon which account the images become ſpecifically lighter than the water, and of courſe aſcend. As the preſſure on the bladder is greater or leſs, ſo muſt the quantity of water which is forced into the images; and therefore, whenever it happens that during the aſcent or deſcent of an image, ſuch a preſſure is made as ſuffices to force in juſt as much water as is requiſite to reduce the image to the ſame ſpecifick gravity with the water, the image ſtops and remains ſuſpended, upon increaſing the preſſure it deſcends, and aſcends if the ſame be leſſened. Some of the images begin to deſcend ſooner, as alſo to riſe later, than others, for one or both of theſe reaſons; firſt, becauſe ſome are ſpecifically heavier than others; and, ſecondly, becauſe the cavities in the legs are greater in ſome images in proportion to their magnitudes, than they are in others; upon both which accounts, a leſs preſſure is requiſite to make ſome deſcend, and to keep them down, than what is neceſſary to produce the ſame effects in others. For, firſt, let us ſuppoſe the ſpecifick gravities of two

images

Lect. XIII. images to be different, but the cavities in their legs, when taken of a given height, to be proportional to their respective magnitudes; since the air is equally dense in both images, it is manifest, that it gives the same opposition in both to the influent water; consequently, the water when forced in by the pressure from above, must rise to equal heights in the cavities of both; since therefore the cavities whose heights are equal, are supposed to be proportional to the magnitudes of the images, it is manifest, that the quantities of water contained in those cavities, must be so too; consequently, each image receives an addition of weight from the influent water proportional to it's magnitude; or in other words, the specifick gravities of the two images are equally augmented; forasmuch therefore as one of the images is supposed to be specifically heavier than the other, it is evident, that when the specifick gravity of the former has received such an addition, from the influent water, as makes it a little exceed the specifick gravity of the water, the specifick gravity of the latter must fall short thereof; consequently, the former must sink and leave the other above.

Secondly, Let us suppose the specifick gravities of the two images to be equal; but let one image be less in proportion to the cavity in it's legs, than the other is in proportion to it's cavity, the height of the cavities being given; since the water does from the same pressure rise to an equal height in both, it is plain, from what I just now said, that the former must receive a greater quantity of water in proportion to it's magnitude, and consequently, a greater addition to it's specifick gravity than the latter, and of course must descend sooner.

From what has been said it follows, that if the proportion which the cavity in the legs bears to the magnitude of the image be given, the greater the

the ſpecifick gravity of the image is, the more apt it will be to deſcend; conſequently, in this caſe the aptitude or promptneſs of an image to deſcend is proportional to, and may be expreſſed by, the ſpecifick gravity. In like manner, if the ſpecifick gravity be given, the greater the proportion is which the cavity in the leg bears to the magnitude of the image, the more apt the image is to deſcend; and therefore in this caſe, the aptitude is proportional to, and may be expreſſed by, the cavity applied to the magnitude of the image. But if neither the ſpecifick gravity of the image, nor the proportion of the cavity to the magnitude of the image, be given, the aptitude of an image to deſcend, is as the ſpecifick gravity into the cavity applied to the magnitude of the image; that is, putting A to denote the aptitude, S the ſpecifick gravity of the image, C the cavity in the leg, (the height whereof is always ſuppoſed to be given) and M the magnitude of the image; $A=\frac{SC}{M}$; or, ſubſtituting the abſolute weight of the image applied to it's magnitude, in the room of the ſpecifick gravity, $A=\frac{WC}{M^2}$; that is, the aptitude an image has to deſcend, is as the weight of the image into the cavity of the leg directly, and the ſquare of the image's magnitude inverſly.

LECT. XIII.

A ſolid ſpecifically heavier than a liquid, being immerſed therein, loſes as much of it's weight as is the weight of a portion of the liquid equal to it in bulk; for it has been already ſhewn, that a ſolid is carried down in a liquid by the exceſs only of it's gravity, above the gravity of a portion of the liquid equal to it in bulk; conſequently, the other part of it's gravity is loſt, as to any effect it has on the body it ſelf; as will appear from the following experiment.

Let

LECT. XIII.

Exp. 2. Let a ſmall cylinder of braſs ſuſpended at one end of a balance and counterpoiſed, be immerſed in water; upon the immerſion it will become lighter, ſuppoſe by 200 grains, which is the weight of as much water as is equal in bulk to the cylinder; for a cylindrical veſſel, juſt large enough to contain the cylinder, being hung at one end of a balance and poiſed, and then filled with water, preponderates with the weight of 200 grains.

Since a ſolid when immerſed in a liquid, loſes as much of it's weight, as is equal to the weight of a portion of the liquid of the ſame dimenſions with the ſolid, it follows, that all bodies whatever, whoſe magnitudes are equal, however different their ſpecifick gravities may be, do ſuffer an equal loſs of weight in the ſame liquid. Thus a cylinder of

Exp. 3. block-tin, equal in dimenſions to the braſs cylinder, but ſpecifically lighter, being immerſed in water, loſes 200 grains, as did that of braſs.

Though a ſolid loſes part of it's weight when immerſed in a liquid, yet it muſt not be imagined that the weight ſo loſt by the ſolid, is actually deſtroyed, but that it is imparted to the liquid, the liquid conſtantly gaining in weight what the ſolid loſes.

Exp. 4. For if the veſſel with the water wherein the cylinders were immerſed, be put into a ſcale and poiſed; upon the immerſion of either cylinder, it will preponderate with the weight of 200 grains, which is what the cylinder loſes.

Solids equal in weight, but of different ſpecifick gravities, being immerſed in the ſame liquid, ſuffer loſſes of weight reciprocally proportional to their ſpecifick gravities; for as the loſs of weight which any body ſuffers in a liquid, is equal to the weight of as much of the liquid as is equal in bulk to the ſolid, the loſs ſuſtained is ever proportional to the magnitude of the body; whatever proportion therefore the magnitudes of bodies have to one another, the ſame will the loſſes of weight have which they

ſuffer; but the magnitudes of bodies equal in weight, but of different ſpecifick gravities, are to one another in the reciprocal proportion of their ſpecifick gravities; conſequently, ſo are the loſſes of weight which they ſuffer. Which is confirmed by the following experiment. LECT. XIII.

Let two cones, one of lead, the other of tin, whoſe ſpecifick gravities are to one another, as 112 to 74, and the weight of each 400 grains, be immerſed in water, after the manner of the cylinders; upon the immerſion, the lead will loſe $35\frac{1}{2}$ grains, and the tin 54; but $35\frac{1}{2}$ is to 54, as 74 to 112, that is, reciprocally as the ſpecifick gravities of the metals. Exp. 5.

From the loſſes of weight being reciprocally proportional to the ſpecifick gravities, it follows, that if two bodies of different ſpecifick gravities, which balance each other in air, be immerſed in water or any other liquor, the *æquilibrium* will be deſtroyed, and that which has the greateſt ſpecifick gravity will deſcend; as will appear, by hanging the cones, one at each end of a balance, and then immerſing them in water, for the lead will preponderate. Exp. 6.

The ſpecifick gravity of a ſolid ſpecifically heavier than a liquid, is to the ſpecifick gravity of the liquid, as the abſolute weight of the ſolid, to the loſs of weight which it ſuffers in the liquid; for the ſpecifick gravities of bodies being as the abſolute weights applied to the magnitudes, where the magnitudes are equal, the ſpecifick gravities are directly as the abſolute weights; if therefore we compare the ſolid with a quantity of the liquid equal to it in magnitude, their ſpecifick gravities muſt be as their weights; but the abſolute weight of ſuch a quantity of the liquid, is equal to the loſs of weight ſuſtained by the ſolid; conſequently, the ſpecifick gravity of the ſolid, is to that of the liquid, as the whole weight of the ſolid, to the loſs which it ſuſtains in the liquid.

Hence

LECT. XIII. Hence we have a method of diſcovering the ſpecifick gravities of ſuch ſolid bodies as are heavier than water; I mean, of diſcovering the proportions of their ſpecifick gravities to the ſpecifick gravity of water. For if we ſuppoſe the ſpecifick gravity of water to be unity, and put L to denote the loſs of weight which any body, whoſe ſpecifick gravity we look for, ſuſtains in water, and W it's whole weight, then $L : W :: 1 : \frac{W}{L}$; conſequently, $\frac{W}{L}$ expreſſes the ſpecifick gravity of the ſolid, that of water being unity; and therefore, in order to know the ſpecifick gravity of any ſolid heavier than water, nothing more is requiſite, but to diſcover the quantities denoted by W and L, and to divide the firſt by the laſt; the firſt is had, by taking the weight of the body in air, and the laſt, by taking the weight in water, and ſubducting it from the weight in air; for the remainder is the loſs of weight, which dividing the weight in air, gives a quotient expreſſing
Exp. 7. the ſpecifick gravity of the body. To apply this to a particular caſe, let it be propoſed to diſcover the ſpecifick gravity of a piece of tin, which being weighed in air, is found to be 300 grains, and in water, $259\frac{1}{2}$, which being ſubducted from the former, leaves $40\frac{1}{2}$ for the loſs of weight; ſo that in this caſe, W denotes 300, and L $40\frac{1}{2}$; and therefore dividing 300 by $40\frac{1}{2}$, we ſhall have $7\frac{4}{10}$ for the ſpecifick gravity of tin, that of water being unity. Whence it appears, that tin, bulk for bulk, is more weighty than water, in the proportion of 74 to ten.

If the body, whoſe ſpecifick gravity is required, be lighter than water; then, foraſmuch as it's gravity is not ſufficient to cauſe a total immerſion, the loſs of weight which it ſuffers in water, cannot be found out by weighing it alone in that liquid; let it therefore be joined to ſome other body ſo weighty

that

that the compound may ſink; but firſt let the loſs of weight which the heavier body alone ſuſtains in water be found out, as before; and then let the loſs of weight which the compound ſuſtains be likewiſe diſcovered, whence deducting the loſs of weight ſuſtained by the heavier, the remainder will exhibit the loſs ſuſtained by the lighter; conſequently, dividing the weight of the lighter by that remainder, the quotient will expreſs the ſpecifick gravity required; that is, putting W for the weight of the body whoſe ſpecifick gravity is ſought, L for the loſs of weight ſuſtained by the compound, and l for the loſs ſuſtained by the heavier body $\frac{W}{L-l}$ expreſſes the ſpecifick gravity of the body. To apply this to a particular caſe; let the weight of a piece of wood ſpecifically lighter than water be 220 grains, and let it be joined to a piece of tin of 160 grains, whoſe loſs in water is found to be 17 grains; then the compound being weighed in water, will be found to loſe 334 grains; ſo that in this caſe, W is equal to 220 grains, L to 334, and l to 17; and L leſs l, is equal to 317 grains. And therefore, dividing 220 by 317, we ſhall have $\frac{694}{1000}$ for the ſpecifick gravity of the wood, that of water being unity. So that that kind of wood is, bulk for bulk, lighter than water, in the proportion of 694 to 1000.

LECT. XIII.

Exp. 8.

If the body whoſe ſpecifick gravity is ſought be diſſolvable in water, then inſtead of water, let ſome other liquor be made uſe of, which will not diſſolve the body; and let the proportion of the ſpecifick gravity of the body to the ſpecifick gravity of that liquor, be diſcovered by the foregoing method; as alſo the proportion of the ſpecifick gravity of that liquor to the ſpecifick gravity of water, by the method which ſhall be ſhewn preſently. Then in whatever proportion the ſpecifick gravity of the liquor exceeds

LECT. XIII. ceeds or falls ſhort of the ſpecifick gravity of water, in the ſame proportion let the ſpecifick gravity of the body with regard to that of the liquor be increaſed or diminiſhed, and it will give the ſpecifick gravity of the body with reſpect to that of water; that is, if we put A for unity or the ſpecifick gravity of water, B for the ſpecifick gravity of the other liquor, and C for the ſpecifick gravity of the body with regard to that liquor; then by ſaying, as A is to B, ſo C to a fourth proportional, we ſhall have $\frac{BC}{A}$ for the ſpecifick gravity of the body with reſpect to that of water; or rejecting the diviſor as being equal to unity, and putting S for the ſpecifick gravity of the body with reſpect to water, we ſhall

Exp. 9. have $S=BC$. To apply this, let the ſpecifick gravity of Roman-vitriol be required; let the weight of a piece in air be 67 grains, and in ſpirit of wine 41 grains; conſequently, it's loſs of weight in the ſpirit is 26 grains, which dividing 67, gives 2.576 for the ſpecifick gravity of the vitriol with regard to the ſpecifick gravity of the ſpirit, which, in this caſe, is ſuppoſed to be unity; but the ſpecifick gravity of the ſpirit, with regard to that of water, is leſs than unity, being only $\frac{87}{100}$, as ſhall be ſhewn preſently; wherefore B is $=0.87$, and C is $=2.576$; conſequently, 2.24 which is the product ariſing from the multiplication of thoſe two numbers, expreſſes the ſpecifick gravity of Roman-vitriol with reſpect to that of water, which is as unity; and therefore, in whole numbers, the ſpecifick gravity of Roman-vitriol exceeds that of water, in the proportion of 224 to 100.

The ſpecifick gravities of liquors are diſcovered by taking the loſſes of weight ſuſtained by one and the ſame ſolid in the ſeveral liquors; for ſince the loſs of weight in each liquor, is equal to the weight

of

of as much of the liquor as is equal in bulk to the body; by taking the loſſes of weight ſuſtained by the ſame body in the ſeveral liquors, we get the abſolute weights of ſuch portions of thoſe liquors as are equal in bulk; and by conſequence, the ſpecifick gravities of the liquors, the ſpecifick gravities of bodies equal in bulk, being to one another as their abſolute weights; wherefore, putting L for the loſs of weight which a body ſuſtains in water, and little l for the loſs of weight ſuſtained by the ſame body in any other liquor; then, by ſaying, as L to l, ſo is unity to a fourth term, we ſhall have $\frac{l}{L}$ for the ſpecifick gravity of the other liquor, that of water being unity; ſo that to diſcover the ſpecifick gravity of any liquor, we have nothing more to do, but to weigh one and the ſame ſolid, both in the liquor whoſe quantity is ſought, and in water, and to divide the loſs of weight which the ſolid ſuffers in the liquor, by the loſs which it ſuſtains in water; for the quotient will expreſs the ſpecifick gravity of the liquor. Thus, a glaſs bubble whoſe weight in air is 1727 grains, being weighed in water, is found to loſe 641 grains, and 558 in ſpirit of wine; and therefore, dividing 558 by 641, we ſhall have a quotient of 0.87 for the ſpecifick gravity of the ſpirit, that of water being unity.

LECT. XIII.

Exp. 10.

When a body ſpecifically lighter than a liquid, is ſet to float upon it, the part immerſed is equal in bulk to a portion of the liquid whoſe weight is equal to the weight of the whole body; for ſince the body ſinks in part, by moving ſome of the liquor out of it's place, and ſince the weight of the body is the power which moves the liquor, the body muſt continue to ſink, till it has removed as much of the liquor as is equal to it in weight; conſequently, the

P part

LECT. XIII. part immerfed muft be equal in magnitude to fuch a portion of the liquor, as is equal in weight to the whole body; which is abundantly confirmed by the following experiment.

Exp. 11. A ball of pear-tree, a wood fpecifically lighter than water, being fet to float on water contained in a glafs veffel, let the veffel be placed in a fcale and counterpoifed; then, taking out the ball, let the veffel be filled up with water to the fame height at which it ftood when the ball was in it, and the fame weight will counterpoife it as before.

From the veffel's being filled up to the fame height at which the water ftood when the ball was in, it is manifeft, that the quantity poured in is equal in magnitude to that part of the ball which was immerfed; and, from the fame weight counterpoifing, it is evident, that the water poured in, is equal in weight to the whole ball.

The part immerfed is to the whole body, as the fpecifick gravity of the body to the fpecifick gravity of the liquid; for the fpecifick gravities of two bodies, being to one another as their abfolute weights applied to their magnitudes, if their weights be equal, their magnitudes are in the reciprocal *ratio* of their fpecifick gravities; fince therefore, fuch a portion of the liquid as is equal in magnitude to the immerfed part of the folid, is likewife equal in weight to the whole folid; the magnitude of the immerfed part is to the magnitude of the whole body, as the fpecifick gravity of the folid to the fpecifick gravity of the liquid.

When the fame body is fet to float fucceffively in different liquors, the parts immerfed are to one another in the reciprocal proportion of the fpecifick gravities of the liquors. For the body defcends in each liquor, till the part immerfed takes up the room of as much liquor as is equal in weight to the whole body; and therefore, fuch portions of the

feveral

ſeveral liquors as are equal in magnitude to the immerſed parts of the body have all equal weights; but the magnitudes of bodies equal in weight, are to one another reciprocally, as their ſpecifick gravities; conſequently, in one and the ſame body floating in different liquors, the parts immerſed are reciprocally as the ſpecifick gravities of the liquors. On this principle is founded the HYDROMETER; which is an hollow glaſs ball, with a ſmall hollow ſtem of about 5 or 6 inches in length, oppoſite to which, on the other ſide of the ball, adheres a ſmaller ball filled in part with mercury, or ſome other weighty body, to the intent, that when the ball is ſet to float in water, or any other liquor, the ſtem may be kept uppermoſt, and in a poſition perpendicular to the ſurface of the liquor; and at the ſame time, that the machine may be ſo far immerſed, as that the ſtem only, or ſome part thereof, may remain above the liquor; the ſtem being graduated from top to bottom, has numbers annexed to every degree, expreſſing the magnitudes of the parts which lie below the ſeveral degrees.

The uſe of this little machine is to diſcover the ſpecifick gravities of liquors, which is done in the following manner. The *hydrometer* being firſt ſet to float in water, the degree to which it ſinks muſt be obſerved, and the number thereto annexed; then being ſet to float in any other liquor, the degree to which it ſinks, with the number annexed, muſt likewiſe be noted; for as this number is to the former, ſo is the ſpecifick gravity of water, to that of the other liquor, as is evident from what was juſt now ſaid. To illuſtrate this in the caſe of water Exp. 12. and ſpirit of wine. The *hydrometer* being dropt into water, ſinks to the degree whoſe number annexed is 87; and being dropt into ſpirit of wine, ſinks to the degree whoſe number is 100; whence it appears, that the ſpecifick gravity of water is to that of ſpirit of wine, as 100 to 87.

Though *hydrometers* may be uſeful in diſcovering the ſpecifick gravities of liquors for looſe and inaccurate computations, yet are they not to be depended on in caſes where great exactneſs is required, and that for two reaſons; Firſt, becauſe it is extremely difficult to graduate the ſtems ſo exactly, as that the numbers annexed ſhall truly expreſs the magnitudes of the parts below them. Secondly, becauſe, partly from the motion of the *hydrometer* in the liquor, and partly from the riſing of the liquor about the ſtem from the attractive force of the glaſs, it is hardly poſſible to determine with exactneſs the degree to which the *hydrometer* ſinks. Upon both which accounts, as alſo becauſe the method of determining the ſpecifick gravities of liquors by means of the glaſs bubble is much more eaſy and exact, this method by the *hydrometer* is intirely laid aſide.

LECTURE XIV.

OF HYDROSTATICKS.

LECT. XIV.

IN this lecture I ſhall give you an account of the flux of water from RESERVOIRS through *orifices* and *pipes*.

If water, flowing out at an orifice in the bottom of a veſſel, be kept conſtantly at the ſame height in the veſſel, by being ſupplied as faſt above, as it runs out below, the velocity wherewith it flows out, is as the ſquare root of the height of the water above the orifice.

For if we ſuppoſe the column of water which ſtands directly above the orifice, to be divided into an indefinite number of plates of an equal but exceedingly ſmall thickneſs, it is manifeſt, that whatever be the force of gravity, wherewith the uppermoſt plate preſſes upon the ſecond, the ſecond preſſes upon the third with a double force, and the third

upon

upon the fourth with a triple force, and so on; so that the plate which is next the orifice is pressed downward by the joint gravities of the several plates which lie above it, and likewise by the force of it's own gravity, inasmuch as there is no other plate beneath it whereon to rest; consequently, from it's own gravity, and that of the several plates above it, it does all at once receive as many equal impressions from gravity, as it would successively in falling down the height of the water; and of course, must pass through the orifice, with the same velocity that it would acquire in falling down that height; but I proved in my lecture upon gravity, that the velocity which a body acquires in falling through any space, is as the square root of the space; consequently, the velocity wherewith the water flows out, is as the square root of the height of the water above the orifice.

LECT. XII.

To confirm this by an experiment; let there be two vessels in all things alike, excepting that one is four times as tall as the other, the height of one being 20 inches, and of the other 5; let each of them have a circular orifice in the bottom a fifth part of an inch in diameter; and being both filled with water, let them be set a running, and let the water be supplied as fast above as it runs out below; the taller vessel will discharge about twenty one ounces in the space of a quarter of a minute, and in the same time the shorter will discharge about 11 ounces. Now, forasmuch as the orifices through which the water flows are equal, and likewise the times of the flux, the quantities discharged are as the velocities, consequently, the velocity wherewith the water flows out of the taller vessel, is to the velocity wherewith it flows out of the shorter, as 21 to 11, that is, nearly as 2 to 1, which are the square roots of the heights of the water above the orifices. Exp. 1.

LECT. XIV. As the preſſure ſuſtained by the lower parts of water from the height of thoſe above, exerts it ſelf with the ſame force laterally that it does downward, it matters not whether the orifice through which the water flows, be at the bottom or ſide of a veſſel; for the water will flow out of both with the ſame velocity, provided they are at equal depths below the upper ſurface of the water; and therefore, the velocity of water flowing out of an orifice in the ſide of a veſſel, is as the ſquare root of the height of the water above the orifice; as will appear, by repeating the laſt experiment with veſſels whoſe orifices are in their ſides; for the quantities diſcharged will be the ſame as before. Exp. 7.

From what has been ſaid it follows, that if an orifice in the ſide of a veſſel, be ſituated as far above an horizontal plane, as it is below the upper ſurface of the water, the water will ſpout from that orifice, to the diſtance of twice the height of the orifice above the plane. Pl. 6. Fig. 12. For inſtance, if AOBC be a veſſel full of water, O an orifice in the ſide, whoſe height OD above the horizontal plane DH, is equal to OA, the diſtance of the orifice from the top of the water; DH the horizontal diſtance to which the water ſpouts, will be double of OD, the height of the orifice above the plane. For the ſpouting water has two motions, one uniform from the preſſure of the water in the veſſel, in the direction OF perpendicular to the orifice, the other accelerated from the force of gravity in the direction OD perpendicular to DH; which two motions do by no means hinder one another, but by their combination cauſe the water to ſpout in the curve of a *parabola*. Now, the velocity wherewith the water moves in the direction OF, being equal to the velocity acquired by a body in falling from A to O, or from O to D; in the ſame time that it falls from O to D, and by ſo doing, reaches the horizontal plane, it will be

carried

carried in the direction OF, through a ſpace equal to twice OD, (inaſmuch as all bodies whatever that move uniformly, with a velocity equal to that which is acquired by a body in falling through any height, do in the ſame time with that of the fall, deſcribe a ſpace double of that of the fall); conſequently, the horizontal diſtance to which the water ſpouts, will be equal to twice the height of the orifice above the plane. Thus, from an orifice in the ſide of a Exp. 3. veſſel, the depth whereof below the ſurface of the water is 20 inches, the water will ſpout to the diſtance of 38 inches on an horizontal plane, whoſe diſtance below the orifice is likewiſe 20 inches; and where the depth of the orifice below the top of the water is five inches, the water will ſpout to the diſtance of $9\frac{1}{2}$ inches on an horizontal plane ſituated at the diſtance of 5 inches below the orifice; ſo that in both caſes the diſtances to which the water ſpouts, are nearly double the diſtances of the planes below the orifices; and they would be exactly double, were it not that the water is retarded a little by the oppoſition it meets with from the air.

The diſtances to which water ſpouts on an horizontal plane, from orifices in the ſides of different veſſels, the orifices being at equal heights above the plane, or to one another as the ſquare roots of the heights of the water above the orifices.

For ſince the orifices are at equal heights above the plane, the times of the deſcent of the water from the ſeveral orifices to the plane muſt be equal; conſequently, the horizontal diſtances to which the water ſpouts, muſt be as the velocities wherewith it ſpouts; but thoſe velocities are as the ſquare roots of the heights of the water above the orifices; conſequently, ſo muſt the horizontal diſtances. Thus, if two veſſels be ſo placed, as that the orifices in Exp. 4. their ſides will be 20 inches above an horizontal plane, the height of the water in one veſſel being 20 inches above the orifice, and in the other 5; the

LECT. XIV. water will ſpout from the former, to the diſtance of 38 inches, and from the latter, to the diſtance of 19 inches; but 38 is to 19, as 2 to 1; that is, as the ſquare roots of the heights of the water above the orifices, for the heights are as 4 and one.

The diſtance to which water ſpouts from an orifice in the ſide of a veſſel, whatever be the height of the orifice above the plane, as alſo of the water above the orifice, may be thus determined; let BR repreſent an horizontal plane, F an orifice in the ſide of a veſſel at any height above the plane, and AB the height of the upper ſurface of the water above the plane. On AB as a diameter, deſcribe the ſemicircle ADB, and at F ſet off FE perpendicular to AB, and meeting the circle in E. The diſtance to which the water ſpouts on the plane BR from the orifice F, is proportional to the line FE. Pl. 6. Fig. 13.

For, from the nature of motion, the ſpace deſcribed, is as a rectangle under the time and velocity; but in this caſe, the time of the motion is as the ſquare root of FB, and the velocity wherewith the water ſpouts, is as the ſquare root of AF; conſequently, the ſpace through which the water runs in the horizontal direction, is as the ſquare root of the rectangle AFB; but, by the nature of the circle, the ſquare root of the rectangle AFB, is equal to FE; conſequently, the horizontal diſtance to which the water ſpouts on the plane BR from the orifice F, is as FE.

Hence it follows, that the diſtance to which the water ſpouts, is as the ſine of the arch AE, whoſe verſed ſine AF, is equal to the height of the water above the orifice. And, foraſmuch as any two ſines, which are equally diſtant from the center are equal, it follows, that the water muſt ſpout to the ſame diſtance from two orifices as F and L, whoſe diſtances from the center are equal; as alſo, that it muſt ſpout to the greateſt diſtance from an orifice in

in the center, the ſine CD being in that caſe equal to *radius*, and conſequently, the greateſt. LECT. XIV.

To confirm what has been ſaid; let a veſſel whoſe height is 16 inches, and which is perforated in the middle, and likewiſe at the diſtance of $5\frac{1}{2}$ inches above and below the middle, be filled with water, and ſet upon an horizontal plane; the water will ſpout from the middle orifice to the diſtance of above 15 inches, and from each of the other two, to the diſtance of about 10 inches. Exp. 5.

All things being ſuppoſed as before, the diſtances to which the water ſpouts, ſetting aſide what little diſturbance may ariſe from the reſiſtance of the air, are equal to twice the ſines of the arches, whoſe verſed ſines are equal to the heights of the water above the orifices. For, the diſtance to which the water ſpouts from the central orifice C, is to the diſtance to which it ſpouts from any other orifice as F, as the ſine CD is to the ſine FE; but foraſmuch as the orifice C is as far diſtant above the plane as it is below the ſurface of the water, the diſtance to which the water ſpouts from that orifice is equal to twice CB, or twice CD; conſequently, the diſtance to which it ſpouts from F muſt likewiſe be equal to twice FE, and ſo of any other orifice.

Water which ſpouts perpendicularly upward, ſets out with ſuch a velocity, as is ſufficient to carry it to the ſame height with the water in the veſſel from which it ſpouts. For the velocity wherewith it ſets out, is equal to the velocity acquired in falling down the height of the water; and, in my lecture upon gravity, I ſhewed, that a body thrown directly upward riſes to ſuch a height, whence if it be let fall, it will by the end of the fall acquire the ſame velocity wherewith it was thrown up; conſequently, the water ſpouts with a velocity ſufficient to carry it to an equal height with the water in the reſervoir; however, it cannot poſſibly arrive at that height, Exp. 6.

by

LECT. XIV. by reafon of the refiftance it meets with from the air; which, as it cannot be taken off, muft leffen the heights of all jets whatever, fo as to make them fall fhort of the heights in the refervoirs; befides the water in the uppermoft part of the jet, when it has loft all it's motion, refts for fome time on the part next below it, and by it's weight obftructs and retards the motion of the whole column, and thereby leffens it's height; and fo great is the refiftance arifing from this caufe, as that the jet is frequently deftroyed by it, the rifing water being by fits and ftarts preffed down to the very orifice from which it fpouts.

Exp. 7. By giving the jet a little inclination, the uppermoft parts, when they have loft their motion upward, are made to fall off from the reft, whereby the refiftance which arifes from their weight is taken off. And this is the true reafon why, *cæteris paribus*, fuch jets as are a little inclined, rife higher than thofe whofe afcents are perpendicular.

The velocity wherewith water flows out of a cylindrical pipe inferted horizontally into the fide of a veffel, is as the fquare root of the height of the water in the veffel above the place of the pipe's infertion directly, and the fquare root of the length of the pipe inverfly; for fince the pipe is cylindrical, the velocity wherewith the water flows out at one end, muft be equal to the velocity wherewith it flows in at the other; but the velocity wherewith it flows in, is in the proportion laid down; for the preffure of the incumbent water in the veffel, cannot make the water which lies next the orifice flow into the pipe, unlefs at the fame time it drives forward all the water contained in the pipe; for which reafon, the water in the pipe may be looked upon, as an obftacle which refifts and impedes the moving caufe. Now, where a caufe

acts

acts under the disadvantage of a clog or impediment, the potency of such a cause is increased, either by diminishing the impediment, or augmenting the absolute strength and vigour of the cause it self; where the strength and vigour of the cause is given, the potency thereof increases in proportion as the impediment lessens, and lessens as that increases; and where the impediment is given, the potency of the cause increases, and lessens in proportion to the increase and diminution of the absolute strength and vigour of the cause; consequently, the potency is in a *ratio* compounded of the strength or magnitude of the cause, and of the weakness or smallness of the impediment; that is, it is as the magnitude of the cause directly, and as the magnitude of the impediment inversly; or as the magnitude of the cause applied to the magnitude of the impediment. Now, in the case before us, where the pressure of the water in the reservoir is the moving cause, and the water in the pipe is the impediment, the magnitude of the former is measured by a rectangle under the height of the water, and the orifice of the pipe, and the magnitude of the latter by a rectangle under the orifice of the pipe, and the length thereof; or rejecting the orifice as being ever the same in both, the magnitude of the moving cause, is as the height of the water, and that of the impediment, as the length of the pipe; and therefore, putting H for the height of the water in the reservoir above the place of the pipe's insertion, and L for the length of the pipe; $\frac{H}{L}$ will denote the pressure of the water in the reservoir, as lessened by the resistance of the water in the pipe; and putting O for the orifice of the pipe, $\frac{HO}{L}$ will express the force which drives the water into the pipe; and forasmuch as the motion generated in any time by a force acting constantly

LECT. XIV.

LECT. XIV. conſtantly and uniformly, is as a rectangle under the force and time; putting T for the time that the water continues to flow into the pipe, $\frac{HOT}{L}$ will be as the motion generated in the water flowing into the pipe; but the motion generated in the influent water, is as the quantity which flows in, multiplied into the velocity wherewith it flows; and therefore, putting Q and V for the quantity and velocity, $\frac{HOT}{L}$ is as QV; or, becauſe the quantity which flows in, is in a *ratio* compounded of the orifice, time and velocity; by ſubſtituting O, T, V, which denote the orifice, time, and velocity, in the place of Q, we ſhall have $\frac{HOT}{L}=OTV^2$; and ſtriking out OT from both ſides, we ſhall have $\frac{H}{L}=V^2$; conſequently, V is as $\sqrt{\frac{H}{L}}$; that is, the velocity wherewith the water flows out of the reſervoir into the pipe, and conſequently, the velocity wherewith it flows out of the pipe, is as the ſquare root of the height of the water in the reſervoir, applied to the ſquare root of the length of the pipe.

Hence it follows, that if the length of the pipe be varied whilſt the height of the water in the reſervoir continues the ſame, the quantities diſcharged in any given time, will be to one another inverſly as the ſquare roots of the lengths of the pipe; for ſince the diameter of the pipe, and the time of the flux are given, the quantities diſcharged muſt be as the velocities wherewith they run out, that is, in the inverſe *ratio* of the ſquare roots of the lengths of the pipe.

To confirm this by an experiment; let a pipe of 16 feet in length, and half an inch in diameter, be

inſerted horizontally into the ſide of a veſſel; and let the water in the veſſel be kept conſtantly at the height of three feet above the place of the pipe's inſertion; the pipe when ſet a running will diſcharge about 161½ ounces in half a minute; let it then be made ſhorter by 12 feet, and ſet a running again, and it will in the ſame ſpace of time diſcharge 321 ounces, that is near twice as much as before; ſo that the quantities diſcharged will be to one another reciprocally as the ſquare roots of the lengths of the pipe, which in this caſe are as 4 to 1.

Table I.

L	Q	T
1	436½	
4	321	¾
9	211¾	1¾
16	161½	3
25	132	5
36	87	9
49	72	14
6	65	20
81	61	29
100	59	42

Table II.

D	Q
1	87¼
4	88½
9	88
16	81
25	74
36	67¼
49	65
64	58¾
81	56
100	54

June

LECT. XIV. *June* the 21ſt, 1722, I made ſeveral experiments concerning the motion and diſcharge of water through pipes, in the following manner.

There was a reſervoir of 3 feet in height, which was kept conſtantly full during the flux of the water; at the bottom was inſerted horizontally a pipe of half an inch in diameter, whoſe length when greateſt was 100 feet, but being compoſed of ſeveral pieces, was capable of being made of ten different lengths; which lengths were the ſquares of the natural numbers. Into this pipe were inſerted horizontallly (as occaſion was) ten other pipes, each of them 6 inches long, and $\frac{1}{4}$ inch in diameter; the places of their inſertion into the main pipe were diſtant from the reſervoir the ſquares of the natural numbers in feet; the axes of the ſmall pipes made an angle of 80 degrees, with that of the main pipe; the reaſon why they were inſerted in ſuch an angle was, that it had been obſerved that the water flowed out of orifices made in the main pipe nearly in that angle.

In TAB. I. L denotes the length of the main pipe (the ſmall pipes not being inſerted), Q the quantity in *troy* ounces diſcharged in half a minute of time, T the time in ſeconds which the water took to flow from the reſervoir to the extremity of the pipe, the ſame having been firſt exhauſted.

In TAB. II. D denotes the diſtance from the reſervoir, at which the ſmall pipe was inſerted into the main pipe; Q the quantity in *troy* ounces diſcharged by the ſmall pipe in half a minute of time, the main pipe being ſtopped.

TABLE

TABLE III.

1	2	3	4	5	6	7	8	9	10	P	Sum.
84¾										63¼	148
	80									59	139
		74								54½	128½
			65½							49¾	115¼
				56¼						48	104¼
					45¾					45	90¾
						41½				42	83½
							31¼			43	74¼
								26		44½	70½
									6½	59½	66
67¼	57	40	27	19½	9½	5	5¼	4	1¾	8	244¼
69¼	63½	56½	28	17½	7¼	5½	5	5	7		264½
69¾	63½	54½	29½	12½	12¾	10¼	8¾	8			269½
68	62	50	24½	27	17¼	9¾	9				267½
69¼	62½	50¾	24¾	28½	18¾	16½					271
69¼	63½	51½	25	31	27¼						267½
69¼	64	55	32	43							263¼
72¾	69	65	43								249¾
76½	75½	75½									227½
82¾	84										166¾
87¼											87¼
	76	50½	36½	27½	13¼	6	5¼	6½	¾	3½	225¾
		55	42	30½	13¼	6½	5¼	7	2	4½	166
			49	32	14¾	15¼	6¾	9	2¼	5¾	135¼
				40	30¼	17¼	8¾	9¼	3	6¼	114¾
					39	21	10½	10¼	3½	8½	92¾
						37½	16¼	11	6¾	12½	84
							25	16½	7	21	69½
								24	8¾	30½	63¼
									9¼	53	62¼
										59	59

In TAB. III. the numbers at the top denote the ten ſmall pipes, P the main pipe, and the numbers below denote the quantities in *troy* ounces diſcharged in half a minute of time, by the pipes denoted by the numbers directly above them. The blanks denote, that the pipes denoted by the numbers directly above them at the top, were ſtopped at the time that the others diſcharged.

LECTURE

LECTURE XV.

OF PNEUMATICKS.

LECT. XV. IN this lecture I shall give an account of the weight and preſſure of the air, and of ſome remarkable effects ariſing from it.

Though the weight of the air which ſurrounds us, is not perceived by reaſon of the equal preſſure which it makes on all parts of our bodies; yet that it is really heavy appears from hence, that veſſels when exhauſted are leſs ponderous than when filled with air. Exp. 1. Thus a glaſs bottle, whoſe contents are nearly 40 cubic inches, being exhauſted by means of the air pump, will be found to ſuffer a ſenſible loſs of weight; when I formerly made the experiment, the loſs of weight amounted to ten grains, and the magnitude of the exhauſted air I found to be 34 cubic inches; for upon immerſing the bottle in water, and opening the valve which covered the mouth, the quantity of water which flowed in and poſſeſſed the place of the exhauſted air, amounted to 8628 grains, which being divided by $253\frac{1}{3}$, the number of grains in a cubic inch of water, give 34 in the quotient; ſo that from this experiment it is manifeſt, that 34 cubic inches of that air, which more immediately ſurrounds us, are equal in weight to ten grains; and that the ſpecifick gravity of the ſame air, is to the ſpecifick gravity of water, as ten to 8628, or, as one to $862\frac{4}{5}$; the ſpecifick gravities of bodies equal in bulk, being to one another as the abſolute weights of the bodies.

As the air riſes above the ſurface of the earth, it grows rarer, and conſequently lighter; a given bulk of air being lighter at the diſtance of a mile, than at the earth's ſurface, and lighter again at the

I diſtance

distance of two miles, and so on continually. And yet notwithstanding this diminution of gravity in the superior parts of air, so great is the height of the atmosphere, as to render the weight of the whole very considerable; as will appear from the following experiment.

Exp. 2. Let a piece of common glass be placed as a cover on the top of a receiver; and upon exhausting the air, the glass will at first be pressed close to the receiver, and at length broken by the weight of the air, which rests upon it.

While the air continues undiminished in the receiver, it does by virtue of it's elasticity press as strongly against the lower surface of the glass, as does the incumbent air by means of it's weight upon the upper surface; as shall be shewn hereafter; consequently, as long as the air remains undiminished in the receiver, the weight of the incumbent air can have no sensible effect on the glass; but upon lessening the quantity, and therewith the spring of the included air, the glass being no longer supported from below, is pressed down, and broken by the weight of the air above; and for the same reason, a square glass phial when exhausted cracks and flies to pieces.

From the weight and pressure of the air on the surface of liquors it is, that they are made to rise in exhausted tubes open at one end, as will appear from the following experiments.

Exp. 3. Let a glass vessel with mercury be placed under a receiver, and let a tube open at one end be suspended above the vessel in such a manner, as that the open end may at pleasure be let down into the mercury; if then, the air being drawn out of the receiver, the tube be let down, the mercury will not rise therein as long as the receiver continues empty; but upon re-admitting the air, it will immediately ascend. The reason of which is, that upon ex-

LECT. XV. haufting the receiver, the tube is likewife emptied of air; and therefore, when it is immerfed in the mercury, and the air re-admitted into the receiver, all parts of the mercury are preffed upon by the air, except that portion which lies beneath the orifice of the tube; confequently, it muft rife in the tube, and continue fo to do, until the weight of the elevated mercury preffes as forcibly on that portion which lies beneath the tube, as the weight of the air does on every other equal portion without the tube. But to proceed to a fecond experiment of the fame kind.

Pl. 6. Fig. 14. Exp. 4. Let two glafs tubes as A and B, each above 30 inches long, of which A is open at one end only, but B at both, be fo contrived, as by means of fcrews, to be let into the little glafs veffel CD, in the manner reprefented in the figure. A being filled with mercury, and then fcrewed into the veffel, let mercury be poured into B, till both that and the veffel are full; let then the veffel be inverted; Pl. 6. Fig. 15. and let the extremity of B be immerfed in a veffel of mercury, the mercury will defcend through B, and continue to do fo, till A is emptied, as alfo fo much of the veffel CD as is above the level of the upper orifice of B. This being done, let A be fo far unfcrewed, as to permit the air to pafs between the threads of the fcrew into the empty part of the veffel; upon the admiffion of the air, the mercury will rife in the tube A. For, from the circumftances of the experiment it is evident, that the part of A which ftands above the level of the mercury remaining in the veffel, is perfectly void of air; confequently, while the mercury all around the tube is preffed by the newly admitted air, that portion which lies beneath the tube, fuffers no preffure from above; and of courfe muft rife, and continue to rife, until the weight of the elevated mercury becomes a balance to the preffure of the air without.

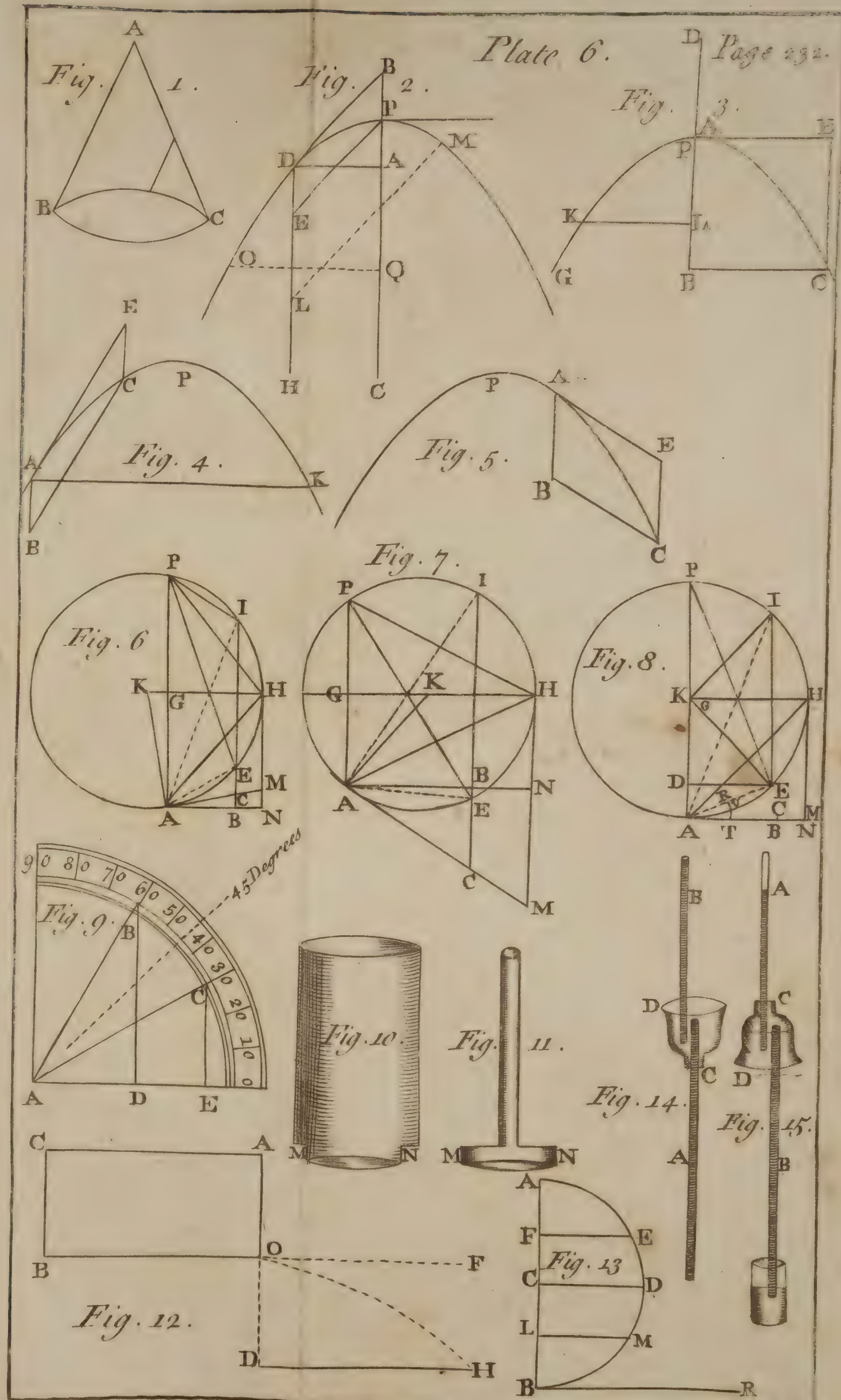


Plate 6.
Page 232.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6
Fig. 7.
Fig. 8.
Fig. 9.
45 Degrees
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13
Fig. 14.
Fig. 15.

By the weight and preſſure of the air, water is raiſed in common pumps, and fire engines, as will appear by conſidering their ſtructures, and the manner in which they work. AB repreſents the body of a pump, which is commonly an hollow cylinder of wood or lead, C a plug fixed near the bottom of the pump, with an hole in the middle, covered by a leathern valve, ſo contrived as to open and give way to the water in paſſing upward, but to ſhut cloſe and obſtruct the paſſage downward; D a ſecond plug of the ſame kind, and perforated in like manner with the former. This plug is commonly called the *ſucker* or *piſton*, and being moveable, is drawn up and thruſt down at pleaſure, by means of the iron rod E to which it is faſtned. The ſides of the ſucker are every where caſed with leather, whereby it is made to fit the cavity of the pump ſo exactly, that neither air nor water can paſs between. At ſome diſtance above the ſucker is an orifice as O in the ſide of the pump, through which the water is diſcharged at the time of working, in the following manner. The ſucker being drawn up, the ſpace between that and the lower plug is left void of air; then foraſmuch as the water, which ſtands about the pump, is every where preſſed by the air, except in that part which anſwers to the hole of the plug, it muſt there give way, and paſs up into the cavity of the pump; and upon depreſſing the ſucker again, as it cannot return downward by reaſon of the valve, which ſhuts cloſe upon the hole, and ſtops the paſſage, it riſes up through the ſucker, and lodges itſelf thereon; ſo that upon the next elevation of the ſucker, it is carried towards the top of the pump, and thrown out at the orifice O.

LECT. XV.

Pl. 7. Fig. 1.

If inſtead of an orifice above the ſucker, we ſuppoſe one juſt above the lower plug, with a valve opening outwardly, ſo as to ſuffer the water to flow out, but not to return. And if we ſuppoſe the

LECT. XV. fucker to be folid without a perforation, the figure will reprefent a forcing pump, or fire engine, in which the water rifes above the lower plug in the fame manner, and from the fame caufe, that it does in a common pump; and by the preffure made upon it by the fucker when thruft down, it is forced out at the orifice, and that fo ftrongly, as by the help of leathern pipes, to be conveyed to the tops of the higheft houfes.

The air in any particular place does not always continue of the fame weight, but is fometimes heavier, and fometimes lighter; which plainly argues a variation in the quantity, inafmuch as the gravity of any body is proportional to the quantity of matter which it contains. From what caufe this variation arifes, is not eafy to determine. Doctor HALLEY is of opinion, that the diminution of the quantity of air in any place, is the effect of two contrary winds blowing from that place, whereby the air is carried both ways from it; and of confequence, the incumbent cylinder of air is diminifhed; as for inftance, if in the *German* ocean it fhould blow a gale of *wefterly* wind, and at the fame time an *eafterly* wind in the *Irifh* Sea; or if in *France* it fhould blow a *foutherly* wind, and in *Scotland* a *northern*; that part of the atmofphere which is impendent over *England* would, he thinks, be thereby carried off and diminifhed. He likewife conceives, that the increafe of the quantity of air in any place, is occafioned by the blowing of two contrary winds towards that place, whereby the air of other places is brought thither and accumulated. And upon this foot, he endeavours to account for what is commonly obferved in this part of the world; namely, that the atmofphere, *cæteris paribus*, is always heavieft upon an *eafterly* or *north-eafterly* wind. This happens, fays he, becaufe, that in the great *Atlantick* ocean, on this fide the thirty fifth degree of *north* latitude, the *wefterly* and *fouth-wefterly* winds blow almoft always; fo that

that whenever the wind comes up here at *eaſt* or *north-eaſt*, it is ſure to be checked by a contrary gale, as ſoon as it reaches the ocean; for which reaſon, the air over us muſt needs be heaped up in greater abundance, as often as thoſe winds blow. To confirm this hypotheſis of contrary winds being the cauſe of the variation in the weight of the air, he obſerves, that within the *Tropicks*, where there are no contrary currents of air, this variation does not obtain; but that the atmoſphere continues much in the ſame ſtate of gravity in all kinds of weather. Now, whether this, or whatever elſe, be the cauſe of it, moſt certain it is, that the weight of the air does vary; and ſo conſiderable is the variation, that the weight of the air in it's heavieſt ſtate, exceeds the weight thereof when it is lighteſt, in the proportion of almoſt ten to nine.

LECT. XV.

The changes which the air undergoes as to it's gravity, are obſerved by means of the *Barometer* or weather-glaſs; which, as it was the invention of TORRICELLIUS, is known among the naturaliſts by the name of the *Torricellian tube or inſtrument*. It conſiſts of a ſmall glaſs tube, about three feet long, cloſed at one end, which being filled with mercury well purged from air, is inverted into a cylindrical box of timber, wherein ſome mercury is lodged; upon the inverſion ſome of the mercury falls out, whereby the upper part of the tube is left empty whilſt the lower part continues full. Now, foraſmuch as it has appeared from experiments, that the ſuſpenſion of the mercury in the tube, is owing to the preſſure of the air on the ſtagnant mercury; the pillar of mercury which is kept up in the tube, muſt always be equal in weight to a pillar of the atmoſphere of the ſame thickneſs; conſequently, as the weight of the atmoſphere varies, the height of the mercury in the barometer muſt do ſo too; the mercury conſtantly riſing as the weight of the air increaſes, and ſinking as that

 leſſens.

LECT. XV. leſſens. That the minute variations in the height of the mercury may be obſerved, that part of the tube which lies between the limits of the leaſt and greateſt height, to wit, from 28 to 31 inches, is graduated; each inch being divided into ten or twelve equal parts by means of a table, whereunto the tube is fixed; whereon likewiſe are inſcribed in their proper places, ſuch conſtitutions of the air and weather, as have been obſerved to accompany different heights of the mercury. In contriving this inſtrument, care muſt be taken to make the box, which contains the ſtagnant mercury, ſo large, as that upon the riſing or falling of the mercury in the tube, the height of that in the box may ſuffer little or no variation; for ſhould the ſtagnant mercury ſink upon the riſing of the mercury in the tube, or riſe as that ſinks, which muſt be the caſe where the box is ſmall; the riſe or fall of the mercury in the tube will appear to be leſs than it really is; as for inſtance, if when the mercury riſes half an inch in the tube, it does at the ſame time fall a quarter in the box, the riſe in the tube, which appears to be only half an inch, is in truth three quarters; becauſe the height of the mercury is always to be computed from the ſurface of that in the box. So, on the other hand, if the mercury by falling half an inch in the tube riſes a quarter in the box, the true deſcent in the tube is three quarters of an inch; inaſmuch as the height of the mercury in the tube above the ſurface of the ſtagnant mercury in the box, is leſs after the fall by three quarters of an inch. By making the circular *area* of the box thirty or forty times greater than that of the tube, (which is generally the caſe, the tubes of moſt barometers being but one fifth of an inch wide, and the boxes an inch and a quarter) the ſtagnant mercury in the box may be kept conſtantly at the ſame height very nearly; the greateſt variation of the height not amounting to more than the tenth or twelfth part of an inch, which is inconſiderable. If

If the tube instead of being continued directly upward, be bent at the height of 28 inches, in the manner here represented, it is then called an *inflected* or *diagonal barometer*; in which the inclined part AB may constitute an obtuse angle of any magnitude with the perpendicular part BC; but the nearer the angle approaches to a right one, the longer must the inclined part be; for it must be continued until the perpendicular altitude thereof AH, above the horizontal line HB, becomes equal to three inches, which is the difference between the greatest and least height of the mercury in the barometer; otherwise, the mercury will not have room to rise to it's utmost height, at such times as the constitution of the air requires it. This barometer shews the minute variations in the weight of the air much more accurately than the former; because the rise or fall of the mercury in the inclined part AB is very sensible, when an alteration in the perpendicular height is scarcely to be perceived. But then the box which contains the stagnant mercury, ought to be much larger in proportion in this than in the former; because in this, a much larger quantity of mercury rises into, and falls out of the tube, upon the changes of the weather. LECT. XV. Pl. 7. Fig. 2.

If the lower part of the tube in the first barometer, instead of being inserted into a box, be turned up in the form of a crook, it is then called a *curved barometer*, in which the crooked part generally terminates in a large bubble open at top. The bubble contains the stagnant mercury, which, as it is pressed upon more or less by the incumbent air, is forced up to a greater or smaller height in the strait part of the tube. In this barometer the bubble ought to be so large in proportion to the tube, as that upon the greatest variation of the height of the mercury in the tube, the height thereof in the bubble may not vary above one tenth of an inch; the Pl. 7. Fig. 4.

 necessity

LECT. XV. neceſſity there is for this, is evident from what was ſaid concerning the magnitude of the box in the firſt kind of barometer.

Beſides the barometers hitherto mentioned, there is the *wheel*, as alſo the *pendant* or *conical barometer*, and others of various kinds; which, however they may differ as to their ſtructures, do all agree in ſhewing the changes in the weight of the air, by the riſing and falling of the mercury in their tubes; wherein it ſometimes, though very rarely, deſcends as low as twenty eight inches; and at others riſes to thirty one; the mean height thereof being twenty nine inches and an half. So that a pillar of the atmoſphere, in the mean ſtate of it's gravity, is equal in weight to a pillar of mercury of the ſame thickneſs, and whoſe altitude is twenty nine inches and an half. Whence it follows, that an inch ſquare of the earth's ſurface, or of any other body contiguous thereto, ſuſtains a preſſure from the incumbent atmoſphere, when in the mean ſtate of it's gravity, equal to ſeventeen pounds, eight ounces, and 374 grains; that being the weight of a ſquare pillar of mercury one inch thick, and twenty nine and an half high.

From this great preſſure of the air it is, that two brazen hemiſpheres, whoſe diameter is three inches and an half, being laid one upon another, and then exhauſted, cling ſo faſt together, as to require above 150 pounds to ſeparate and draw them aſunder. And it muſt be obſerved, that as the globe in this experiment cannot be perfectly exhauſted, that ſmall portion of air which remains within, by expanding itſelf, contributes to the ſeparation of the hemiſpheres; for which reaſon, they are drawn aſunder by a leſs weight than that wherewith the air preſſes them together; for the diameter of the ſphere being three inches and an half, the *area* of it's greateſt circle is nine ſquare inches and three fifths nearly; conſequently, the weight of that pillar of

air which preſſes the hemiſpheres together, is not leſs than 162 pounds, even in it's lighteſt ſtate, when the mercury in the barometer ſtands at the height of 28 inches only. If the globe, after it has been exhauſted, be hung within a receiver, upon drawing the air out of the receiver, the lower hemiſphere will fall off from the other; which plainly ſhews, that their coheſion is owing to nothing elſe but the weight and preſſure of the air upon them.

Since the atmoſphere even in it's lighteſt ſtate is ſo ponderous, as that a ſquare pillar of it one inch thick, weighs ſixteen pounds, nine ounces, and 461 grains; it follows, that a middle ſized man, the ſurface of whoſe body is generally allowed to contain about fifteen ſquare feet, ſuſtains a preſſure from the atmoſphere, when in it's lighteſt ſtate, equal to the weight of 31144 pounds; which preſſure on larger bodies, and in heavier ſtates of the air, is ſtill greater; and therefore it may well be aſked, how it comes to paſs, that we are not ſenſible of this preſſure, great as it is. In anſwer to which it muſt be obſerved, that ſuch preſſures only are perceived by us, as do in ſome meaſure move our fibres, and put them out of their natural ſituation. Now, the preſſure of the air being equal on all parts of the body, cannot poſſibly move the fibres of any one part, or force them from their ſituation; but on the contrary, muſt by reaſon of it's uniformity keep all the fibres in their proper places, and as ſo doing, cannot be perceived. And that this is the caſe is evident from hence, that if the preſſure of the air be taken off from one part of the body, the preſſure on the neighbouring parts immediately becomes ſenſible. Thus, if a man covers the top of an open receiver with his hand, upon exhauſting the receiver, and thereby taking off the preſſure of the air from the palm of the hand, he will perceive a weight

a weight on the back of his hand, and that ſo great as to put him to pain, and almoſt endanger the breaking of his hand.

LECTURE XVI.

OF PNEUMATICKS.

LECT. XVI. BY the elaſticity of the air, whereof I intend to treat in this lecture, we are to underſtand that force wherewith the particles of air expand themſelves, and recede from each other, whenever the preſſure from without, which keeps them together, is taken off. The method which I ſhall obſerve in treating of this force is, Firſt, to ſhew from experiments, that the air is really indued with ſuch a force; and, Secondly, to enquire into its nature and laws.

As to the firſt; if a little warmed ale, or any other liquor ſomewhat glutinous be put into a glaſs and included in a receiver, upon exhauſting the receiver the liquor will riſe in large frothy bubbles, and run over the glaſs.

Exp. 1. As the liquor is glutinous, it retains a great number of airy particles, which upon the removal of the outward air, and therewith the preſſure which it makes on the liquor, dilate and expand themſeves; and foraſmuch as they cannot readily extricate themſelves from the liquor by reaſon of it's clamminess, they raiſe it up, and carry it over in the form of froth. And for the ſame reaſon it ſeems to be, that mead, cyder, and moſt other domeſtick wines, after they have been bottled a while, upon drawing the cork, ſpurt out and fly. For as they are all in ſome meaſure glutinous, they retain a good quantity of air; which upon corking the bottle is condenſed

densed by reason of the condensation of the air which is lodged in the neck of the bottle; besides, by the slight fermentation which such liquors commonly undergo in the bottle, a fresh supply of air is generated, equal in density to the former. When therefore upon drawing the cork, the extraordinary pressure arising from the condensed air in the neck of the bottle is taken off, the air which is dispersed through the liquor, expands it self with great force, and not finding a ready passage between the parts of the liquor, which by reason of their clamminess do not easily separate, drives the liquor before it in the manner of a spout. But to proceed; LECT. XVI.

The expansive force of the air is likewise evident from the following experiment. Let a glass bottle of a globular form, and containing a small quantity of water, have a small glass tube open at both ends, inserted into it so far as that the lower end may be below the surface of the water; and let the insertion be made by means of a screw and a collar of leathers, in such a manner as that no air may pass into or out of the bottle; let then the whole *apparatus* be placed under a tall receiver, and upon exhausting the air out of the receiver, the water will rise up through the tube in the form of a jet, which will be higher or lower in proportion as the receiver is more or less exhausted; the reason of which is, that the air included in the bottle by endeavouring to expand it self, presses upon the surface of the water, which therefore must rise in the tube, as soon as the pressure of the outward air which keeps it down is lessened; and the greater the diminution of that external pressure is, the higher the water must be thrown. Exp. 2.

If a bladder wherein a small quantity of air is included, be placed under a receiver, upon drawing the air out of the receiver the bladder will swell, and the swelling will be greater or less in proportion as the receiver is more or less emptied; Exp. 3.

which

LECT. XVI. which plainly argues an expanſive force in the included air; as does likewiſe the burſting of a full blown bladder in an exhauſted receiver; as alſo the cracking of a ſquare glaſs phial when cloſe ſtopped.

Exp. 4. Exp. 5. Exp. 6.

If a ſmall ſiphon, having a weight faſtened from the handle of the piſton, and being cloſed at the end ſo as that upon drawing up the piſton no air can get in, be ſuſpended in an inverted poſition with the weight downward, and then covered with a receiver; upon drawing part of the air out of the receiver, the weight will deſcend, and draw down the piſton; and upon the re-admiſſion of the air it will riſe again.

When part of the air is drawn out of the receiver, that portion which remains within expands it ſelf, whereby it's ſpring is ſo far weakened, as not to be able to ſtand againſt and ſupport the weight, for which reaſon the weight deſcends; whereas, upon the return of the air which was carried off, the elaſtick force is ſo far increaſed, as to become an overbalance for the weight, and upon that account drives it up.

From this and the foregoing experiments it fully appears, that the air is indued with an expanſive force. Whence that force ariſes, and what the law of it's action is, comes now to be conſidered.

The naturaliſts were formerly of opinion, that the elaſticity of the air was owing to the ſhape and figure of it's parts; for they ſuppoſed each particle of air to conſiſt of ſeveral branches, which being of a pliable nature, were capable of being compreſſed and ſqueezed together by any outward force, and of expanding and ſpreading themſelves abroad upon the removal of the compreſſing force; and this has been thought by ſome to be a full and ſatisfactory account. But that great philoſopher Sir ISAAC NEWTON, was of opinion, that the expanſive force of the air is altogether inexplicable on the foot of this,

this, or indeed any other hypothesis, except that of the air's being indued with a repelling power, whereby the particles recede and fly from each other; his words are these. LECT. XVI.

"That there is a repulsive virtue, seems also to "follow from the production of air and vapour. "The particles when shaken off from bodies by "heat or fermentation, so soon as they are beyond "the reach of the attraction of the body, receding "from it, and also from one another with great "strength, and keeping at a distance, so as some-"times to take up above a million of times "more space than they did before in the form "of a dense body; which vast contraction and "expansion seems unintelligible, by feigning the "particles of air to be springy and ramous, or "rolled up like hoops, or by any other means "than a repulsive power."

Now, supposing this to be the case, and that the repelling power of each particle exerts it self on the next adjacent particles only, as Sir ISAAC seemed to imagine, I shall shew you what the law of this repelling power is, or, in other words, how this power is varied, by varying the distance of the particles; and in order thereto, shall lay down the following PROPOSITION.

PROP. *If a fluid be composed of particles endued with a repulsive power, so as that each particle repels those, and those only, which are next it, and if the force wherewith two adjacent particles repel each other, be in a given reciprocal* ratio *of the interval of their centers; that is, putting* I *for the interval of the centers, and* P *for the index of the given power of that interval; I say, if two adjacent particles repel each other with a force that is as* $\frac{1}{I^P}$, *the force which compresses the fluid, is as the cubic root of that power of the*

LECT. XVI. *the density of the fluid, whose index is* P *increased by* 2, *or* $P+2$; *that is, putting* F *for the compressing force, and* D *for the density of the fluid,* F *is as* D *raised up to the power whose index is* $\frac{P+2}{3}$.

Exp. 7. Pl. 7. Fig. 5. For the proof of this, let a portion of the fluid be contained in a given cubic space, whose upper surface is denoted by the square ABCG, the compressing force being applied to that surface.

The elastick force of the fluid, which withstands the compressing force, and is exactly equal thereto, is the force of those parts only which compose the upper surface; because the repelling forces of the particles are supposed to exert themselves on those particles only which lye next them, and not to extend to particles more remote. But the force of the superficial parts, is as the number of particles in the surface, and the force wherewith any two adjacent particles repel each other conjointly. Now, the number of particles in the given square surface, is reciprocally as the square of the distance of the centers of two adjacent parts; that is, as $\frac{1}{I^2}$; and by supposition, the force wherewith two particles repel each other, is as $\frac{1}{I^P}$; and therefore, the elastick force of the fluid, and of consequence the compressive force, or F, is as $\frac{1}{I^{P+2}}$. The density of the fluid contained in the given cubical space, is inversly as the cube of the distance between the centers of the particles; that is, D is as $\frac{1}{I^3}$, and I is as $\frac{1}{D^{\frac{1}{3}}}$; and therefore, by substituting $\frac{1}{D^{\frac{1}{3}}}$ in the

room

room of I, F is as $D^{\frac{P+2}{3}}$; that is, the compressive force is as the cube root of that power of the density of the fluid, whose index is P+2. LECT. XVI.

COROL. From this proposition it follows, that if the density of an elastick fluid be as the force which compresses it, the particles repel one another with forces that are inversly as the distances of their centers.

For since F is as D, $\frac{P+2}{3}$ is equal to unity, and so likewise is P; consequently, the P power of I, whose reciprocal expresses the repulsive force of the particles, is equal to I.

Hence the particles of air must repel one another with forces reciprocally proportional to the distances of their centers, because the density of the air is proportional to the force which compresses it; as will appear from the following experiment.

Let an inflexed tube as AB, open at both ends, Exp. 8. be filled up with mercury to some small height, suppose DC; then stopping the end B, so as that Pl. 7. the Air may not get out when it is compressed, and Fig. 6. measuring the length of BC, that part of the shorter leg that is filled with air, which air, it is evident, is compressed by the weight of the atmosphere; let mercury be poured in at A, till the height thereof in the longer leg above the height of the same in the shorter, becomes equal to the height at which it stands in the barometer, by which means the air in the shorter leg will be compressed by twice the weight of the atmosphere; let then the length of that part of the leg which is possessed by the air under this double pressure be measured, and it will be found to be just one half of BC; whence it appears, that the spaces which a given quantity of air possesses under different pressures, are reciprocally proportional to the pressures; and consequently, inasmuch

LECT. XVI. much as the denſities of bodies where the quantity of matter is given are reciprocally as their magnitudes, the denſity of the air is directly as the compreſſing force. From this property of the air, COTES has deduced a method for determining the denſity thereof at any height; what he has delivered concerning this matter, is contained in the 5th chapter of his *Harmonia Menſurarum*, which I ſhall endeavour to explain to you; and in order thereto, ſhall lay before you ſuch properties of the logarithmic curve, as I ſhall have occaſion to make uſe of, referring you for their demonſtrations to the forementioned author, and others who have wrote of that curve.

Pl. 7. Fig. 7. Let then BDGI be a logarithmic curve, AH it's aſymptot, that is, a right line ſo ſituated with reſpect to the curve, as not to meet it till it is drawn to an infinite, or rather indefinite length, BA, DC, and GF, ordinates, that is, right lines perpendicular to the aſymptot at the points A, C, and F, and terminating in the curve. BC a tangent to the curve at the point B. The properties of this curve, which I ſhall have occaſion to mention, are theſe four.

Firſt, Any portion of the aſymptot intercepted between two ordinates, is the logarithm or meaſure of the *ratio* which thoſe ordinates bear one to the other; thus AC meaſures the *ratio* of BA to DC; and CF meaſures the *ratio* of DC to GF; and ſo likewiſe, AF meaſures the *ratio* of BA to GF. And if AC, AF, and AH be in arithmetick proportion, then DC, GF, and IH are in geometrick proportion; and if any portion of the aſymptot be a given quantity, then is the *ratio* of the two ordinates which intercept that portion, likewiſe given.

Secondly, That portion of the aſymptot as AC, which is intercepted between a tangent and an ordinate, drawn to the ſame point of the curve as B, is a given quantity; or in other words, to whatever

ever point of the curve the tangent and ordinate are drawn, the portion of the aſymptot which they intercept, is always of one and the ſame length. The portion ſo intercepted is called the *ſubtangent*, and it is the module, or that which regulates the magnitudes of all the logarithms in the ſame ſyſtem; for they are greater or leſs in proportion to the magnitudes of the ſubtangent; ſo that if in two logarithmic curves, the ſubtangent of one be double or triple the ſubtangent of the other, the meaſures of the ſame *ratios* are likewiſe twice or thrice as great in the former as they are in the latter.

Thirdly, The indefinite *areas* comprehended between the curve and the aſymptot, drawn on to an indefinite length beyond HI, are to one another as the ordinates which bound them in their wideſt parts; thus, the indefinite *Areas* BAHI, DCHI, and GFHI, are to one another, as the ordinates BA, DC, and GF.

Fourthly, The indefinite *area* BAHI, is equal to the parallelogram BACE, comprehended under the ordinate BA, and the ſubtangent AC.

Theſe things being premiſed, let AB repreſent the earth's ſurface, and let AH be a line perpendicular thereto; then, foraſmuch as the denſities of the air at different heights, are as the preſſures of the incumbent atmoſphere, and the ordinates in the curve, as the indefinite *areas* which lie beyond them; if the indefinite *area* BAHI be made to denote the weight or preſſure of all the air, and AB it's denſity at the ſurface of the earth, then by the third property of the curve, the indefinite *area* DCHI, will denote the weight or preſſure of all the air which lies above C, and the ordinate DC will denote the denſity of the air at that height; and thus it is with regard to any other height, ſo that at all heights, the denſities of the air will be denoted by the reſpective ordinates; wherefore, by

R the

LECT. XV. the firſt property of the curve, the difference between any two heights, is the meaſure of the *ratio* which the denſities of the air bear to one another at thoſe heights; thus CF meaſures the proportion which the air's denſity at the height C, bears to it's denſity at the height F. Let us now ſuppoſe the force of gravity to ceaſe, and that the air is ſo compreſſed by ſome external force, as to be every where from top to bottom of the ſame denſity, as it is at the ſurface of the earth; it's weight or preſſure which before was denoted by the indefinite *area* BAHI, may now be denoted by the parallelogram BACE, inaſmuch as by the fourth property of the curve, that *area* and this parallelogram are equal. Since then two fluids which balance each other muſt have their heights inverſly as their ſpecifick gravities, if we put unity to denote the ſpecifick gravity of the air at the ſurface of the earth, and ſay, as unity to 11890, which is the ſpecifick gravity of mercury with reſpect to that of air, ſo is $2\frac{1}{2}$ feet, which is the height of the mercury in the barometer, to a fourth number, we ſhall have 29725 feet for the height of the homogeneal atmoſphere; and this height is equal to the ſubtangent AC. For ſince the preſſure of this homogeneal atmoſphere is as it's denſity into it's height, and likewiſe as the rectangle BACE; and ſince the denſity is denoted by BA, the height muſt be denoted by AC, the module in this ſyſtem of logarithms. Hence we have a method for determining the denſity of the air at any height; for putting H to denote the height at which the denſity of the air is required, by the ſecond property of the curve, we have this analogy, as the integral number marked A, which is the module of this ſyſtem, is to the fractional number marked B, which is the module of BRIGGS's ſyſtem, ſo is H expreſſed in feet, to a fourth number, which in BRIGGS's tables is the logarithm of the

the *ratio* of the denfity of the air at the earth's fur-face, to it's denfity at the height H, anfwerable to which in the tables is the natural number expreffing that *ratio*.

LECT. XVI.

$$\overset{A}{29725} : \overset{B}{0.43429448} :: H : \frac{0.43429448 \times H.}{29725}$$

$$\frac{26400 \times \overset{B}{0.43429448}}{\underset{29725}{A}} = \overset{D}{0.385661.} \qquad \overset{D}{2.4303.}$$

Thus, for inftance, if the denfity of the air at the height of five miles, or 26400 feet, be required, by multiplying that number by the fractional number marked B, and dividing the product by the integral number marked A, we fhall have the logarithm marked C, anfwerable to which in the tables is the natural number marked D, expreffing the *ratio* of the air's denfity at the furface of the earth, to it's denfity at the height of five miles; whence it appears, that at the furface of the earth, the air is denfer than it is at the height of five miles, in the proportion of almoft $2\frac{1}{2}$ to one; but then, this on fuppofition that the force of gravity continues the fame at all heights, whereas in truth, that force decreafes in the recefs from the earth's center in the duplicate *ratio* of the diftance, which caufeth the denfities of the air at different heights to be fomewhat different from what they would be in cafe the force of gravity did not vary.

In order therefore to determine the denfities more accurately, let S be the earth's center, and AB, equal to AB in the laft figure, the earth's furface, and let F be the height at which the denfity of the air is required; let SK be a third proportional to SF and SA, and at the point K, let the ordinate KG be drawn

Pl. 7. Fig. 8.

LECT. XVI. drawn, denoting the denſity of the air at F, then taking the point M at an indefinitely ſmall diſtance above F, let SL be a third proportional to SM and SA; and at the point L, let the ordinate LN be drawn, denoting the denſity of the air at M; this being done, it will appear, that the curve BGN which paſſeth through the points G and N, is the ſame logarithmic curve with the former, but in an inverted poſition. For ſince SL is to SA, as SA to SM, and ſince SK is to the ſame SA, as SA to SF, then by equality of *ratio*, SL is to SK, as SF to SM, and by diviſion and permutation, KL is to FM, as SK to SM; or becauſe FM is indefinitely ſmall, as SK to SF, that is, as SA^q to SF^q; whence reducing that analogy into an equation, and dividing by SF^q, we ſhall have $KL = \frac{SA^q \times FM}{SF^q}$; and rejecting SA^q, as being a given quantity, we ſhall have KL as FM directly, and SF^q inverſly; but FM is as the quantity of air in the indefinitely little ſpace FM, and SF^q inverſly is as the gravitation of the ſame air, and KG is as it's denſity; conſequently, the rectangle under KL and KG, or the *area* KGNL, is as the gravitation, the quantity, and denſity of that air conjointly, that is, as it's preſſure on the air beneath it; and the ſum of all the ſimilar *areas* below KG, is as the ſum of all the preſſures above F, that is, as the denſity of the air at F, or as KG, which denotes that denſity; and KGNL which is the difference of the two ſums of all the ſimilar *areas*, one of which ſums begins from the point K, and the other from the point L, is as the difference of the air's denſities at F and M, that is, as KG—LN. Let now KL be given; that is to ſay, let the ſmall portion intercepted between KG and LN be always of one and the ſame length, in whatever parts of the line AS the points K and L are taken; then KG will be as the *area* KGNL, and

and confequently, as KG—LN; whence by divi- fion, KG will be as LN, fo that the *ratio* of KG to LN is given, and of courfe the given line KL will be the meafure of that given *ratio*; whence by the firft property of the logarithmic curve, the curve which paffeth through the points G and N is a logarithmic curve; and it is alfo the fame with the former; for taking AO the height above the earth's furface indefinitely fmall, it is evident, that the force of gravity is the fame at O that it is at A, confequently, the denfity of the air at O will come out the fame, whether the law of gravity be taken into the confideration or left out; let then the ordinate OP be drawn in the former curve, and at the fame diftance from A in the latter curve, let the ordinate PQ be drawn. Now, fince one and the fame denfity of the air at the earth's furface, is denoted in both curves by the equal ordinates BA, it is evident, that the ordinates OP and PQ, which in the two curves denote one and the fame denfity at O, muft likewife be equal; whence it follows, that both curves have the fame curvature, as alfo the fame inclination of their tangents at the points B, and their fubtangents equal; that is, the latter curve is the fame with the former, but in an inverted pofition. Now, forafmuch as BA in the latter curve denotes the denfity of the air at the furface of the earth, and GK it's denfity at F, it is evident by the firft property of the curve, that in this fyftem, AK is the meafure of the *ratio* which the denfity at the furface has to the denfity at F; the firft thing therefore which muft be done, in order to difcover the denfity at F, is to find out the line AK, and this is done by diminifhing AF in the fame proportion that the earth's femidiameter SA is lefs than SF, the diftance of F from the earth's center; for by the conftruction, SF is to SA, as SA to SK; whence by divifion, SF : SA : : AF : AK. AK being thus obtained, let it be called H; then, by the fame

LECT. XVI. procefs as before, we may difcover the denfity of the air at the height F.

$$4005 : 4000 :: 5 : \frac{4000 \times 5}{4005} = \overset{E}{4.99375} = \overset{\text{feet.}}{26367}.$$

$$\frac{26367 \times \overset{B}{0.43429448}}{\overset{A}{29725}} = \overset{F}{0.385232}. \qquad \overset{G}{2.4279}.$$

For inftance, if the denfity of the air at the height of five miles be required as before; then by faying, as 4005 miles, that is SF, is to 4000 miles, that is SA, fo is five miles, that is AF, to a fourth, we fhall have the number marked E, expreffing miles, and parts of a mile, equal to 26367 feet, which being multiplied by the fractional number marked B, and the product divided by the integral number marked A, we fhall have the fractional number of BRIGGS's tables marked F, anfwerable to which is the natural number marked G, expreffing the *ratio* of the air's denfity at the furface of the earth, to it's denfity at the height of five miles. After the fame manner may the *ratio* of the air's denfity at the furface, to it's denfity at any height be computed. The refult of fuch computations I have fet down in the annexed table; the firft column of which contains the heights of the air in *Englifh* miles, whereof 4000 make a femidiameter of the earth. The numbers in the fecond column exprefs the *ratio* of the air's denfity at the furface, to it's denfity at the refpective heights, and they likewife denote the rarity or expanfion of the air at thofe heights. The third column contains the denfities and compreffions at the feveral heights. The numbers at the bottom of the fecond column included in crotchets denote, that fo many figures are to be annexed to the five preceding, and thofe included in the crotchets at the bottom of the third column

column denote, that so many decimal cyphers are to be prefixed to the five following figures. LECT. XVI.

Heights of the air in *English* miles.	Rarity and expansion.	Compression and density.
0	1	1
$\frac{1}{4}$	1.0454	0.95676
$\frac{1}{2}$	1.0928	0.91509
$\frac{3}{4}$	1.1424	0.87535
1	1.1943	0.85405
$1\frac{1}{4}$	1.2429	0.80456
$1\frac{1}{2}$	1.3052	0.76616
$1\frac{3}{4}$	1.3644	0.73290
2	1.4263	0.70118
$2\frac{1}{4}$	1.4871	0.67244
$2\frac{1}{2}$	1.5586	0.64160
$2\frac{3}{4}$	1.6292	0.61379
3	1.7031	0.58716
$3\frac{1}{4}$	1.7883	0.55919
$3\frac{1}{2}$	1.8596	0.53775
$3\frac{3}{4}$	1.9460	0.51387
4	2.0336	0.49173
$4\frac{1}{4}$	2.1257	0.47043
$4\frac{1}{2}$	2.2221	0.45002
$4\frac{3}{4}$	2.3226	0.43012
5	2.4279	0.41187
10	5.9182	0.16897
20	34.288	0.029164
30	198.34	0.0050418
40	1136.	0.00088028
50	6449.2	0.00015505
100	33584[3]	0. [7] 26798
400	11271[24]	0. [28]88723
4000	19316[150]	0.[154]51770
40000	33097[276]	0.[280]30214
400000	32859[301]	0,[305]30433
1000000	22002[303]	0.[307]45450
Infinite.	37311[304]	0.[308]26802

 COROL.

LECT. XVI. COROL. Since SF is by conſtruction equal to $\frac{SA}{SK}$, and ſince from the nature of muſical proportion, the quotients ariſing from the diviſion of one and the ſame quantity by quantities in arithmetick progreſſion, conſtitute a ſeries of muſical proportionals, it follows, that if ſeveral diſtances from the earth's center as SF, be taken in muſical progreſſion, their reciprocals as SK, muſt be in arithmetick progreſſion; and by the firſt property of the logarithmic curve, the denſities of the air as KG, muſt be in geometrick progreſſion.

Since the denſity of the air is proportional to the compreſſing force, and ſince the compreſſing force is equal to the elaſtick force, it is manifeſt, that if the denſity of the air be increaſed, the elaſticity will likewiſe increaſe in the ſame proportion; and on this principle are founded artificial fountains, which play by means of condenſed air; they are of two kinds, ſingle and double. The

Pl. 7. Fig. 9. ſingle fountain is made of braſs, and is every where ſhut, excepting that through the middle of the baſon BB, there paſſes down a pipe PP, whoſe lower end reaches nearly to the bottom of the fountain, and to the upper end is fitted a ſtop-cock, by help of which the pipe may be ſhut or opened at pleaſure.

Exp. 8. Some part of the fountain as ADC, being filled with water poured in through the pipe, a condenſing or forcing ſyphon is ſcrewed to the top of the pipe above the cock, by means whereof, a great quantity of air is thrown into the pipe; which as it cannot return back, by reaſon of a valve which ſhuts cloſe upon the hole of the ſyphon, forces it's way through the water into the upper part of the fountain, and there remains in a ſtate of condenſation, greater than that of the outward air. When therefore the condenſer is taken off, and the cock opened, the included air preſſing ſtrongly on the water which lies

beneath

beneath it, throws it up through the pipe, and thereby makes a jet.

The force wherewith the water is thrown up, is proportional to, and may be expreſſed by the exceſs of the denſity of the included air above that of the external air. For if the included air be equally denſe with that without, it's elaſtick force muſt be equal to the compreſſive force of the atmoſphere; conſequently, thoſe two forces will balance one another, and the water will continue at reſt, being preſſed as ſtrongly downward by the weight of the external air, as it is upward by the expanſive force of the included air; but if the included air be more denſe than the external, it's elaſtick force will exceed the compreſſive force of the atmoſphere, in the ſame proportion that it's denſity exceeds the denſity of the outward air; conſequently, that part of the expanſive force of the included air which raiſes the water, is proportional to, and may be expreſſed by, the exceſs of the denſity of the included air above that of the external air. So that putting F for the force which raiſes the water, D for the denſity of the included air, and 1 for the denſity of the air without, F is as $D-1$.

The height in feet to which the water riſes, ſetting aſide all impediments, is equal to the product ariſing from the multiplication of 33 into the exceſs of the denſity of the included air above that of the outward air; that is, putting H for the height of the jet, and x for 33, $H=x\,D-x$.

For as water which is driven out of a reſervoir by the preſſure of the incumbent water, if it ſpouts directly upward, riſes to the ſame height with the water in the reſervoir; ſo if it be driven by any other force, it muſt riſe to an equal height with a pillar of water whoſe preſſure is equal to that of the driving force; foraſmuch therefore as the atmoſphere makes an equal preſſure with a height of water of 33 feet, the

LECT. XVI. the water will be thrown to the height of 33 feet by the compressive force of the atmosphere; wherefore if we put 1 for the pressure of the atmosphere, and say, as one is to 33 or x, so is D—1, which expresses that part of the pressure of the included air which drives out the water, to a fourth proportional, we shall have xD—x, or $x \times \overline{D-1}$, for the height to which the water is thrown; whence it appears, that if D—1 be equal to unity, which is the case when the air within is as dense again as that without, the water will rise to the height of 33 feet; and if D—1 be equal to 2, which is the case when the included air is thrice as dense as the external, the height of the jet will be 66 feet, and so on.

The double fountain consists of two single fountains, whose bottoms are fastened to an hollow brass cylinder, one at each end, in the manner represented in the figure, wherein AA and BB denote the two fountains with their basons; CC the hollow cylinder, which plays upon the pins DD as upon an axle; each has a pipe as P, whose lower end reaches nearly to the bottom of the fountain. From the bason of the fountain AA, there issues another pipe as T, which passing through AA, and likewise the hollow cylinder CC, without communicating with either, opens at E into the fountain BB. And in like manner such another pipe issuing from the bason of BB, and passing through that fountain and the cylinder, opens into the fountain AA. The hollow cylinder being placed in an upright posture by means of the carriage which supports it, and the pipes of the lower fountain being stopped, water is conveyed into it through the pipe T, which issues from the bason of the upper fountain; by the running in of the water the air contained in the lower fountain is crowded into a smaller space, and thereby condensed; if then both the pipes of the upper fountain

Pl. 7. Fig. 10.

Exp. 9.

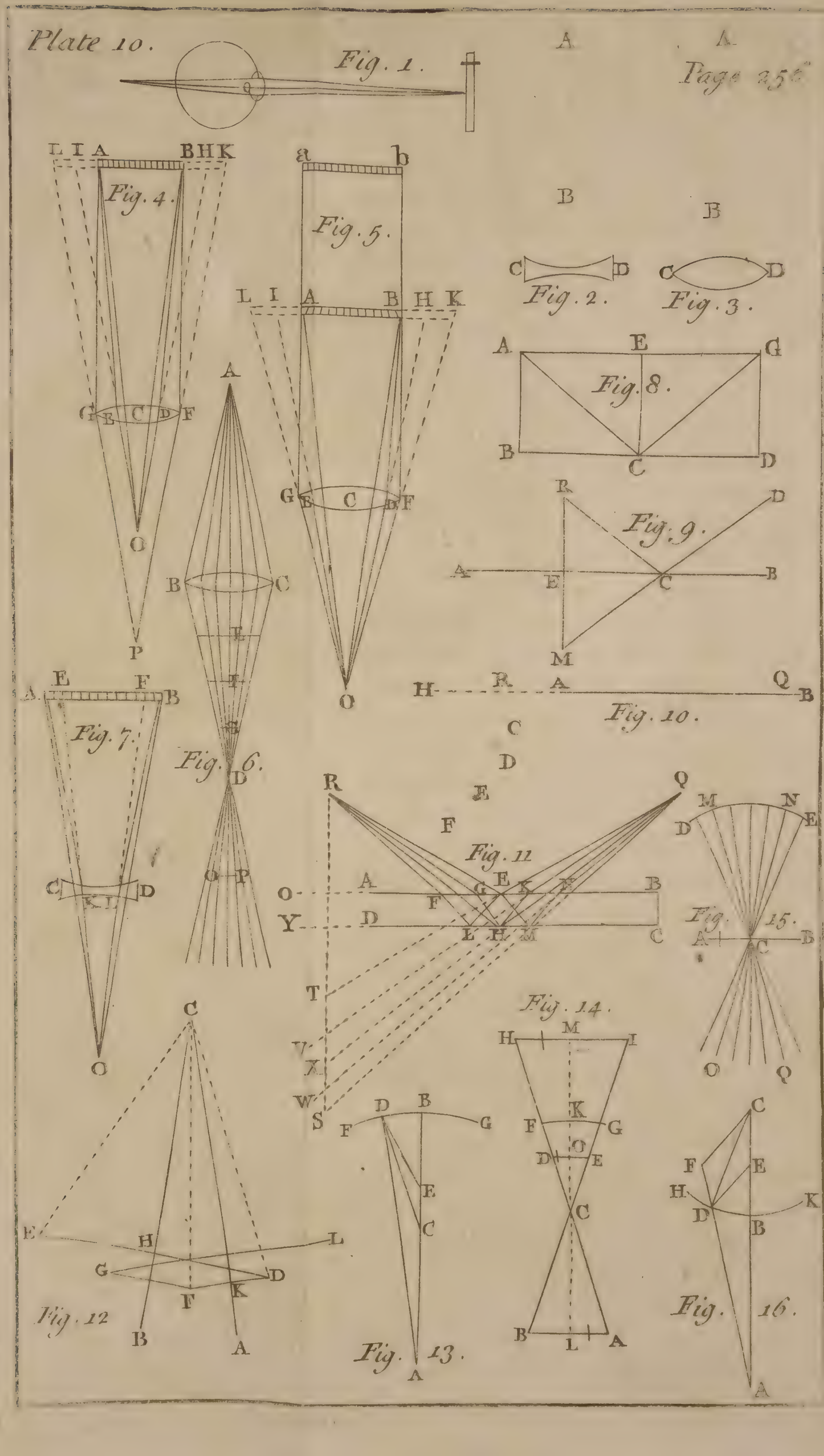


Plate 10.
Page 256
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11
Fig. 12
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.

fountain be ſtopped, and the lower fountain be brought into the place of the upper, by turning the cylinder on it's pins, the water which it contains will fall to it's bottom, and the lower end of the pipe P will be immerſed therein, in the manner repreſented in the upper fountain; ſo that upon opening that pipe, the water will be driven through it by the expanſive force of the condenſed air; and as it falls into the baſon, it will be conveyed thence by the pipe T into the lower fountain; and when the upper is exhauſted and ceaſes to play, then ſtopping it's pipes, and changing the places of the fountains as before, the other may be ſet a going in the ſame manner.

LECTURE XVII.

OF SOUNDS.

IN this lecture I ſhall firſt explain to you the NATURE OF SOUNDS, and then treat of the VIBRATIONS OF MUSICAL STRINGS. LECT. XVII.

That SOUNDS have a neceſſary dependance on the air, will appear from the following experiment.

Let a bell be placed under a receiver in ſuch a manner as that it may be rung at pleaſure; and upon drawing the air out of the receiver, the ſound of the bell will grow leſs and leſs audible in proportion to the degrees of exhauſtion, ſo as at laſt almoſt to die away, and ſcarcely to be heard at all; and upon re-admitting the air, the ſound will revive again, and increaſe in proportion to the quantity of air that is taken in. Exp. 1.

As this experiment proves the air to be neceſſary to the production of ſounds, ſo the tremblings which great guns, bells, drums, and many other ſounding bodies communicate, by means of the

 intermediate

LECT. XVII. intermediate air, to ſuch bodies as are near them, plainly ſhew, that ſounds depend on tremulous motions of the air; which therefore I ſhall endeavour to explain to you, together with the cauſe and manner of their production. When the parts of a bell, a muſical ſtring, or any other elaſtick body are ſet in motion by a ſtroke, they vibrate, that is, they go forward and return backward alternately through very ſhort ſpaces; in going forward they propel, and thereby compreſs and condenſe the air which lies next them; and in returning backward, they ſuffer the compreſſed air to recede and expand it ſelf; ſo that the parts of the air which are contiguous to the trembling body, go and return in the ſame manner with the parts of the body; and as they are endued with a repulſive power, they muſt by means thereof excite the ſame vibrations in thoſe parts which lie next beyond them; and theſe again, muſt in like manner agitate the parts beyond them, and ſo on continually; ſo that by one ſingle vibration of an elaſtick body, a motion is excited in the air, and propagated directly forward, by which ſome parts go forward, whilſt others return back, and that alternately, as far as the motion reaches.

Pl. 7. Fig. 11. That this motion may more readily be conceived, let ST repreſent an elaſtick ſtring, ſtretched and made faſt at both ends; and by a force applied to the middle point H, let it be drawn into the poſition SET; upon the removal of the force which inflects it, it will by vertue of it's elaſticity return to it's former poſition SHT; and foraſmuch as the reſtitutive force acts conſtantly upon it during the time of it's motion from E to H, it's motion through that ſpace muſt be continually accelerated, and the velocity thereof muſt be greateſt at H. When the ſtring has recovered the poſition SHT, it will not remain therein; but by vertue of the velocity acquired in moving from E to H, it will be carried forward till it has moved through a ſpace as HK, equal

to EH, and then it's motion forward will ceaſe; for as it moves towards K, the elaſtick force acts continually upon it in drawing it back, and by ſo doing, retards the motion from H to K, in the very ſame manner that it accelerated the motion from E to H; conſequently, by the time that the ſtring has moved from H to K, it will have loſt all that velocity which it acquired in moving from E to H; as ſoon as it ceaſes to go forward, it will be brought back again from K to H by the force of elaſticity; with an accelerated motion, in the very ſame manner as it was at firſt from E to H; and when it has arrived at H, it will by vertue of the acquired velocity, go on to E, with a retarded motion, in the ſame manner as it did from H to K. The motion of the ſtring from E to K and back again, is called a *vibration*; and it is evident from what has been ſaid, that ſetting aſide all external impediments, a ſtring which has made one vibration, muſt continue to vibrate for ever through the ſame ſpace; but, whereas it meets with continual reſiſtance from the air, the ſpace through which it vibrates, muſt on that account grow leſs and leſs continually, and at length vaniſh; and yet, notwithſtanding this variation in the ſpace, the times of the vibrations are all equal, as I ſhall demonſtrate before the cloſe of this lecture; but I take notice of it in this place, becauſe one of the chief properties of the pulſes of the air, whereof I ſhall have occaſion to make mention preſently, has a neceſſary dependance thereon.

LECT. XVII.

When the ſtring is drawn into the poſition SET, if we ſuppoſe A, B, C, and ſo forth, to be particles of air placed in a right line one beyond another, and that the diſtance of the firſt particle from the ſtring at E, is equal to the interval of any two adjacent particles, as it muſt needs be, on ſuppoſition that the particles of the air fly from other bodies with the ſame force that they repel one another; upon letting

LECT. XVII. ting the ſtring go, as it cannot move forward without approaching to the particle A, it muſt in the very next inſtant after it begins it's motion, propel that particle; which for the ſame reaſon, muſt in the next inſtant after it begins to move propel the particle B, and that muſt in the ſame manner propel C, and C propel D, and ſo on; ſo that the ſtring, and the ſeveral particles of air taken in their order, will begin to move forward ſucceſſively one after another, at very ſmall intervals of time. And whereas the ſtring is accelerated in it's motion from E to H, and retarded in it's motion from H to K, the particle A muſt likewiſe be accelerated in one half of it's progreſs, and retarded in the other; for ſince A is equally diſtant from the ſtring, and from B, before the vibration commences, and ſince it begins to move forward a little later than the ſtring; it is evident, that upon the firſt motion of the ſtring, the diſtance between that and A, muſt become leſs than the diſtance between A and B; and foraſmuch as the increments of velocity which are continually generated in the ſtring by the action of it's elaſticity, are not communicated to the particle A, in the inſtant of time wherein they are generated, but a little later; it is manifeſt, that the ſtring during it's motion from E to H, muſt continually be nearer to A than A is to B; and conſequently, muſt act more forcibly in driving A forward, than B does in driving it backward, and by ſo doing accelerate it's motion. After the ſtring has arrived at H the middle point of it's progreſs, and ceaſed to be accelerated, in the very next moment A likewiſe reaches the middle point of it's progreſs, and ceaſes to be accelerated, being driven as ſtrongly backward by B, as it is forward by the ſtring. But however, by vertue of the acquired motion, it continues to go forward, but with a retarded motion; and is at length ſtopped by the repulſive

repulſive power of B; in the ſame manner that the ſtring in moving from H to K is retarded, and at laſt ſtopped by the action of it's elaſtick force. After the ſtring has reached K, the utmoſt limit of it's progreſs, in the very next moment does A likewiſe reach the utmoſt limit of it's progreſs, and then turning back, purſues the ſtring, which had likewiſe turned back the moment before. And as the ſtring is accelerated during it's return from K to H, and retarded from H to E; ſo the particle A during the firſt half of it's return, being nearer to B than it is to the ſtring, muſt be accelerated; and during the latter half, being nearer to the ſtring, is thereby retarded, and at length ſtopped upon it's arrival at the place from whence it ſet out, which happens immediately after the ſtring has returned to E; and there it continues at reſt, unleſs by a ſecond vibration of the ſtring it be again driven forward in the ſame manner as before. As this particle is made to go and return through a very ſhort ſpace, by the impulſe of the ſtring, ſo likewiſe are the ſeveral ſucceeding particles, by the impulſes of the foregoing; and as the ſtring, and the ſeveral particles taken in their order, begin their motions forward, ſucceſſively one after another at very ſmall intervals of time, ſo likewiſe do they begin to return in their order at the ſame intervals of time; whence it follows, that ſome of them muſt go forward, at the ſame time that others return back. As the particles which go forward begin their motions ſucceſſively one after another, they muſt neceſſarily come nearer together; that is, they muſt be condenſed. And it muſt be obſerved, that the condenſation goes forward continually; for in the very next inſtant after any particle as D, has made it's neareſt approach to E, E muſt make it's neareſt approach to F; and in the next inſtant F muſt make it's neareſt approach to G, and ſo on continually; ſo that the condenſation

LECT. XVII. ſation muſt paſs forward ſucceſſively in a regular manner through the ſeveral particles of air.

But that I may explain this vibratory motion of the air more particularly, it muſt be obſerved, that as the ſtring during the firſt half of it's progreſs from E to H is continually accelerated, it's diſtance from the particle A muſt conſtantly grow leſs; and foraſmuch as during the latter half of it's progreſs from H to K, it is continually retarded, and that in the ſame uniform manner that it was accelerated from E to H, it's diſtance from A muſt conſtantly be inlarged, and that in the ſame regular manner that it was diminiſhed during the progreſs of the ſtring from E to H; ſo that by the time it has arrived at K, the utmoſt limit of it's progreſs, it is juſt as far diſtant from the particle A, as it was when it firſt ſet out. Upon the return of the ſtring, inaſmuch as it is continually accelerated from K to H; it's diſtance from the particle A muſt ſtill be inlarged; and foraſmuch as it is retarded in it's motion from H to E, in the very ſame manner as it was accelerated from K to H, it's diſtance from A muſt conſtantly grow leſs in the ſame regular manner that it was inlarged during it's motion from K to H, ſo that upon it's return to E, it is again juſt as far diſtant from the particle A, as it was at it's firſt ſetting out. From what has been ſaid, it is evident, that the ſtring during the time of it's progreſs, is always nearer to the particle A, than it was before it's motion began, and that it's leaſt diſtance from the particle is at H, the middle point of it's progreſs; it is likewiſe manifeſt, that during the time of it's return, it is always more diſtant from the particle than it was before it's motion began; and that it's greateſt diſtance from the particle is at H, the middle point of it's return. And what has been thus ſhewn of the ſtring with reſpect to the particle A, is in like manner true of that particle with reſpect

ſpect to the particle B, and of B with reſpect to C, and ſo on of every particle, with reſpect to that which lies immediately beyond it, as far as the motion reaches; ſo that each particle with regard to that which lies immediately beyond it, is in a ſtate of condenſation during it's progreſs, and of rarefaction during it's return, it's greateſt condenſation being at the midſt of it's progreſs, and it's greateſt rarefaction at the midſt of it's return. What proportion theſe rarefactions and condenſations bear to the denſity of the air in it's natural ſtate, in every point of that ſmall ſpace through which a particle of air vibrates, ſhall be ſhewn in my next lecture, as alſo the law of this vibratory motion.

As the parts which go forward, do in their progreſſive motion ſtrike ſuch obſtacles as they meet in their way, they are for that reaſon called *pulſes*; and the ſenſations which are excited in the mind by the ſtrokes of theſe pulſes on the drum of the ear are called *ſounds*; ſo that ſounds as conſidered in their phyſical cauſes, are nothing elſe but the pulſes of the air. In order therefore to explain the nature of ſounds, I ſhall lay before you the chief properties of theſe pulſes.

The firſt of which is, that they are propagated from the trembling body all around in a ſphærical manner. For though the parts of the body, by whoſe vibrations the pulſes are generated, do go and return according to certain directions, yet foraſmuch as every impreſſion which is made on a fluid is propagated every way throughout the fluid, whatever be the direction wherein it is made, the pulſes muſt ſpread and dilate, ſo as to form themſelves into concentric ſphærical ſurfaces, or rather thin ſhells, whoſe common center is the place of the ſounding body. And hence appears the reaſon why one and the ſame ſound may be heard by ſeveral perſons, though differently ſituated with reſpect to the ſounding body.

LECT. XVII. A ſecond property of the pulſes is, that they grow leſs and leſs denſe as they recede from the ſounding body, and that in the ſame proportion with the ſquares of their diſtances from the body. For whatever be the force wherewith the ſounding body acts on the firſt ſphærical ſhell of air, with the very ſame force does that ſhell act upon the ſecond, and that again upon the third, and ſo on continually; ſo that the force which condenſes the air in the ſeveral ſhells is given; conſequently, the condenſations which it produces in thoſe ſhells; muſt be inverſly as the reſiſtances it meets with; but the reſiſtances are as the ſhells; and therefore, ſince thoſe increaſe continually in the ſame proportion with the ſquares of their diſtances from the center, their denſities muſt decreaſe in the ſame manner.

By reaſon of this diminution in the denſities of the pulſes, thoſe which are farther removed from the ſounding body, make ſligher impreſſions on the drum of the ear, than thoſe which are leſs diſtant; and hence it is, that ſounds grow leſs and leſs audible, the farther they go from the ſounding body; and at certain diſtances become ſo weak as not to be heard at all.

A third property of the pulſes is, that all of them, whether denſer or rarer, move equally ſwift, ſo as to be carried through equal ſpaces in equal times, as I ſhall demonſtrate in my next lecture.

From this property it follows, that all ſounds, whether they be loud or low, grave or acute, move equally ſwift, the ſofteſt whiſper making equal ſpeed with the noiſe of a cannon, or the loudeſt thunder-clap; and it has been found by experiment, and I ſhall likewiſe demonſtrate in my next lecture, that ſounds move at the rate of 1142 feet in a ſecond of time or thereabouts; for the velocity is not preciſely the ſame in all ſeaſons of the year, but is ſomewhat greater in *ſummer* than in *winter*, on account of the heat which renders the air more elaſ-

tick

tick in proportion to it's denſity, than it is in the cold *winter* ſeaſon. LECT. XVII.

A fourth property of the pulſes is, that all thoſe which are excited by the vibrations of one and the ſame body, are at equal diſtances from one another. For ſince each pulſe is excited by one ſingle vibration of the ſounding body, and ſince all the pulſes move with equal and uniform velocities, it is manifeſt, that they muſt ſucceed one another at diſtances proportional to the times of the vibrations; but the times of the vibrations of one and the ſame body are all equal; conſequently, the intervals of the pulſes are ſo too. And it muſt be obſerved, that the interval between two pulſes, which is by ſome called the *length*, and by others the *breadth* of a pulſe, is that ſpace through which the motion of the air is carried, during the time, wherein any one particle performs it's vibratory motion in going forward and returning back.

On the intervals of the pulſes depend the tones of ſounds; and here I muſt obſerve to you, that all the variety there is in ſounds, reſpects either their *ſtrength* or their *tone*; with regard to their ſtrength, they are diſtinguiſhed into *loud* and *low*; and with reſpect to their tone, into *grave* and *acute*, otherwiſe called *flat* and *ſharp*. The ſtrength of any ſound depends on the magnitude of the ſtroke, which is made by a pulſe on the drum of the ear; the greater the ſtroke is, the louder is the ſound which it excites, and the weaker the ſtroke, the lower the ſound; and whereas all the pulſes move with equal velocities, the magnitude of the ſtroke, and conſequently the ſtrength of the ſound, muſt be as the quantity of matter in the pulſe; that is, as a rectangle under the denſity and breadth of the pulſe; and ſuppoſing the breadth of the pulſe to be given, it muſt be as the denſity.

The tone of a ſound depends on the duration of a ſtroke; the longer a ſtroke is which a pulſe makes

LECT. XVII. on the drum of the ear, the more grave is the found which it produces; and the fhorter the ftroke, the more acute is the found; but fince all the pulfes move equally fwift, the duration of a ftroke muft be proportional to the interval between two fucceffive pulfes; and of confequence, a found is more or lefs grave or acute in proportion to the length of that interval. Hence it follows, that all the founds from the loudeft to the loweft, which are excited by the vibrations of one and the fame body, are of one tone. It likewife follows, that all thofe founding bodiés, whofe parts perform their vibrations in equal times, have the fame tone; as alfo, that thofe bodies which vibrate floweft, have the graveft or deepeft tone; and on the contrary, thofe which vibrate quickeft have the fharpeft or fhrilleft tone.

As there may be an infinite variety in the times wherein founding bodies perform their vibrations, fo may there likewife in the tones of the founds which depend thereon; and yet amidft this great variety, muficians acknowledge but feven principal notes in an *octave*; for though the eighth be requifite to complete the feven intervals in an octave, yet are there in truth but feven notes; for that which is called the *eighth*, becomes the bafe or ground note in the next octave afcending; and as it ftands in the limits of the two octaves, it is called the *eighth* with refpect to the bafe note below it, and the ground or bafe note with refpect to the 15th which is above it; which 15th is likewife the bafe in the next afcending octave; for by a repetition of notes, wherein the proportions of the times of the notes in the firft octave are preferved, the octaves may be continued on both ways, afcending and defcending, and that in *infinitum*; and yet, notwithftanding this infinite progreffion in the octaves, the number of harmonic founds is limited. Mr. SAUVEUR is of opinion, that all the harmonic founds, that is, fuch founds as can be heard diftinctly and with pleafure, and

and in whose tones a difference can be clearly perceived by the air, lie within the compass of ten octaves; as also, that all sounds whatever, from the lowest harmonic sound, to the highest that the human ear can well bear, are contained within the limits of two octaves more. And if this be the case, it follows, that that body which gives the shrillest sound that the ear can bear, makes 4096 vibrations in the same time that one vibration is performed by that body which gives the gravest harmonic sound; for since in every octave, the time of the eighth is $\frac{1}{2}$ of the time of the base note, if $\frac{1}{2}$ be raised up to the 12th power, it will exhibit the time of the shrillest sound, that of the gravest being unity; but the 12th power of $\frac{1}{2}$ is the 4096 part of an unite; consequently, the time of the shrillest sound that the ear can well bear, and likewise of the vibration which produces it, is to the time of the gravest harmonic sound, and of the vibration whereby it is produced, as 1 to 4096; but the times of the vibrations of two bodies are inversly, as the numbers of vibrations which they perform in a given time; consequently, the body which gives the shrillest sound performs 4096 vibrations in the same time that the body which gives the gravest harmonic sound performs one; and forasmuch as Mr. SAUVEUR has found by some experiments which he made on organ pipes, of which I shall give you an account in my next lecture, that a body which gives the gravest harmonic sound, vibrates 12 times and an half in a second, the shrillest sounding body must perform 51100 vibrations in the same time; which argues great swiftness in the vibrating parts; and yet, great as it is, it has nothing extraordinary or surprising in it, if compared with the velocity of some other motions; for if we suppose the parts in each vibration to run through a space equal to the 10th part of an inch, though it is highly probable, that the lengths they run are much shorter;

LECT. XVII. and if we ſuppoſe them to move with the ſame velocity during the whole time of their motion; it follows, that they are carried at the rate of 425 feet and ten inches in a ſecond; conſequently, they do not move with much more than two third parts of the velocity wherewith a ball flies from the mouth of a cannon.

The fifth and laſt property of the pulſes is, that they may be propagated together in great numbers from different bodies, without diſturbance or confuſion; as is evident from conſorts, wherein the ſounds of the ſeveral inſtruments are conveyed diſtinctly to the ears of the audience; as they move along, ſome of them coincide and ſtrike the drum of the ear at one and the ſame time, and thereby excite a ſmooth and regular motion, that is pleaſing and agreeable; whilſt others which do not mix and unite, at leaſt not frequently, ſtrike the ear at different inſtants of time, and thereby diſturb each other's motions, ſo as to render them harſh, grating, and offenſive. And hereon depends almoſt the whole of concords and diſcords in muſick; ſuch ſounds, generally ſpeaking, being deemed concords, as are excited by pulſes which have frequent coincidences; and on the other hand, ſuch ſounds being called *diſcords*, as ariſe from pulſes which coincide but rarely.

The frequency or infrequency of the coincidences depends on the proportions which the intervals of the pulſes bear one to another; as I ſhall ſhew you in relation to the ſeveral notes in an octave; in doing of which, inſtead of the pulſes and their intervals, I ſhall conſider the vibrations of the bodies which excite the pulſes, and the times of thoſe vibrations; becauſe the number of pulſes is always equal to the number of vibrations in the ſounding bodies, and the intervals of the pulſes proportional to the times of the vibrations.

If two vibrating bodies begin their motions together and vibrate in equal times, it is manifeſt, that their vibrations

brations muſt keep pace together, and conſtantly coincide. But if the vibrations be performed in unequal times, it is plain, that they cannot conſtantly keep pace together; for which reaſon ſome of them only will coincide; and which thoſe are may be determined from the times of the vibrations; for ſince the numbers of vibrations, which are performed in a given time, are inverſly as the times of the vibrations, if the numbers which expreſs the times of the vibrations of two bodies be taken reciprocally, they will exhibit the coincident vibrations of the reſpective bodies. For inſtance, if the time of the vibrations of one body, be to the time of the vibrations of another, as 8 to 9, which is the caſe of two bodies, whereof one ſounds a ſecond or tone major to the other, every ninth vibration of the former coincides with every height of the latter. So again, if the times of the vibrations be to one another, as 5 to 6, which is the caſe, where one body ſounds a leſſer 3d to the other, every ſixth vibration of the former falls in with every fifth of the latter.

In this ſcheme, I have ſet down thoſe fractional numbers which expreſs the proportions that the times of the vibrations of thoſe bodies, which ſound the ſeveral notes in an octave, bear to the time of the vibration of that body which ſounds the baſe note; by the help of which numbers the coincident vibrations may be readily diſcovered. For in each fraction, the denominator exhibits the coinciding vibration of that body which ſounds the note, and the numerator the coinciding vibration of the body which ſounds the baſe note.

$\frac{1}{2}$	*Eight.*
$\frac{8}{15}$	*Greater ſeventh.*
$\frac{5}{9}$	*Leſſer ſeventh.*
$\frac{3}{5}$	*Greater ſixth.*
$\frac{5}{8}$	*Leſſer ſixth.*
$\frac{2}{3}$	*Fifth.*
$\frac{3}{4}$	*Fourth.*
$\frac{4}{5}$	*Greater third.*
$\frac{5}{6}$	*Leſſer third.*
$\frac{8}{9}$	*Second or tone major.*
1	*Baſe note.*

LECT. XVII. Having thus explained the nature and properties of ſound, I come now to give you an account of the VIBRATIONS of MUSICAL STRINGS, and to ſhew you in what proportions the times of the vibrations are varied, by varying the length, thickneſs, or tenſion of the ſtrings; and in order thereto, I ſhall lay down the following PROPOSITION.

Pl. 7. Fig. 13. *Let an elaſtick ſtring as* AB, *faſtened at* A, *and paſſing over a ſmall pin or pulley at* B, *be ſtretched by an appending weight as* P, *(which I ſhall call the tending force ;) and by a force applied at the middle point* C, *(which I ſhall call the* inflecting force,) *let it be drawn into the poſition* ADB; *if the diſtance between* C *and* D *be exceedingly ſmall in proportion to the length of the ſtring, or, to ſpeak in the mathematical phraſe, if* CD *be a naſcent quantity, the inflecting force will be meaſured by a rectangle under the ſpace* CD, *and the tending force applied to the length of the ſtring.* For ſince the tending force acts upon the ſtring in the direction DB, it may be denoted by that line, and being ſo denoted, it may be reſolved into two forces, whereof one acts in pulling the ſtring horizontally in the direction CB, and is therefore to be expreſſed by CB; whilſt the other acts in drawing the ſtring perpendicularly upward from D towards C, and is therefore to be expreſſed by the line DC; ſo that that portion of the tending force which acts in moving the ſtring upward, is to the whole force, as DC to DB; or, becauſe D and C are ſuppoſed to be indefinitely near, as DC to CB; but the force which acts in drawing the ſtring upward, is equal to the inflecting force, becauſe they balance each other; conſequently, the inflecting force is to the tending force, as CD to CB; and turning this analogy into an equation, by multiplying the extremes and means, and then dividing by CB, we ſhall have the inflecting force equal to a rectangle under the tending force, and the line CD, applied

to

to half the length of the ſtring; and therefore, forasmuch as whole quantities are in proportion as their halves, the inflecting force will be as a rectangle under the tending force and the line CD, applied to the length of the ſtring; ſo that putting F for the inflecting force, P for the tending force, S for the line CD, and L for the length of the ſtring, F is as $\frac{SP}{L}$. Hence it follows, that if P and L, that is, if the tending force and length of the ſtring be given, the inflecting force is as the line CD, as will appear from the following experiment. LECT. XVII.

Let a ſmall braſs wire three feet long, faſtened at one end, and paſſing over a pin ſo as that when ſtretched it may be in an horizontal poſition, be tended by a weight of three pounds; and let half an ounce, and an ounce, be appended ſucceſſively to the middle of the wire; in the former caſe, the point of ſuſpenſion will be drawn down $\frac{4}{10}$th parts of an inch, and in the latter $\frac{8}{10}$ths. Exp. 2.

Since the force which inflects a ſtring of a given length, and tended by a given force, is as the ſpace CD, through which the ſtring is bent; the force wherewith the ſtring reſtores it ſelf, muſt likewiſe be as CD, becauſe the reſtitutive force is in all caſes equal to the inflecting force; conſequently, the point D is carried towards C, by a force that varies with the diſtance; and therefore, whatever be the diſtance at which it begins it's motion, the time wherein it arrives at C will ſtill be the ſame; as I proved in my lecture on the pendulum. Whence it follows, that the vibrations of one and the ſame ſtring, whether they be through larger or ſmaller ſpaces, are all performed in equal times.

If L and S be given, F is as P; that is, if the length of the ſtring, and the ſpace through which it is bent be given, the inflecting force is as the tending force; or, in other words, one and the ſame ſtring,

being

LECT. XVII. being tended by different forces, will upon the inflexion be drawn down equal ſpaces by inflecting forces, which are to one another in the ſame proportion with the tending forces.

Exp. 3. Let the ſame wire as before be tended by a weight of ſix pounds, and it will require one ounce to draw it down $\frac{4}{10}$ths of an inch, and two ounces to draw it down $\frac{8}{10}$ths; whereas, when it was tended by a weight of three pounds only, it was drawn down the ſame ſpaces by half an ounce, and an ounce.

If P and S be given, F is as $\frac{1}{L}$; that is, if the force which tends the ſtring, and the ſpace through which it is bent be given, the inflecting force is inverſly as the length of the ſtring; or, in other words, if ſtrings of different lengths be tended by equal forces, they will be drawn through equal ſpaces by inflecting forces, which are to one another inverſly as the lengths of the ſtrings.

Exp. 4. Let a ſmall braſs wire a foot and an half long be tended by a weight of three pounds, and it will require an ounce to bend it down $\frac{4}{10}$ths of an inch, whereas half an ounce was ſufficient to give the ſame bent to the wire which was of a double length, and under the ſame tenſion.

The time of a vibration of an elaſtick ſtring is meaſured by a rectangle, under the length and diameter of the ſtring, applied to the ſquare root of the tending force. For if, as in the caſe of gravity, we ſuppoſe the force wherewith the inflected ſtring reſtores it ſelf to act uniformly, as we ſafely may, becauſe the ſpace through which it acts is exceedingly ſmall; then the motion generated will be as a rectangle under the force and the time of it's acting; ſo that putting M for the motion, F for the reſtitutive force, and T for the time of it's acting, M is as FT;

FT; but the motion is as the quantity of matter moved into the velocity wherewith it moves; and in this caſe, the quantity of matter is as a product under the length of the ſtring, and the ſquare of it's diameter; wherefore, putting D, L, and V, to denote the diameter, length, and velocity, FT is as D^2LV; and dividing both ſides by F, T is as $\frac{D^2LV}{F}$; but the reſtitutive force of the ſtring being equal to the force which inflects it, and that having been proved to be as $\frac{SP}{L}$, wherein S denotes the ſpace through which the ſtring is bent, P the tending force, and L the length of the ſtring; if inſtead of F we ſubſtitute $\frac{SP}{L}$, T will be as $\frac{D^2L^2V}{PS}$; but the velocity applied to the ſpace is inverſly as the time, that is, $\frac{V}{S}$ is as $\frac{1}{T}$; and therefore, inſtead of that, ſubſtituting this, and multiplying both ſides by T, we ſhall have T^2, as $\frac{D^2L^2}{P}$; and therefore, extracting the root, T is as $\frac{DL}{P^{\frac{1}{2}}}$; that is, the time of a vibration, is as a rectangle under the diameter and length of the ſtring, applied to the ſquare root of the tending force.

Lect. XVII.

Hence it follows, that if D and P be given, T is as L; that is, if the diameter of the ſtring and the tending force be given, the time of the vibrations varies with the length of the ſtring; as is manifeſt from the diviſion of the *monochord*, wherein the parts of the chord which ſound the ſeveral notes in an octave, have the ſame proportions to the whole chord, that the times of the reſpective notes have to the time of the baſe note; as for inſtance, one half of the chord ſounds an octave to the whole, whoſe

Exp. 5.

LECT. XVII. whofe time is one half of the time of the bafe note; and $\frac{2}{3}$ of the chord found a fifth, the time whereof is $\frac{2}{3}$ of the time of the bafe note, and fo of all the reft.

If P and L be given, then T is as D; that is, if the tending force and length of the ftring be given, the time of the vibration is as the diameter of the ftring; as will appear, if two wires of equal lengths be tended by equal weights, the diameter of one being the 90th part of an inch, and that of the other the 45th part; for the former will found an octave to the latter.

Exp. 6.

If D and L be given, then T is inverfly as the fquare root of P; that is, if the diameter and length of the ftring be given, the time of the vibration is inverfly as the fquare root of the tending force; as will appear, if eleven wires equal as to length and thicknefs be tended by weights, whofe fquare roots are to one another inverfly as the times of the notes, in an octave; for the wires fo tended will found the refpective notes.

Exp. 7.

Eighth	$\frac{1}{2}$	240
Greater feventh	$\frac{8}{15}$	$210\frac{15}{16}$
Leffer feventh	$\frac{5}{9}$	$194\frac{2}{5}$
Greater fixth	$\frac{3}{5}$	$166\frac{2}{3}$
Leffer fixth	$\frac{5}{8}$	$153\frac{3}{5}$
Fifth	$\frac{2}{3}$	135
Fourth	$\frac{3}{4}$	$106\frac{2}{3}$
Greater third	$\frac{4}{5}$	$93\frac{3}{4}$
Leffer third	$\frac{5}{6}$	$86\frac{2}{3}$
Tone major, or fecond	$\frac{8}{9}$	$75\frac{15}{16}$
Bafe note	1	60

In the left hand column of this table, the numbers exprefs the times of the feveral notes; and the numbers in the right hand column, exprefs the weights in ounces, whereby the wires which found the refpective notes are tended; the fquare roots of

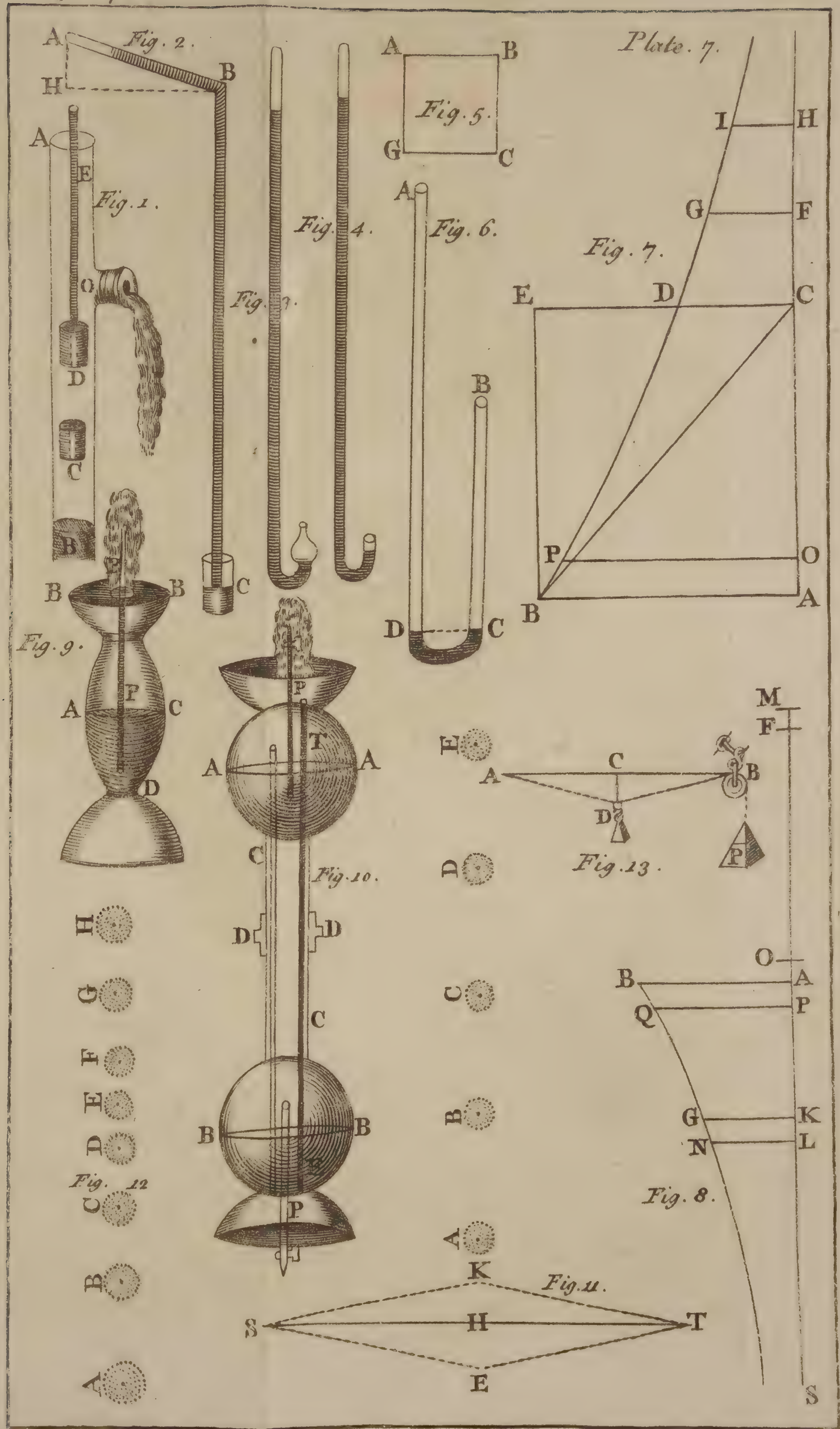


Plate. 7.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12
Fig. 13.

which weights are to one another inverſly as the times of the reſpective notes; as for inſtance, the weight which tends the ſtring that ſounds the octave, is to the weight whereby the ſtring that ſounds the baſe note is tended, as 4 to 1, whoſe ſquare roots are as 2 to 1, that is, inverſly as the time of the octave, to the time of the baſe note; and ſo of all the reſt.

LECTURE XVIII.

OF THE MOTION OF SOUND.

IN my laſt lecture, wherein I treated of that motion of the air, which is productive of ſounds, I ſhewed you, that each particle of air in going forward and returning back, is twice accelerated, and as often retarded; but I did not then enquire into the law of that acceleration and retardation. I likewiſe told you, that all the pulſes of the air move equally ſwift, the demonſtration of which I promiſed to give you in this lecture. LECT. XVIII.

Now, Sir ISAAC NEWTON, having in a moſt elegant manner, in the 47th *Propoſition* of the *Second Book* of his *Principles* demonſtrated, that each particle of air, during it's vibratory motion, is accelerated and retarded, in the very ſame manner as a pendulum vibrating in a cycloid; and having likewiſe, in the 49th and 50th *Propoſitions* of the ſame book, determined the velocity of ſound, I ſhall in this lecture lay before you what he has ſaid, in relation both to the one and the other, in the cleareſt light that I am able.

As to the firſt, let the line AB denote the length of a pulſe, or that ſpace through which the motion of the air is propagated, during the time that a particle performs it's vibration, by going forward and returning Pl. 8. Fig. 1.

LECT. XVIII. returning back; and let E, F and G, be three particles, or physical points of air situated in the right line at equal distances, and at rest; and let EQ, FR, and GT, be three equal, but exceedingly short spaces, through which these particles go and return in their vibrations; which spaces, though they be here taken of some length, to avoid confusion in the scheme, are in reality so exceedingly small, as to bear no proportion to AB, the length of a pulse. Let x, y, and z denote any intermediate points, in which the particles are found during their motion forward or backward. Let EF, and FG be small physical lines, or little portions of air, situated in strait lines between those physical points; which lines are successively moved into the places xy, yz, and QR, RT. Let the right line PS be drawn equal to EQ, and on that line as a diameter, let the circle SIPi be described; and let the circumference of that circle denote the time of the vibration of a particle, and the parts of the circumference, the proportional parts of the time; so as that after any time as PH, or PHSh, if right lines as HL and hl be drawn from the points H and h perpendicular to SP, and Ex be taken equal to PL, or Pl, the particle E may be found at x. By this means the particle or physical point E, in moving forward through x to Q, and thence back again through x to E will be accelerated and retarded, in the same manner with a pendulum vibrating in a cycloid; inasmuch as in my lecture on the pendulum, I shewed you, that the spaces described by such a pendulum, and the times of describing those spaces, are (as we have now supposed them to be in the case of the air's motion) as the versed sines and arches of a circle, whose diameter is equal in length to the whole cycloid.

Pl. 8. Fig. 2. Fig. 1.

Pl. 8. Fig. 1.

Now, in order to prove that the several little portions of air are agitated in the forementioned manner

2 by

by their elafticity, which in this cafe is the true moving caufe, let us fuppofe them to be fo moved by fome caufe or other, be that caufe what it will; and their elafticity will be found to be fuch in every point of their progrefs and return, as muft of neceffity produce in them the fame degrees of acceleration and retardation, that gravity does in a pendulum vibrating in a cycloid. LECT. XVIII.

In the circumference of the circle, let the equal arches HI and IK, or hi and ik, be taken, bearing the fame proportion to the whole circumference, that the little right lines EF and FG, do to AB the length of a pulfe; and drawing the lines IM and KN, or im and kn perpendicular to PS, inafmuch as the points or particles E, F and G, are moved in the fame manner fucceffively one after another, the motion beginning with E, and each of them performs it's intire vibration, in going forward and returning back, in the fame time that the motion is propagated through a fpace equal to AB, the length of a pulfe; if PH or PHSh denotes the time from the beginning of E's motion, PI or PHSi, will denote the time from the beginning of F's motion; and in like manner PK, or PHSk, will denote the time from the beginning of G's motion. And if the points E, F and G be found at x, y and z; the lines Ex, Fy, and Gz, in the firft figure, will be refpectively equal in the fecond, to PL, PM, and PN, in the progrefs of the points; and in their return, equal to Pl, Pm, Pn, thofe being the verfed fines of the arches which denote the times. Whence it follows, that xz, which is equal to the difference between Ex, and the fum of EG and Gz, is in the progrefs of the points, equal to EG—LN, and to EG+ln in the return; but xz is as the expanfion of the little portion of air EG, when it is in the place xz; confequently, that expanfion is to the mean ordinary expanfion, or that expanfion which it has when at reft before it is put into

Pl. 8. Fig. 1. Fig. 2. Fig. 1.

LECT. XVIII. into it's vibratory motion, as EG—LN, to EG, when that portion of air in going forward is found in the place xz; and it is as EG+ln; or, because LN and ln are equal, as EG+LN, to EG, when the portion of air in returning back, is found in the same place. Let now ID be drawn from the point I, perpendicular to HL, and the nascent triangle HID, will be similar to the triangle OIM, because the angles at D and M are right ones, and the angles at I are equal, as being each of them the complement of one and the same angle DIO, to a right one; consequently, DI, or its equal LM, is to HI, as IM to the *radius* OI, equal to OP; and double LM equal to LN, is to double HI equal to HK, as IM to OP; and by the construction, HK is to EG, as the circumference of the circle, to AB; or putting R for the *radius* of a circle, whose circumference is equal to AB, as OP to R; whence reducing these two analogies into equations, we shall have $\frac{LN}{HK}=\frac{IM}{OP}$, and $\frac{HK}{EG}=\frac{OP}{R}$; wherefore, multiplying these equations together, we shall have $\frac{LN}{EG}=\frac{IM}{R}$; and resolving this into an analogy, we shall have LN : EG : : IM : R; and of course, by substituting IM and R, in the places of LN and EG, the expansion of the small portion of air EG, or of the physical point F, when in the place xz or y, is to it's mean ordinary expansion, as R—IM to R, in it's going forward, and as R+im, to R, in it's returning; and forasmuch as it's elasticity is inversly as it's expansion, it's elasticity when at the point y, is to it's ordinary elasticity, as $\frac{1}{R-IM}$ to $\frac{1}{R}$ in it's progress, and in it's regress in the same point, as $\frac{1}{R+im}$ to $\frac{1}{R}$; and

Pl. 8. Fig. 2.

and by the fame way of arguing, the elaftick forces of the phyfical points E and G, when in going forward they are found at x and z, will be to their ordinary elafticity, as $\frac{1}{R-HL}$, and $\frac{1}{R-KN}$ to $\frac{1}{R}$; and by fubducting the latter of thefe quantities from the former, the difference of thofe forces will be as $\frac{HL-KN}{R^2-R\times HL-R\times KN+HL\times KN}$ to $\frac{1}{R}$; or, rejecting all the terms of the divifor except the firft, as being indefinitely fmall with refpect to that, as $\frac{HL-KN}{R^2}$ to $\frac{1}{R}$; or, multiplying both fides by R^2, as HL—KN to R; but forafmuch as R is a given quantity, HL—KN is as unity; confequently, the difference of the forces is as HL—KN. But from the fimilarity of triangles, HL—KN is to HK, as OM to OI or OP; confequently, fince HK and OP are given, HL—KN is as OM; or, becaufe SP and EQ are equal, if EQ be biffected in C, as cy. And by the fame way of reafoning, the difference of the elaftick forces of the fame points, when in their return they are found at x and z, is as the fame cy; but that difference, or the excefs of the elaftick force of the point x above the elaftick force of the point z, is the force by which the little line or portion of air xz, which lies between thofe points is accelerated in it's progrefs; and on the other hand, the excefs of the elaftick force of the point z above that of the point x, is the force by which the fame little line or portion of air is accelerated in it's return; fo that the force by which that little portion is accelerated, is every where as it's diftance from C, the middle point of it's vibration; confequently, during it's vibratory motion, it muft be accelerated and retarded in the fame manner with a pendulum vibrating in a cycloid; inafmuch as I proved in my lecture on the pendulum, that the

LECT. XVIII.

T force

LECT. XVIII. force which agitates the pendulum in the foremen- tioned manner, is every where as it's diſtance from the middle or loweſt point of the vibration. And what has been thus proved of the little portion EG, is in like manner demonſtrable of every other little portion of air, through which the motion is propagated.

As to the velocity of ſound, or what amounts to the ſame thing, of the pulſes of the air, if a pendulum be made equal in length to the height of an homogeneal atmoſphere, whoſe weight is equal to that of our atmoſphere, and it's denſity the ſame with that of the air at the ſurface of the earth; which height is, as I ſhewed you in a former lecture, equal to 29725 feet, and which I ſhall now denote by the letter H; in the ſame time that ſuch a pendulum performs an intire vibration by going forward and returning back, a pulſe of the air will move through a ſpace equal to the circumference of a circle deſcribed with the *radius* H. For if the little portion of air EG, vibrating through a ſmall ſpace as PS, be acted upon at P and S, the extremities of the ſpace through which it vibrates by an elaſtick force equal to it's gravity, it will perform it's vibrations in the ſame time that it would in a cycloid whoſe length is equal to PS; becauſe equal forces muſt of neceſſity move equal bodies through equal ſpaces in equal times. Since then, the times of vibrations are in the ſubduplicate *ratio* of the lengths of the pendulums, and the length of any pendulum is equal to half of the cycloid, wherein it vibrates; the time in which the ſmall portion of air would vibrate by the force of it's gravity in a cycloid equal in length to PS, muſt be to the time of the vibration of a pendulum whoſe length is H, in the ſubduplicate *ratio* of PO to H. But the elaſtick force which acts upon the little portion of air in the extreme points P and S, was proved to be to it's whole or ordinary elaſtick

Pl. 8. Fig. 2.

force

force, as HL—KN to R; that is, in the caſe before us where the point K coincides with P, as HK to R; for upon the coincidence of K and P, KN vaniſhes, and HL, which in this caſe is their difference, becomes the ſine of HK, and equal to it, inaſmuch as HK is a naſcent arch. And the whole elaſtick force of that little portion of air, or which is the ſame thing, the weight which compreſſes it, is to it's own weight, as the height of the homogeneal atmoſphere or H, to the ſmall line EG; whence putting e to denote the elaſtick force, which agitates the ſmall portion of air in the extreme points of it's vibration P and S, and w for it's weight, W for the whole elaſtic force, or the weight of the compreſſing atmoſphere, and reducing the two laſt analogies into equations, we ſhall have $\frac{HK}{R} = \frac{e}{W}$ and $\frac{W}{w} = \frac{H}{EG}$; whence multiplying the two middle terms together, and likewiſe the extremes, we ſhall have $\frac{e}{w} = \frac{HK \times H}{R \times EG}$; and by ſubſtituting PO and R for HK and EG, to which they are proportional, $\frac{e}{w}$ is equal to $\frac{PO \times H}{R^2}$; that is, by reſolving this equation into an analogy, the elaſtick force which agitates the little portion of air in the extreme points of the ſpace through which it vibrates, is to it's weight, as PO×H to R^2; ſince then, from the nature of motion, the times wherein equal bodies are moved through equal ſpaces, are reciprocally in the ſubduplicate *ratio* of the moving forces, it follows, that the time wherein the little portion of air performs it's vibration by vertue of the elaſtick force denoted by e, is to the time wherein it can vibrate through an equal ſpace by the force of it's gravity, in the ſubduplicate *ratio* of R^2 to PO×H, and of courſe, to the time of the vibration of a pendulum whoſe length is H, in a *ratio*

LECT. XVIII.

LECT. XVIII. compounded of the laſt mentioned *ratio*, and of the ſubduplicate *ratio* of PO to H; that is, as $R^2 \times PO$ to $H^2 \times PO$; that is, by dividing by PO, and extracting the ſquare roots in the ſimple *ratio* of R to H. But in the time that the little portion of air performs one vibration by going forward and returning back, the pulſe is carried through a ſpace equal to AB; conſequently, the time in which a pulſe moves from A to B, is to the time in which a pendulum whoſe length is H, ſwings forward and backward, as R to H, or as BC, the circumference of a circle whoſe *radius* is R, to the circumference of a circle whoſe *radius* is H; but the time of the pulſe's motion from A to B, is to the time in which it moves through a ſpace equal to the circumference of a circle whoſe *radius* is H, in the ſame proportion; wherefore, in the ſame time that a pendulum whoſe length is H, ſwings forward and backward, a pulſe will move through a ſpace equal to the circumference of a circle whoſe *radius* is H, which was the thing to be proved.

As a *Corollary* it follows, that the pulſes move with ſuch a velocity as a heavy body acquires in falling down half the height denoted by H; for in the ſame time with the fall, they will with a velocity equal to that acquired by the fall, deſcribe a ſpace double that of the fall, that is, a ſpace equal to H; and of conſequence, in the time that the pendulum vibrates forward and backward, they will run through a ſpace equal to the circumference of a circle whoſe *radius* is H. For, in my lecture on the pendulum, I ſhewed you, that the time of the fall through half the length of the pendulum, is to the time of one vibration, as the diameter of a circle, to it's circumference; and of courſe, to the time of a double vibration, as the *radius* to the circumference. Since then it has been proved, that the pulſes move with ſuch a velocity as carries them through a ſpace equal to the circumference of a circle whoſe

whose *radius* is H, in the same time that a pendulum whose length is H, performs a double swing; and since it appears that the velocity acquired by a heavy body in falling down half the height H, will carry the pulses through the same space in the same time, it is manifest, that they move with that velocity.

As a second *Corollary* it follows, that the velocity of the pulses is in a *ratio* compounded of the subduplicate *ratio* of the air's elasticity directly, and of the subduplicate *ratio* of it's density inversly; for since the velocity wherewith they move, is such as a body acquires in falling down half the height H, and since the velocities acquired by falling bodies, are in the subduplicate *ratios* of the heights from which they fall, it is manifest, that the velocity of the pulses is as the square root of H, but the height H is directly as the air's elasticity, and inversly as it's density; consequently, the velocity of the pulses is in the subduplicate *ratio* of the air's elasticity directly, and the subduplicate *ratio* of it's density inversly. Whence it appears, that the velocity of the pulses is given, forasmuch as, *cæteris paribus*, the elasticity is as the density. In the *winter* time indeed, the motion of the pulses is somewhat slower than in *summer*, because the coldness of that season does in some measure weaken the elasticity, and at the same time increase the density. From what has been said, the space through which sound moves in any given time, may readily be determined; for since it is known by experience, that a pendulum $39\frac{1}{5}$ inches long, performs a double vibration by going forward and returning back in two seconds of time, a pendulum whose length is H, that is 29725 feet long, will perform a like double vibration in $190\frac{3}{4}$ seconds; consequently, in that time sound will move through a space equal to the circumference of a circle whose *radius* is 29725 feet; that is, it will move through 186768 feet, which being

 di-

LECT. XVIII. divided by $190\frac{3}{4}$, gives a quotient of 979 feet, for the ſpace through which ſound moves in one ſecond of time. But it muſt be obſerved, that in this computation no regard has been had to the thickneſs of the ſolid particles of air, through which ſound is propagated in an inſtant; if that therefore be allowed for, the velocity of ſound will come out greater in the proportion of about ten to nine; for ſince the ſpecifick gravity of air is to that of water, as 1 to 870, if we ſuppoſe the particles of air to be equally denſe with thoſe of water, and that the greater rarity of air is owing to the greater interval between it's particles, it follows, that that interval is about nine times as great as the diameter of a particle; conſequently, a tenth part of the ſpace through which ſound is propagated is poſſeſſed by the particles of air; if therefore to 979 feet, which is the ſpace through which ſound would move in a ſecond, in caſe the particles of air had no magnitude, we add a ninth part, or 109 feet more on account of the thickneſs of the particles, we ſhall have 1088 feet for the ſpace through which ſound is carried in a ſecond of time. Beſides, as there are vapours diſperſed through the air, which being of a different tone and elaſticity, do not partake of that motion of the true air by vertue whereof ſound is propagated, the moving cauſe having on that account fewer particles of matter to agitate, muſt of neceſſity give them a greater velocity; and from the nature of motion it is evident, that the velocity will be greater in the inverſe ſubduplicate *ratio* of the quantity of matter to be moved; that is to ſay, if we ſuppoſe the atmoſphere to conſiſt of ten parts of true air, and one part of vapours, the motion of ſound will be quicker in ſuch an atmoſphere, than in an atmoſphere conſiſting intirely of true air, in the ſubduplicate *ratio* of 11 to 10, or in the ſimple *ratio* of about 21 to 20. If therefore the velocity laſt found be augmented in that proportion, we ſhall have

1142

1142 feet for the ſpace through which ſound moves in one ſecond of time; and this agrees with the moſt accurate experiments that have been made, for diſcovering the velocity of ſound. LECT. XVIII.

The ſpace through which ſound moves in a ſecond of time being thus diſcovered, the length of the pulſes excited by the vibrations of a ſounding body may likewiſe be found, provided the number of vibrations performed by the ſounding body in a given time, can by any method be determined; for ſince each vibration excites a new pulſe, all that is requiſite to be done, is to divide 1142 by the number of vibrations which the ſounding body performs in a ſecond, and the quotient will expreſs the length of a pulſe in feet. Now, the number of vibrations which a ſounding body performs in a given time, has been determined by Mr. SAUVEUR, in the following manner; "Muſicians having frequently "obſerved, that if two organ pipes which are nearly "unifons, be made to ſound together, there are "certain inſtants of time, and thoſe, as well as "they can be judged of by the ear, at equal inter-"vals, wherein their joint ſound is ſtronger, than "in the intermediate times." This Mr. SAUVEUR, with great appearance of reaſon, thinks is owing to the coincidence of their vibrations at thoſe inſtants; for when by the coincidence of their vibrations, they ſtrike the ear at one and the ſame inſtant, they muſt needs make a ſtronger impreſſion upon it, than when they ſtrike it ſeparately one after another. Taking this for granted, he, by the help of a pendulum, took the time between two ſucceſſive coincidences in the vibrations of two pipes of conſiderable lengths, and nearly of the ſame tone; he made choice of long pipes, becauſe the coincidences of their vibrations are rarer, and conſequently, the intervals between the coincidences are more eaſily meaſured, in long pipes than in ſhort ones. Having thus found the time which paſſed between two

LECT. XVIII. successive coincidences, he readily found the number of vibrations performed by each pipe in the same time, they being inversly as the numbers expressing the proportion of the tones of the pipes; as for instance, if the time between two successive coincidences was found to be the sixth part of a second, and the numbers which expressed the proportion of the tones of the pipes were 45 and 46, the longer pipe performed 45 vibrations, and the shorter 46, in the sixth part of a second. From these experiments he found, that a pipe, whose length was about five *Parisian* feet, had the same tone with a string that vibrates an hundred times in a second; consequently, of the pulses excited by the sounding of such a pipe, there are about one hundred in the space of 1142 *English*, or 1070 *Parisian* feet; and of course, the length of one pulse is about 10 *Parisian* feet and $\frac{7}{10}$ths, that is about twice the length of the pipe; whence it is probable, that the lengths of the pulses excited by the soundings of open pipes, are in all cases equal to twice the length of the pipes.

In a former lecture, speaking of the increase which motion received by being communicated from a smaller elastick body to a larger, I took occasion to give a reason for the augmentation of sound in speaking trumpets; I shall close this lecture, by acccounting for it from the nature of the pulses of the air. From what has been said in relation to the properties of those pulses, it is manifest, that the greater their condensation is, the stronger is the sound which they excite; now, when the voice acts upon a portion of air confined within a trumpet, it must necessarily make a stronger impression upon it, and of course condense it more, than when it acts upon it in an unconfined state; inasmuch as in the former case, the force of the voice is wholly imployed in giving motion to that small portion of air which lies within the trumpet, whereas

in

in the latter caſe, not only that portion of air is put in motion by the force of the voice, but likewiſe all that body of air which immediately ſurrounds it; the air then in the trumpet being by reaſon of it's confinement, more ſtrongly agitated and more cloſely condenſed, than it would otherwiſe be, muſt at the *exit* of the trumpet, communicate to the air without greater degrees of condenſation; and of conſequence, produce a louder ſound, than could poſſibly be excited by the ſame force of the voice, were it immediately impreſſed on the unconfined air.

LECTURE XIX.

OF LIGHT.

LIGHT, whereof I intend to treat in this lecture, is a moſt ſubtile fluid, conſiſting of particles exceedingly ſmall, but of different magnitudes, as ſhall be ſhewn hereafter, which are thrown off from luminous bodies by the vibrating motions of their parts, with a velocity ſurpriſingly great; for they do not ſpend above ſeven or eight minutes of an hour in paſſing from the ſun to the earth, as was obſerved firſt by Mr. ROMER, Profeſſor of Aſtronomy to the late King of *France*; and after him by others, by means of the *eclipſes* of the *ſatellites* of JUPITER; for theſe eclipſes, when the earth is between the ſun and JUPITER, are obſerved to happen about ſeven or eight minutes ſooner than they ought to do by the aſtronomical tables; and on the contrary, when the earth is beyond the ſun with reſpect to JUPITER, they happen about ſeven or eight minutes later than they ought to do; ſo that in the latter ſituation of the earth, they are obſerved to happen fourteen or ſixteen minutes later than in the former; foraſmuch therefore as the ſatellites LECT. XIX.

lites

LECT. XIX. lites cannot diſappear, but muſt continue viſible to the eye of an obſerver, till all that light which they reflect before their immerſions has paſſed by the place of obſervation, it follows, that the reflected light of the ſatellites ſpends fourteen or ſixteen minutes in paſſing from one end of the diameter of the earth's orbit to the other; and conſequently, half that time in moving from the ſun to the earth. Hence, if the diſtance of the ſun from the earth be 70 millions of miles, as it muſt be on ſuppoſition that it's horizontal parallax is twelve ſeconds of a degree, and ſuch the moſt accurate obſervations of the lateſt aſtronomers make it; then light moves at the rate of about 150 thouſand miles in a ſecond of time, and it's velocity exceeds the velocity of ſound, in the proportion of above ſeven hundred thouſand to one.

The motion of light is in it's own nature rectilineal, as is evident from the ſhadows which all opaque bodies caſt when placed in the light of the ſun, or of any other luminous body; and yet the beams or rays of light in paſſing out of one tranſparent body or medium into another of a different denſity, are bent and turned out of their way; or to ſpeak more properly, they are made to change the direction of their motion; and this bending or change of direction is commonly called *refraction*; and it has been found by experience, that the rays in paſſing out of a rarer *medium* into a denſer, are bent in ſuch a manner as to be brought nearer to a line drawn perpendicular to the refracting ſurface at the point of incidence; and on the contrary, in their paſſage out of a denſer medium into a rarer, they decline from the perpendicular.

Pl. 8. Fig. 3. For the illuſtration of which, let AB repreſent a ray of light moving in air from A to B, and paſſing into water at B, and let HK be perpendicular to the ſurface of the water at the point B; when the ray goes into the water, it does not continue it's

motion ſtrait forward in the line BC, but in ſome other line as BD, which is more inclined to the perpendicular BK. And on the other hand, if the line DB be ſuppoſed to be a ray of light moving in water from D to B, and there paſſing into air, inſtead of continuing it's motion in the direction BE, it goes on in ſome other direction as BA, which being leſs inclined to, is more diſtant from, the perpendicular BH; as will appear from the following experiment. Let an empty veſſel as BCDE, have a ſmall object as A, placed at it's bottom; and let it be ſo ſituated as that the ſight of the object may be interceped by the ſide of the veſſel from an eye placed at Q; let then the veſſel be filled with water, and the ray AB, which before the pouring in of the water, moved in a right line from A to K, and by ſo doing paſſed above the eye, will upon it's emerſion out of the water be bent downward, ſo as to ſtrike upon the eye, and thereby render the object viſible.

LECT. XIX.

Exp. 1. Pl. 8. Fig. 4.

This bending of the rays in their paſſage out of one medium into another, ſeems to be owing to the attractive force of the denſer medium acting upon the rays at right angles to the ſurface, as may appear by conſidering the conſequences of ſuch an attraction.

Let then AC be a ray of light moving from A to C, and there entring into a denſer medium, the ſurface which ſeparates the two mediums being denoted by the line HK. The motion of the ray in the direction AC, being reſolved according to the known method into two, one in the direction AD, and the other in the direction AB or DC, whereof the former is parallel, and the latter perpendicular to HK; it is manifeſt, that as the ray enters into the denſer medium at C, it's perpendicular motion muſt be accelerated by the attraction, whilſt it's parallel motion continues the ſame; let then the line CG be taken in the ſame proportion to CD, that the

Pl. 8. Fig. 5.

LECT. XIX. the velocity of the perpendicular motion after refraction has to the velocity thereof before the refraction; and forasmuch as the parallel motion is the same before and after refraction, let CE be taken equal to AD or BC, and letting fall EF equal and parallel to CG, and drawing the diagonal CF, the ray after refraction will describe the line CF in the same time that it moved from A to C before the refraction; and forasmuch as GF is equal to AD, LM, that is, the sine of the angle MCL, must be less than AD, the sine of ACD; consequently, by the attraction of the denser medium, the ray in passing into that medium is brought nearer to the perpendicular.

Again, let FC denote the motion of a ray in the denser medium from F to C, and let this motion be resolved into two others, one in the direction FG or EC, and the other in the direction FE or GC, the former being parallel, and the latter perpendicular to HK; when the ray passes into the rarer *medium* at C, the parallel motion does not suffer any change from the attraction; but the perpendicular motion is retarded by the attractive force, which in this case acts in direct opposition to it; let then CD be to GC, as the perpendicular velocity of the ray in the rarer *medium*, to the perpendicular velocity thereof in the denser; and let DA be drawn equal and parallel to FG, in order to denote the parallel motion of the ray after refraction; and the diagonal CA will be the line described by the ray after refraction, in a space of time equal to that wherein it described the line FC before refraction; and forasmuch as AD is equal to GF, it must be greater than LM; consequently, the Angle ACD is greater than FCG; and therefore, the ray in passing out of a denser *medium* into a rarer, is by the attraction of the denser *medium*, bent from the perpendicular; so that in both cases, the refraction seems to be owing to the attractive force of the den-

fer

ſer medium, acting upon the rays at right angles to it's ſurface; and what farther confirms this opinion is, that the denſer any medium is, and conſequently, the ſtronger it's attraction, the greater, *cæteris paribus*, is it's refractive power; thus oil of vitriol, whoſe denſity exceeds the denſity of water in the proportion nearly of three to two, acts more forcibly than water on the rays of light, in bending and turning them out of their way; as will appear from the following experiment; let the ſixth figure repreſent a Quadrant, whoſe *radius* AB is parallel to the horizon; and let A be a ſmall coloured object, placed on the limb of the quadrant at the extremity of the horizontal *radius*; this being viewed through an empty glaſs veſſel as C, of a priſmatick form, placed at the center of the quadrant, with it's real place at A. Let then the veſſel be filled with water, and let the object be raiſed on the limb of the quadrant as high as D, that is to ſay, to the height of fifteen degrees and twenty minutes, and the rays as DB, which go from it towards the priſm, will be ſo bent in paſſing through the water as to enter the eye in a direction parallel to the horizon, and repreſent the object as if placed at A. And the ſame thing will happen when the veſſel is filled with oil of vitriol, excepting only, that the object muſt be raiſed to a greater height ſuppoſe to E, ſo as to have an elevation of 20 degrees and eight minutes; which plainly ſhews, that the rays are more bent, and ſuffer a greater refraction under the ſame circumſtances from oil of vitriol, than they do from water.

LECT. XIX.

Exp. 2. Pl. 8. Fig. 6.

Refractions

LECT. XIX.

Refractions taken by the Quadrant and prismatic vessel.

	Density	Degr. and minutes	Sines
Water	1	15.20	2644342
Oil of vitriol	1.497	20.8	3442060
Salt water	1.2	17.52	3068029
Spirit of hartshorn	1.011	16.	2756374
Spirit of wine	0.835	17.	2923717
Oil of turpentine	0.869	22.34	3837582
Oil of linseed	0.939	22.57	3889277

The denser medium begins to attract the rays at some distance from it's surface, and it acts upon them more and more forcibly in proportion as their distance from it's surface lessens; but however, in what follows I shall suppose the attractive force to act with the same vigour in all parts of the space through which it extends itself; because, as that space is indefinitely small, no sensible error will arise from such a supposition. If then CD be the surface of the denser medium, and AB the space through which the attractive force extends itself from A to B; a ray of light in passing from B to A will be accelerated in such a manner as that the perpendicular velocity thereof at the point A will be equal to the square root of the sum of the square of the perpendicular velocity of the ray at it's incidence on the point B, and of the square of the perpendicular velocity

Pl. 8. Fig. 7.

locity which it would have at A, ſuppoſing it began it's motion at B, from a ſtate of reſt. For ſince the attractive force is ſuppoſed to act uniformly through the ſpace BA, the motion which it generates will as to it's properties correſpond with the motion ariſing from gravity; if therefore the triangle EGH be taken to denote the ſpace BA, GH will expreſs the velocity of a ray at A, on ſuppoſition that from a ſtate of reſt it begins it's motion at B; but if at B it has a velocity expreſſed by any right line as IK, parallel to GH, let the triangle be continued on till the portion IFLK becomes equal to EGH, and FL will expreſs the velocity of the ray at the point A; and foraſmuch as the triangle EFL, is equal to the ſum of the two triangles EGH and EIK, FL is equal to the ſquare root of the ſum of the ſquares of GH and IK; that is, the perpendicular velocity of the ray at A, is equal to the ſquare root of the ſum of the ſquare of the perpendicular velocity of the ray at it's incidence on the point B, and of the ſquare of the perpendicular velocity which it would have at A, on ſuppoſition that it began it's motion at B from a ſtate of reſt. And this being ſo, the courſe and velocity of a ray of light after refraction, in paſſing out of a rarer *medium* into a denſer, may be determined in the following manner. Let Z be a rarer *medium*, and X a denſer, ſeparated by the common ſurface EF, on which let a ray of light as AC, fall obliquely, and let AC meaſure the velocity of the ray in the rarer *medium*; which velocity is the ſame, whatever be the inclination of the ray. From the center C with the *radius* CA, let a circle be deſcribed, in which let NM be drawn through the center perpendicular to EF, and from A let fall AQ perpendicular to EF, as alſo AO perpendicular to NC. The motion of the ray in the direction AC being reſolved into two others, one in the direction AO or QC, and the other in the direction AQ or OC; the line OC will

LECT. XIX.

Pl. 8. Fig. 8.

Pl. 8. Fig. 9.

LECT. XIX. will meaſure the velocity of the perpendicular motion; and therefore, if CP be taken to denote the perpendicular velocity generated by the attraction of the denſer medium, the line PO will meaſure the perpendicular velocity of the ray in the denſer medium; and foraſmuch as the velocity of the parallel motion is no way altered by the attraction, if CV be taken equal to QC, and VB be drawn parallel to CM, and equal to PO, it is evident, that the ray after refraction, will deſcribe the line CB, and that the velocity of it's motion will be meaſured by that line.

As a *Corollary*, from what has been proved it follows, that the velocity of the refracted ray in the denſer medium is no way varied by varying the inclination of the incident ray; for the ſquare of BC being equal to the ſum of the ſquares of BV and CV, or of PO and AO, and the ſquare of PO being equal to the ſum of the ſquares of CO and PC, the ſquare of CB is equal to the ſum of the ſquares of AO, CO, and PC; but the ſquares of AO and CO are equal to the ſquare of CA or CN; conſequently, the ſquare of CB is equal to the ſum of the ſquares of PC and CN, which quantities continue unvaried, whatever be the inclination of the incident ray; and therefore PN or CB is a given quantity; that is, the meaſure of the velocity, and of conſequence, the velocity wherewith the rays move after refraction in the denſer medium, is always the ſame, however differently inclined the rays may be to the ſurface of the denſer medium at their incidence thereon.

The angle ACN, which the line deſcribed by the incident ray, contains with the perpendicular to the refracting ſurface at the point of incidence, is called the *angle of incidence*; and the angle BCM, which the line deſcribed by the refracted, contains with the perpendicular to the refracting ſurface at the point of incidence, is called the *angle of refraction*.

I As

As a second *Corollary*, from what has been proved it follows, that the sines of these angles are to one another in a given *ratio*; or in other words, that whatever proportion the sine of any one angle of incidence bears to the sine of the corresponding angle of refraction, the same does the sine of any other angle of incidence bear to the sine of the respective angle of refraction. For since CB is cut by the circle in the point T, if from B and T, BS and TR be drawn perpendicular to the *radius*, BS will be equal to AO, which is the sine of the angle of incidence, and TR will be the sine of the angle of refraction; and from the nature of similar triangles BS is to TR as CB to CT; that is, the sine of incidence is to the sine of refraction in the same proportion with two standing quantities; consequently, that proportion is given, whatever be the inclination of the incident ray. And what has been thus proved, with respect to the sines of incidence and refraction, when rays pass out of a rarer medium into a denser, is in like manner demonstrable of those lines, when the rays move out of a denser medium into a rarer, with this difference only, that whereas in the former case the angle of incidence exceeds the angle of refraction, in the latter it is exceeded by it; for as the attraction of the denser medium by accelerating the perpendicular velocity of the rays in their passage from a rarer medium turns them out of their way, so as to bring them nearer the perpendicular, so on the other hand, by retarding their perpendicular velocity in their passage into the rarer medium, it turns them out of their way so as to remove them farther from the perpendicular, as has been already shewn; and forasmuch as the rays are turned out of their way in both cases by one and the same cause acting in the same uniform manner, it is manifest, that in both cases, they must be equally bent; consequently, as

Pl. 8. Fig. 9.

LECT. XIX. much as the angle of incidence exceeds the angle of refraction when a ray passes out of the rarer medium into the denser, so much must it be exceeded by it, when the passage of the ray is made the contrary way.

Now that the sine of the angle of incidence is to the sine of the angle of refraction in a given *ratio*, whatever be the inclination of the incident ray, may be proved experimentally in the following manner. Pl. 8. Fig. 10. Let a brass quadrant graduated on both sides, and fixed at it's center to a perpendicular pillar in the manner represented, have two indices as A and B, one on each side, moveable on the center C; and let the index A, whereof the stem D is a continuation, be made to point to the 15th degree, and the index B to the 15th minute of the 20th degree; let then the pillar be immersed in water, so far as that CE the horizontal edge of the quadrant may touch the surface of the water, and upon viewing the stem D which lies within the water, it will by reason of the refraction, seem to have changed it's situation, and appear to lie in the same plane with the index B. And the same thing will likewise obtain, if the index A be set at the 30th degree, and B at the 30th minute of the 42d degree. Now in both these cases, the angle of incidence is equal to the angle contained between FC, the perpendicular edge of the quadrant, and the index A; and the angle of refraction is the angle made by the perpendicular edge of the quadrant, and the index B; so that one of the angles of incidence is 15 degrees, and the other 30, and the corresponding angles of refraction are nineteen degrees, fifteen minutes, and 41 degrees, 30 minutes; and 25, which is the sine of the lesser angle of incidence, is to 33, the sine of the corresponding angle of refraction, as 50, the sine of the greater angle of incidence, to 66, the sine of the angle of

refraction which correſponds thereto, as in the following TABLE.

	Angles of incidence.	Sines.	Angles of refraction.	Sines.
Out of water into air.	d. 15.	2588	d. m. 19. 15	3296
	d. 30.	5000	d. m. 41. 30	6626
Out of oil of turpentine.	d. 15.	2588	d. 22.	3746
	d. 30.	5000	d. 47.	7313

LECTUUE XX.

OF COLOURS.

NATURALISTS were formerly of opinion, that LIGHT was in it's own nature ſimple and uniform, without any difference or variety in it's parts. And that COLOURS, which are to be the ſubject of this lecture, were nothing elſe than certain changes or modifications of light cauſed by *refractions*, *reflections*, and *ſhadows*. But Sir ISAAC NEWTON, to whom we are indebed for almoſt every thing that we know with certainty concerning the nature of light, has ſhewn from experiments, that notwithſtanding the uniform appearance of light, the particles whereof it is compoſed are of different colours; and that the colour of each particle is laſting and permanent, ſo as not to be changed either by refraction or reflexion. He has likewiſe ſhewn, that thoſe particles which differ as LECT. XX.

LECT. XX. to colour, differ alſo in degrees of refrangibility; by means whereof, the rays of different colours may be ſeparated from each other, and exhibited apart.

Exp. 1. Pl. 8. Fig. 11. Let a beam of the ſun's light paſs into a darkened chamber through a round hole as H, about the ſixteenth or twentieth part of an inch wide, ſo as to fall directly on the middle of a double *convex lens* as L, ground to a *radius* of five or ſix feet, and placed at the diſtance of ten or twelve feet from the hole; by which means the image of the hole will be projected to I, on the other ſide of the *lens*, at the diſtance of ten or twelve feet more, and there appear white and round. Let then a priſm of ſolid greeniſh glaſs as P, be placed cloſe behind the *lens*, and in ſuch a poſture as that the beam of light may fall upon it perpendicular to it's axis, which is an imaginary ſtrait line, running through the middle from one end to the other parallel to it's edges; this being done, the image of the hole, inſtead of being round and white, and projected to I, will be long and coloured, and caſt ſideways from I; and the colours of the image taken in their order from that which lies neareſt to I, will be *red*, *orange*, *yellow*, *green*, *blue*, *purple*, and *violet*; as in the image MN, where the ſeveral colours are denoted by their initial letters.

From the lengthening of the round image by the refraction of the priſm, it is evident, that of the particles of light which form the image, ſome are more refrangible than others; for were they all alike refrangible, the diſtances to which they are thrown ſideways from their firſt ſituation at I, would be all equal, and of conſequence, the ſecond image would be round as the firſt.

As in the coloured *ſpectrum* the *red* lies neareſt to, and the *violet* fartheſt from I, it is manifeſt, that the red particles in their paſſage through the priſm, are puſhed out of their way leſs, and the violet more,

more, than any other; and consequently, that the red particles have the smallest degree of refrangibility, and the violet the greatest; and that the particles of intermediate colours have intermediate degrees of refrangibility, greater or less in proportion as they lie nearer to the one or the other of the two extremes.

This difference of refrangibility in the particles of light, argues a difference likewise in their magnitudes; for since one and the same cause, to wit, the attraction of the glass, acting upon them all with equal force, and under like circumstances, produces unequal changes in the directions of their motions, it must needs be that they move with unequal forces, and consequently, that their quantities of motion are unequal, which inequality of motion can arise from nothing else but the different size of the particles, in case they all move equally swift, as they are generally supposed to do; and that they are all perfectly solid, as their power of penetrating and dissolving the densest bodies, without suffering any change themselves, seems to require; consequently, the particles of light which differ as to colour, differ also in magnitude; those of violet being smallest, and the particles of other colours increasing continually one above another, as they are more and more removed from the violet, and approach nearer to the red, whose particles are largest of all; and here it will not be improper to observe, that as the red particles are of all others the largest, they must on that account act with the greatest force, and excite the strongest vibrations in the nervous coat of the eye; which may be one reason why reds are found to be more offensive to the eyes, than any other colour whatever.

The seven colours whereof the long image is composed are permanent and lasting, and cannot possibly be changed, either by refraction or reflexion, as will appear from the following experiments.

LECT. XX. Let a ſmall hole be made in the paper whereon the coloured image is formed, through which, let each of the ſeven colours paſs ſucceſſively, and falling upon a priſm, be again refracted, and they will be found to continue the ſame, without the leaſt change or alteration; thus the *red*, when refracted, will continue totally of the ſame red colour as before; neither *orange*, *yellow*, *green*, *blue*, nor any other new colour, will ariſe from the refraction; and the like conſtancy and immutability will be found in the other ſix colours, when refracted ſingly and apart from the reſt. And as theſe colours are not changeable by refraction, ſo neither are they by reflection; for if bodies of different colours be placed in the red light, they will all appear red, and in the blue light, they will appear blue, in the green light, green, and ſo of the other colours; in the light of any one colour, they will all appear totally of that ſame colour, with this difference only, that in ſome the colour will be more ſtrong and full, in others more faint and dilute, every body appearing moſt ſplendid and luminous in the light of it's own colour.

Exp. 2.

Exp. 3. Thus, for inſtance, if a deep red, as *carmine*, and a full blue, as *ultramarine*, be held together in the red light, they will both appear red; but the *carmine* will appear of a ſtrongly luminous and reſplendent red, and the *ultramarine* of a faint obſcure and dark

Exp. 4. red; and on the other hand, if they be held together in the blue light, they will both appear blue; but the *ultramarine* will appear of a ſtrongly luminous and reſplendent blue, and the *carmine* of a faint dark blue.

Since the colours of the rays are not capable of being changed either by refraction or reflexion, it is manifeſt, that if the ſun's light conſiſted of but one ſort of rays, there would be but one colour in the world; and by conſequence, that the variety of colours depend upon the compoſition of light. It is likewiſe manifeſt, that the permanent colours of natural

natural bodies arife from hence, that fome bodies reflect fome fort of rays, and others other forts more copioufly than the reft, and upon that account appear of this or that colour. Thus *minium*, and other red bodies, reflect the red rays moft copioufly, and thence appear red; *violets*, and all other bodies of the like colour, reflect the violet rays in greater abundance than the reft, and thence have their colour; and fo of other bodies, every body reflecting the rays of it's own colour more copioufly than the reft, and deriving it's colour from the excefs and predominancy of thofe rays in the reflected light; for though all bodies appear of the fame colour, when placed together in the light of any one colour, yet every body looks more fplendid and luminous in the light of it's own colour than in that of any other, which puts it paft difpute, that every body reflects the rays of it's own colour in greater abundance, than it does the reft, and thence has it's colour.

LECT. XX.

As natural bodies appear of divers colours, accordingly as they are difpofed to reflect moft copioufly the rays originally indued with thofe colours, fo from the different proportions which the predominant rays bear to the reft of the reflected light, arife different fhades or degrees in thofe colours. Where the predominant rays are very numerous in proportion to the reft, the colour appears ftrong and full; but as the excefs of the predominant rays leffens the colour from the mixture of the other rays, abates of it's livelinefs, and becomes more faint and dilute; and when all the rays are equally reflected, fo as that no one kind predominates, the colour becomes white; for whitenefs is a mixture of all the colours, and it is more or lefs intenfe in proportion as the reflected rays are more or fewer in number; all *grays*, *duns*, *ruffets*, *browns*, and other dark and dirty colours, down to the deepeft *black*,

LECT. XX. being but ſo many leſſer degrees of *white*, and differing from perfect whiteneſs on no other account but that they conſiſt of a leſſer quantity of light, and conſequently appear leſs glaring and luminous.

The reaſon why bodies reflect this or that kind of ray more copiouſly than the reſt, and conſequently appear of this or that colour, depends altogether on the ſize and denſity of the particles whereof the bodies are compoſed. Particles of coloured bodies reflecting rays of different colours according to their different magnitudes and denſities, as has been fully proved by Sir ISAAC NEWTON, from experiments and obſervations made on the colours of thinned bodies of *air*, *water*, and *glaſs*; by the help of which he has, in the ſecond book of his *Opticks*, given us a table containing *ſeven orders* or *ſeries* of colours, together with the thickneſſes of the particles of air, water, and glaſs, which exhibit the ſeveral colours in each order; which thickneſſes are expreſſed in parts, whereof ten hundred thouſand make an inch. The firſt part of that table is here laid before you; and by inſpection thereof it will be found, that in each order of colours, the *red* is reflected by particles of the greateſt thickneſs, and that the thickneſſes of the particles which reflect the other colours, grow leſs and leſs, as the colours which they reflect are more and more removed from the *red*. It is likewiſe manifeſt from the ſame table, that among the particles which reflect one and the ſame colour, thoſe which have the greateſt denſity, have the leaſt thickneſs; thus for inſtance, the thickneſs of a particle of glaſs which reflects the *ſcarlet* of the *ſecond order*, is but $12\frac{2}{3}$; whereas the thickneſs of water which reflects the ſame colour, is $14\frac{3}{4}$, and that of air ſtill greater, to wit $19\frac{2}{3}$; ſo that the thickneſſes of the particles which reflect any colour, increaſe as their denſities leſſen; for which reaſon, particles

particles of the ſame thickneſs may reflect different colours, provided their denſities be unequal; thus the particles of air which reflect the *violet* of the *ſecond order*, have very nearly the ſame thickneſs with particles of water which reflect the *green*, as alſo with the particles of glaſs which reflect the *orange* of the ſame *order*. LECT. XX.

		Thickneſs of		
		Air.	*Water.*	*Glaſs.*
Their colours of the *firſt order.*	*Very black*	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{10}{31}$
	Black	1	$\frac{3}{4}$	$\frac{20}{31}$
	Beginning of black	2	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{11}{20}$
	White	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$
	Yellow	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange	8	6	5^{1}
	Red	9	$6\frac{3}{4}$	$5\frac{4}{5}$
Of the *ſecond order.*	*Violet*	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{3}$
	Indico	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue	14	$10\frac{1}{2}$	9
	Green	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$
	Yellow	$16\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$
	Orange	$17\frac{2}{9}$	13	$11\frac{1}{9}$
	Bright red	$18\frac{1}{3}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$

From what has been ſaid concerning the colours of natural bodies, it follows, that if any change be made in the ſize or denſity of the particles whereof a body is compoſed, the colour of the body will likewiſe be changed; for which reaſon, if two colourleſs liquors be mixed together, they may in the mixing ſuffer ſuch changes in the ſize and denſity of their parts from their mutual actions one upon another, as to become opaque and coloured; and ſuch

LECT. XX.

such liquors as are coloured, may for the same reason, when mixed together, either become transparent and colourless, or of such a colour as is different from the colour of either, before the mixture; as will appear from the experiments now to be made.

Colours produced by the mixture of liquors void of colour.

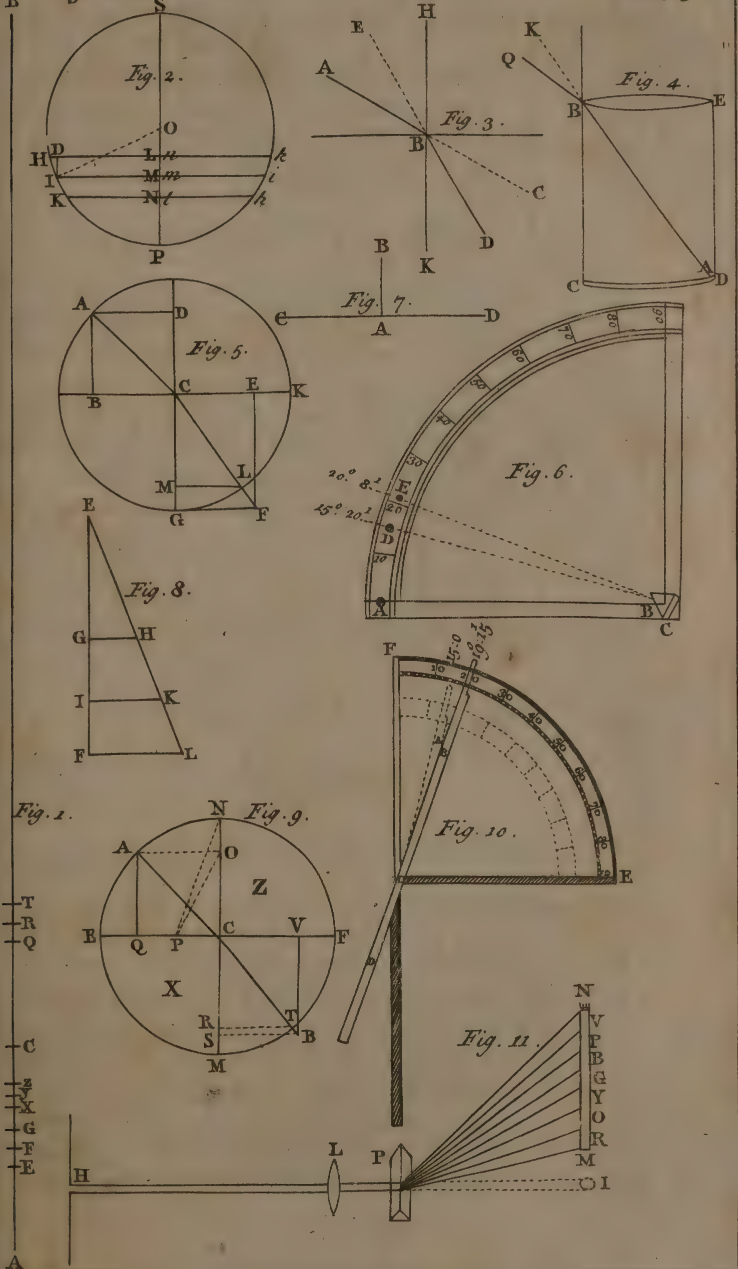
1. Rosated spirit of wine, and spirit of vitriol, a *Red.*
2. Solution of mercury, and oil of tartar, *Orange.*
3. Solution of sublimate, and lime water, *Yellow.*
4. Tincture of roses, and oil of tartar, *Green.*
5. Tincture of roses, and spirit of urine, *Blue.*
6. Solution of copper, and spirit of sal armoniack, *Purple.*
7. Solution of sublimate, and spirit of sal armoniack, *White.*
8. Solution of sugar, of led, and the solution of vitriol, *Black.*

Colours arising from the mixture of such liquors as are coloured.

1.	*Yellow.*	Tincture of saffron	*Green.*
	Red.	Tincture of red roses	
2.	*Blue.*	Tincture of violets	*Crimson.*
	Brown.	Spirit of sulphur	
3.	*Red.*	Tincture of red roses	*Blue.*
	Brown.	Spirit of hartshorn	
4.	*Blue.*	Tincture of violets	*Violet.*
	Blue.	Solution of copper	
5.	*Blue.*	Tincture of violets	*Purple.*
	Blue.	Solution of Hungarian vitriol	
6.	*Blue.*	Tincture of cyanus	*Green.*
	Blue.	Spirit of sal. armon. coloured	

7. *Blue.*

Page 304. Plate 8.



7.	*Blue.* Solution of Hungarian vitriol *Brown.* Lixivium		*Yellow.*
8.	*Blue.* Solution of Hungaritn vitriol *Red.* Tincture of red roſes		*Black.*
9.	*Blue.* Tincture of cyanus *Green.* Solution of copper		*Red.*

Colours changed and reſtored.

1. A ſolution of copper, which is *green*, by ſpirit of nitre is made *colourleſs*, and is again reſtored by oil of tartar.

2. A limpid infuſion of galls is made *black* by a ſolution of vitriol, and *tranſparent* again by oil of vitriol, and then *black* again by oil of tartar.

3. Tincture of red roſes is made *black* by a ſolution of vitriol, and becomes *red* again by oil of tartar.

4. A ſlight tincture of roſes, by ſpirit of vitriol becomes of a fine *red*, then by ſpirit of ſal armoniack turns *green*, and then by oil of vitriol becomes *red* again.

5. Solution of verdegreaſe, from a *green* by ſpirit of vitriol becomes *colourleſs*, then by ſpirit of ſal armoniack turns a *purple*, and then by oil of vitriol becomes *tranſparent* again.

Among the various *phænomena* of colours, there is none more remarkable than that of the *rainbow*, which is an appearance obſervable in thoſe places only where it rains in the ſunſhine, and where the ſpectator is placed in a due poſition between the ſun and the rain, with his back to the former; for which reaſon it is generally allowed, that the bow is made by the refraction of the ſun's light in drops of falling rain; the manner wherein it is formed, has in ſome meaſure been explained by ANTONIUS DE DOMINIS, archbiſhop of *Spalato*, and after him by DES CARTES; but as neither of them underſtood the

LECT. XX. the true origin of colours, it was impossible for them not to be defective in their accounts; and therefore Sir ISAAC NEWTON, after he had discovered the true nature and rise of colours, set himself to the consideration of this subject, and towards the latter end of the first book of his *Opticks*, has given a full and satisfactory account of the whole matter; the substance of what he has there delivered concerning the rainbow, is as follows.

Pl. 9. Fig. 1. Let a drop of rain, or any other sphærical transparent body be represented by the sphere BNFG, and let AN be one of the sun's rays, incident upon it at N, and thence refracted to F, where let it either go out of the sphere by refraction towards V, or be reflected to G; and there let it either go out by refraction to R, or be reflected to H, where let it go out by refraction towards S, cutting the incident ray in Y; let AN and RG be produced till they meet in X. Parallel to the incident ray AN, let the diameter BQ be drawn, and let BL be a quadrant, on every point of which let us suppose a ray to fall parallel to BQ; as the point of incidence removes from B towards L, the angle AXR which the rays AN and RG contain, will first increase, and then decrease; and on the other hand, the angle AYS, contained between the rays AN and YS, will first decrease and then increase. This being so, if we suppose N to be that point of the quadrant BL, whereon if the incident ray AN falls, it makes the greatest angle with the ray GR, which emerges after one reflexion; then all the rays which fall on each side at a very little distance from N, and go out after one reflexion, will emerge parallel or very nearly parallel to GR; whereas those which fall on the quadrant at greater distances from N, will notwithstanding their parallelism before their incidence be scattered, and diverge from one another after their emergence. If therefore an eye be situated in the direction of the former rays

which

which go out parallel, they will enter it so copiously as to exhibit the image of the sun in the drop of rain which reflects them; but if the eye be so placed as to receive the latter rays which go out diverging, those which enter the eye will be too few to excite any sensation; and of consequence, the image of the sun will not appear in the drop to an eye so situated. LECT. XX.

If N be the point, whereon if the incident ray AN falls it makes the smallest angle with the ray HS, which emerges after two reflexions; then, as before, all the rays which are incident near N, and which emerge after two reflexions, will go out parallel, and for that reason, will exhibit the sun's image to an eye situated in their direction; but those rays which are incident at any sensible distance from N, and which emerge after two reflexions, will be scattered as they go out, and upon that account will be too few, and consequently too feeble to excite any sensation in the eye of the spectator.

Now, forasmuch as the rays which are of different colours have likewise different degrees of refrangibility, the greatest angle AXR which can be made by the incident rays, and those which go out after one reflexion, will be of different magnitudes in rays of different colours; so likewise will the smallest angle AYS, that can be made by the incident rays, and those which go out after two reflexions; and it has been found by computation, that in the least refrangible or red rays, the greatest angle AXR, is 42 degrees and two minutes; and the least angle AYS, 50 degrees and 57 minutes; and in the most refrangible or violet rays, the greatest angle AXR, has been found to be 40 degrees and 17 minutes; and the least angle AYS, 54 degrees and 7 minutes.

Suppose now that O is the spectator's eye, and OP a line drawn parallel to the sun's rays; and let Pl. 9. Fig. 8.

LECT. XX. POE be an angle of 40 degrees and 17 minutes, POF of 42 degrees 2 minutes, POG of 50 degrees 57 minutes, and POH an angle of 54 degrees 7 minutes; and theſe angles turned about their common ſide, ſhall with their other ſides OE, OF, OG, and OH, deſcribe the verges of two rainbows AFBE and CHDG. For if E, F, G, and H, be drops of rain placed any where in the conical ſurfaces deſcribed by OE, OF, OG, and OH, and be illuminated by the ſun's rays SE, SF, SG, and SH, the angle SEO being equal to the angle POE, or 40 degrees and 17 minutes, ſhall be the greateſt angle in which the moſt refrangible rays can after one reflexion be refracted to the eye; and therefore, all the drops in the line OE, ſhall ſend the moſt refrangible rays moſt copiouſly to the eye, and thereby ſtrike the ſenſes with the deepeſt *violet* colour in that region. And in like manner, the angle SFO being equal to the angle POF, or 42 degrees 2 minutes, ſhall be the greateſt in which the leaſt refrangible rays after one reflection can emerge out of the drops; and therefore, thoſe rays ſhall come moſt copiouſly to the eye from the drops in the line OF, and ſtrike the ſenſes with the deepeſt *red* colour in that region. And by the ſame argument, the rays which have intermediate degrees of refrangibility, ſhall come moſt copiouſly from drops between E and F, and exhibit the intermediate colours in the order which their degrees of refrangibility require, that is, in the progreſs from E to F, or from the inſide of the bow to the outſide in this order, *violet*, *indigo*, *blue*, *green*, *yellow*, *orange*, and *red*.

Again, the angle SGO being equal to the angle POG, or 50 degrees and 51 minutes, ſhall be the leaſt angle in which the leaſt refrangible rays can after two reflexions emerge out of the drops, and therefore the leaſt refrangible rays ſhall come moſt copiouſly to the eye from the drops in the line OG,

and ſtrike the ſenſe with the deepeſt *red* in that region. And the angle SHO being equal to the angle POH, or 54 degrees and 7 minutes, ſhall be the leaſt angle, in which the moſt refrangible rays after two reflexions, can emerge out of the drops; and therefore, thoſe rays ſhall come moſt copiouſly to the eye from the drops in the line OH, and ſtrike the ſenſes with the deepeſt *violet* in that region. And by the ſame argument, the drops in the regions between G and H, ſhall ſtrike the ſenſes with the intermediate colours, in the order which their degrees of refrangibility require, that is, in the progreſs from G to H, or from the inſide of the bow to the outſide in this order, *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, and *violet*. And ſince theſe four lines OE, OF, OG, and OH, may be ſituated any where in the abovementioned conical ſurfaces, what is ſaid of the drops and colours in theſe lines, is to be underſtood of the drops and colours every where in thoſe ſurfaces. Thus then ſhall there be made two bows of colours, an interior and ſtronger by one reflexion in the drops, and an exterior and fainter by two (for the light becomes fainter by every reflexion,) and their colours ſhall be in a contrary order to one another, the *red* of both bows bordering upon the ſpace GF, which is between the bows. The breadth of the interior bow meaſured croſs the colours, ſhall be one degree and 45 minutes, and the breadth of the exterior, ſhall be 3 degrees 10 minutes, and the diſtance between them, ſhall be 8 degrees 55 minutes; the greateſt ſemidiameter of the innermoſt, or the angle POF, being 42 degrees and 2 minutes, and the leaſt ſemidiameter of the outermoſt, or the angle POG, being 50 degrees and 57 minutes. And theſe are the meaſures of the bows as they would be were the ſun but a point; for by the breadth of his body, the breadth of the bows will be increaſed, and their diſtance

LECT. distance lessened by half a degree; and so the
XX. breadth of the interior will be 2 degrees 15 minutes, and that of the exterior 3 degrees 40 minutes, and their distance 8 degrees 25 minutes; the greatest semidiameter of the interior bow 42 degrees 17 minutes, and the least of the exterior 50 degrees 42 minutes; and such Sir ISAAC NEWTON says he has found the dimensions of the bows in the Heavens, when he measured the same. This explication of the rainbow is confirmed by the following experiment; let a glass globe filled with water, as AB, be hung up in the sun-shine, with a black cloth placed behind it, and let IS be one of the sun's rays incident thereon; let the eye of a spectator whose back is to the sun, be placed at O, and let it be directed to such a point in the lower part of the globe suppose C, as that a strait line drawn from the eye through that point, and continued on till it meets the incident ray likewise produced, may therewith make an angle OXI, of 42 degrees 2 minutes; and the spectator shall then see a full *red* colour in that side of the globe opposed to the sun as at F; let then the eye be raised up gradually to P, till the angle PZI becomes equal to 40 degrees and 17 minutes, and as the eye rises, it will perceive other colours, to wit, *yellow*, *green*, and *blue*, successively in the same side of the globe.

Pl. 9. Fig. 3.

Again, let the eye be placed at Q, and let it be directed to such a point in the upper part of the globe suppose D, as that a strait line, drawn from the eye through that point and meeting the incident ray protracted, may therewith make an angle QSI of 50 degrees and 57 minutes, and there will appear a faint red colour in that side of the globe towards the sun; let then the eye be gradually depressed to R, till the angle RTI is 54 degrees 7 minutes, as the eye sinks, the *red* will turn successively to the other colours, *yellow*, *green*, and *blue*, as in the former case upon the raising of the eye.

LECTURE

LECTURE XXI.

Of DIOPTRICKS.

INTENDING in my next lecture to enquire into the Nature of Vision, where I shall have occasion to take notice of *defective Eyes*, I shall in this lecture, by way of preparation, lay before you some of the chief properties of such *lenses* or glasses as are most commonly in use for assisting defective eyes; and they are of two sorts, first, such as are equally convex on both sides, and secondly, such as are on both sides equally concave. The former sort is represented in the fourth figure, and the latter in the fifth. Lect. XXI.

Let ABC be an object placed before the double *convex lens* HK, at any distance greater than the *radius* of the sphere, whereof the *lens* is a segment; the rays, which issue from the several points of the object, and fall upon the *lens*, will, in their passage through it, be so bent by the refractive power of the glass, as to be made to convene at so many other points behind the *lens*, and at the place of their concourse, they will form an image or representation of the object; and this image will be inverted, because the rays which flow from A, the uppermost point of the object, are united at F, the lowermost point of the image, whilst those which flow from C, the lowest point of the object, are brought together again at D, the highest point of the image. So likewise those rays which issue from the right side of the object, are united in the left side of the image, whilst those which proceed from the left side of the object, concur in the right side of the image; as will appear by placing a lighted candle before a double *convex lens*, at such a dis- Pl. 9. Fig. 6.

 tance

LECT. XXI. tance as that the image thereof may be formed on a piece of white paper placed at a due diſtance behind the *lens*; for the flame will appear inverted with it's point downward; and if either ſide of the flame be intercepted by the interpoſition of a dark body, the contrary ſide of the image will be obſcured.

With regard to this experiment, I muſt obſerve to you, that though there is one certain diſtance, at which the paper muſt be placed, in order to exhibit the image with the greateſt diſtinctneſs, yet may the diſtance be a little varied without rendring the image confuſed; and it is remarkable, that when the image is projected on the paper at the neareſt diſtance that it can with any degree of diſtinctneſs, it appears bordered all around with red; which redneſs continually decreaſes, as the paper is more and more removed from the *lens*; and when it is removed to ſuch a diſtance as is requiſite to give the image the greateſt advantage in point of diſtinctneſs, the redneſs intirely vaniſhes, and leaves the image equally white all over; but upon a farther removal of the paper, the edges of the image which at the neareſt diſtance were tinged with red, do now appear tinged with blue. Pl. 9. Fig. 7. If a candle, which is placed at A before the *convex lens* CD, has it's image projected on a paper at EF, ſuppoſing that to be the leaſt diſtance at which it can be projected diſtinctly, it's edges will appear red, but upon the removal of the paper to GH, they will become white; and when the paper is removed to IK, they will appear blue; the reaſon of theſe different appearances is this, the rays of light as AC and AD, which flow from the candle, being compounded of particles of different colours, whereof the red are leaſt refrangible, and the blue moſt ſo, upon paſſing through the *lens*, the blue rays are made to convene ſooneſt, and the red lateſt; as in the figure where the blue are denoted by the pricked lines, and the

red by the continued; so that an image is formed at EF, by the concurrence of some of the more refrangible rays, and it is tinged around it's edges by the red rays, which converging more slowly than the rest lie outermost. LECT. XXI.

After the blue rays have concurred, they cross one another, and go on diverging towards GH, where meeting with the red rays which have not yet concurred, and there mixing with them and the rays of other colours, they produce a white image, whiteness resulting from a due mixture of all the colours; as they proceed forward toward IK, they, by reason of their greater divergence, spread themselves on all sides beyond the other rays, and by so doing, tinge the outlines of the image blue.

On the formation of pictures by means of a double *convex lens*, depend the appearances of the *camera obscura*, which is a small square box with a tube issuing horizontally from one side, at the extremity whereof is fixed a double *convex lens*; within the box is placed a looking-glass in a slanting position, so as to be at half right angles with the bottom of the box, which is parallel to the horizon. On the top of the box is placed horizontally a plate of glass rough on one side, whereon the pictures of objects are represented in the following manner.

Let AB be an object placed before CD, the *lens* fixed in the tube which issues from the box; GH the looking-glass inclined to the bottom of the box, in an angle of 45 degrees, LM the plate of rough glass covering the top of the box horizontally. The rays which flow from A, the uppermost point of the object, after they have passed the *lens*, converge towards F, and would actually meet at that point, but that they are intercepted by the looking-glass GH, which reflects them, and throws them upward; and forasmuch as the inclination of the rays towards one another is no way altered by the reflexi- Pl. 9. Fig. 8. Exp. 2.

LECT. XXI. on, they muſt meet at ſome point as K, as far diſtant above the *ſpeculum*, as the point F is behind it. In like manner, the rays which flow from B, the loweſt point of the object, and which after they have paſſed the glaſs are tending towards E, being reflected upward by the *ſpeculum*, are made to convene at I, whoſe diſtance above the *ſpeculum*, is equal to the diſtance of E behind the *ſpeculum*; and as the rays from the extreme points A and B, are made to convene at K and I, ſo thoſe which flow from the intermediate points of the object, are brought together at correſponding points between K and I, whereby the image is projected horizontally, but with it's right and left ſides correſponding to the contrary ſides of the object; as may appear by placing a man before the *lens*, and cauſing him to ſtir one of his hands; for in the image the other hand will appear to move.

The diſtance of the image behind the glaſs is always varied by varying the diſtance of the object before the glaſs; the image approaching as the object recedes, and receding as that approaches. For if we ſuppoſe A and C to be two radiating points, from which the rays AH, AK, and CH, CK fall upon the *lens* HK, it is manifeſt, that the rays from the more diſtant point diverge leſs than thoſe from the nearer point, the angle at A being leſs than that at C; conſequently, when they paſs through the glaſs they muſt be brought together ſooner, and muſt convene at ſome point as B, leſs diſtant from the *lens*, than is the point D, whereat the more diverging rays from the point C are made to convene.

Pl. 9. Fig. 9.

Where the diſtance of the object, and the *radius* of the *lens's* convexity are given, and where the thickneſs of the *lens* is but ſmall, as is commonly the caſe; the diſtance of the image from the *lens* is determined very nearly, by ſaying, as the diſtance

of

of the object from the *lens*, lessened by the *radius* of the *lens*'s convexity, is to the *radius*, so is the distance of the object from the *lens*; to the distance of the image from the *lens*; that is, putting D for the distance of the object, R for the *radius* of the convexity, and F for the distance of the image, $D-R : R :: D : F$; consequently, $F=\frac{RD}{D-R}$. LECT. XXI.

The truth of this rule is demonstrated by the writers of DIOPTRICKS; but as all the demonstrations which I have hitherto met with are tedious and intricate, I shall not at present trouble you with them, but shall proceed to confirm the rule by experiments.

Let then the flame of a candle be placed at the distance of twelve feet and an half from a double *convex lens*, the *radius* of whose convexity is four feet two inches; that is, let the distance of the flame from the glass be equal to thrice the *radius*, and the image will be projected behind the *lens* at the distance of six feet three inches, that is, at the distance of a *radius* and an half; for in this case, R being put equal to unity, RD is three, which being divided by D—R, that is, by two, gives one and an half in the quotient. Exp. 3.

If the flame be brought nearer to the *lens*, the image will move farther from it, and when the distance of the flame becomes equal to twice the diameter of the *lens*'s convexity, the distance of the image will be equal to that of the flame, the *lens* standing in the midway between them; for in this case D—R is equal to R, and of consequence, F is equal to D. Exp. 4.

The flame being placed at the distance of the *radius*, the distance of the image becomes infinite. For in this case D—R is nothing, and F is equal to $\frac{DR}{0}$, which expression denotes an infinite quantity; Exp. 5.

LECT. XXI. ſo that in this caſe, there will not be any image of the flame; but the rays of light which flow from the candle, after they have paſſed through the *lens*, will go on parallel to one another; and by ſo doing, form a bright circular image, equal in ſize to the *lens*, and the magnitude thereof will remain the ſame at all diſtances from the glaſs.

Where the diſtance of the flame is leſs than the *radius* of the convexity, D—R becomes a negative quantity, and ſo of conſequence does the quotient ariſing from the diviſion of DR by D—R; which ſhews, that the place at which the rays meet, lies on the ſame ſide of the *lens* with the flame; or to ſpeak more properly, that the rays after they have paſſed the *lens*, proceed diverging from one another in ſuch a manner, as if they had flowed from a point before the *lens*, more diſtant than the place of the flame. For the eaſier underſtanding of which, let the rays AB and AC flow from the point A, whoſe (Pl. 9. Fig. 10.) diſtance from the *lens* BC, is leſs than the *radius* of the *lens*'s convexity; after they have paſſed the glaſs, they will not continue to go on in the directions BD and CE, but in the directions BF and CG, as if they had proceeded from ſome point as H, more diſtant from the *lens* than is the point A, from which they really flow; ſo that in this caſe, the rays after they paſs the glaſs, go on diverging from one another, but however they do not diverge as much as they did before they paſſed the glaſs.

When the diſtance of the flame from the glaſs is ſo great, as that neither the breadth of the *lens*, nor the *radius* of it's convexity bears any ſenſible proportion to it, then D—R is equal to D; and of conſequence, F is equal to R; that is, the diſtance of the image is equal to the *radius* of the glaſs's convexity, and this is the leaſt diſtance at which an image can be projected by ſuch a *lens*; and foraſmuch as the rays of the ſun, which by reaſon of the immenſe diſtance of his body, are always united

at

at the ſmalleſt diſtance, are apt to burn at the place of their union; that place is uſually called the *focus* or *burning point*, and ſometimes the *abſolute focus*, in contradiſtinction to thoſe places whereat the images of leſs remote objects are formed, and which are frequently called the *reſpective foci*. LECT. XXI.

The length or breadth of an object, is to the length or breadth of it's image, as the diſtance of the object from the *lens*, to the diſtance of the image from the *lens*. For if AC be the length or breadth of an object, and DF the length or breadth of it's image; AB, which is one half of AC, is to FE, which is one half of FD, as BL to EL, the triangles ABL and FEL being ſimilar. Hence it follows, that the nearer an object approaches the *lens*, the larger is it's image, the image receding, and conſequently inlarging, as the object approaches; and thus it appears to be from experiments; for the flame of a candle being placed at a diſtance greater than the diameter of the *lens*'s convexity, in which caſe the diſtance of the image is leſs, appears larger than the image; but being brought within the diſtance of the diameter, the image, which in that caſe is at the ſame diſtance, becomes equal to it; and upon bringing the flame ſtill nigher, the image becomes larger in proportion to the ſquare of it's greater diſtance. Pl. 9. Fig. 6. Exp. 6.

The ſame thing is likewiſe evident from the magick lantern; which is a lantern out of which iſſues an horizontal arm, capable of being lengthened or ſhortened at pleaſure, by means of one part ſliding in and out of the other; to the extremity of the moveable part is fitted a double *convex lens*; and to that part of the arm which joins the lantern is adapted a glaſs, plane on one ſide, and *convex* on the other, the plane ſide looking towards the lantern; in the body of the lantern there is placed a candle, whoſe diſtance from the *plano-convex* glaſs is ſomewhat leſs than the focal diſtance; ſo that the light Exp. 7.

LECT. XXI. which paſſes through that glaſs, is thrown very ſtrongly upon little images painted in dilute colours on pieces of plain thin glaſs; which being fixed in a ſlider that moves to and fro acroſs the arm, are placed at a ſmall diſtance behind the *plano-convex* glaſs in an inverted poſition, and by means of the *lens* in the moveable part of the arm, are projected in an erect poſition, on a paper or white cloth placed at a proper diſtance; if by drawing out the moveable part of the arm, the pictures be removed to a greater diſtance from the *lens*, the lantern muſt be brought nearer to the cloth, in order to a diſtinct repreſentation; becauſe, as the object recedes from the *lens*, the image approaches, and at the ſame time the images will be diminiſhed. But on the other hand, if by thruſting in the arm the pictures be brought nearer the *lens*, the lantern muſt be removed farther from the cloth, and in this caſe, the images will appear larger.

As convex glaſſes cauſe the rays of light to converge and unite, ſo thoſe which are concave make them ſeparate and diverge; for which reaſon, if diverging rays fall upon a concave *lens*, they will diverge more after they have paſſed through it, than they did before; and ſuch rays as converge before their incidence, will after their paſſage converge leſs; for inſtance, if the rays AB and AC, which diverge from A, paſs through the concave *lens* BC, they will not go on in the directions BD and CE, but in ſome other directions as BH and CG, ſo as to widen faſter than before. On the other hand, if HB and GC, be two rays converging towards K, after they have paſſed through the glaſs, they will not go on towards K, but towards a more diſtant point as A, ſo as to converge more ſlowly than before. All which is fully confirmed by experiments. For a candle being placed before a *convex lens*, ſo as to have it's image projected on a white paper, placed at

Pl. 9. Fig. 11.

Exp. 8.

a due

a due diſtance behind the *lens*, if a concave glaſs be placed between the *convex* and the image, ſo as that the rays which are converging towards the image may paſs through it, the image will thereby be thrown to a greater diſtance behind, the rays being made to converge more ſlowly, and of conſequence, to meet at a greater diſtance than they did before the concave was interpoſed; and it muſt be obſerved, that as the image is thrown to a greater diſtance, it muſt for that very reaſon be inlarged; and foraſmuch as the larger image is compoſed of the ſame number of rays, or rather fewer, ſome of the rays being reflected by the concave *lens*, it muſt on that account appear leſs bright and luminous than the ſmaller. If by the removal of the *convex lens*, the rays which flow from the candle be ſuffered to fall diverging on the concave, and a white paper be placed cloſe behind the glaſs, there will appear thereon a dark circle of ſome breadth, occaſioned by the ſhadow of the hoop which contains the glaſs; and the circular *area* contained within the ſhadow, will be inlightened by the rays which paſs through the glaſs; and becauſe all the rays which fall upon the glaſs, do not paſs through it, ſome of them being reflected, the circular *area* will appear ſomewhat darker than the other parts of the paper, which are expoſed to the light of the candle, without the interpoſition of the glaſs; upon removing the paper gradually from the glaſs, the circular *area* will gradually inlarge, and as that inlarges, the ſhadow which environs it will grow narrower, and at length vaniſh; and upon the vaniſhing of the ſhadow, if the paper be removed a little farther, there will ariſe a bright circle all around the circular *area*, which will grow broader, but leſs bright, as the paper is more and more removed from the glaſs; and at the ſame time, the circular *area* will continue to widen, and grow darker. All which appearances are the natural and neceſſary conſequences of the

LECT. XXI. the divergency or ſpreading of the rays, occaſioned by their paſſage through the glaſs; for the farther they go from the glaſs, the more they muſt diverge, and by ſo doing, muſt on all ſides ſpread themſelves into the place of the ſhadow, and render it equally luminous with the reſt of the *area*; and when they have ſpread themſelves a little beyond the limits of the ſhadow, they fall upon ſuch parts of the paper as were before inlightned, and there, by their additional light, exhibit that bright circle which ſurrounds the darker *area*; and the bright circle, by the farther ſpreading of the rays, as the paper is more and more removed from the glaſs, grows broader and leſs luminous; as does likewiſe the circular *area*, from the ſpreading of the rays wherewith it is enlightned.

Though concave glaſſes do not collect the rays of light, and conſequently, have not a real *focus*; yet inaſmuch as the rays after they have paſſed through ſuch glaſſes, do flow in ſuch a manner as that they either tend to ſome point behind the glaſs, or appear to flow from ſome point before it, thoſe points are uſually called the *foci*; and in double concaves of equal concavities, the *foci* for converging rays are found, by ſaying, as the *radius* of the glaſs's concavity leſſened by the diſtance of the point of convergence from the glaſs, is to the *radius*, ſo is the diſtance of the point of convergence to the *focus*. And the *foci* for diverging rays are found, by ſaying, as the ſum of the *radius* and the diſtance of the point of divergence from the glaſs, is to the *radius*, ſo is the diſtance of the point of divergence to the *focus*. So that putting F for the *focus*, R for the *radius*, and D for the diſtance of the point of convergence, or divergence, $F=\frac{DR}{D\mp D}$; the negative ſign being to be prefixed to D when the rays converge, and the affirmative where they diverge,

The

The demonſtration of this *Theorem*, I ſhall for the preſent omit, on account of it's tedioufueſs and intricacy, and ſhall cloſe the lecture with this obſervation; that if rays which are converging towards a *focus* be intercepted by a concave *lens*, whoſe diſtance from the *focus* is equal to the *radius* of it's concavity, after they have paſſed through the glaſs, they will ceaſe to converge and become parallel, for R and D being equal, R—D is o; conſequently, F is infinite; that is, the point to which the rays converge, is at an infinite diſtance, and the rays of courſe muſt be parallel.

LECTURE XXII.

OF VISION.

LECT. XXII.

MY deſign in this lecture, is to explain the manner of VISION with the naked eye; and likewiſe to ſhew you, what aſſiſtances the ſight receives from glaſſes; and in order thereto, I ſhall give you a ſhort deſcription of the eye.

If a ſmall portion be cut off of a globe, and in the room thereof a portion of a ſmaller globe, but of an equal circular baſe be ſubſtituted, the compound will exhibit the true figure of the eye; for it is of a globular form, but more *convex* before than in any other part. It conſiſts of ſeveral membranes which lie contiguous one to another, of which the outermoſt is called the *tunica adnata*, or *conjunctiva*; it has it's riſe from that membrane which inveſts the ſcull, and it covers the whole ball of the eye, except the foremoſt tranſparent part; that portion of it which is viſible, is called the *white of the eye*. Beſides, this membrane, which is not reckoned among the proper coats of the eye, there are three others, which conſtitute the proper coats; the firſt of which is called the *ſclerotica*, it is a tough membrane

LECT. XXII. membrane derived from the *dura mater*, which paſſes to the eye from the brain along with the *optick nerve*, and is thence propagated over the whole globe of the eye; on the fore part it becomes tranſparent like thin poliſhed horn, which has given anatomiſts occaſion to make two membranes of it, and to call the tranſparent part *cornea*; this part is repreſented by ABF.

Pl. 9. Fig. 12. The ſecond membrane, called *tunica choroides*, is derived from the *pia mater*, and tranſmitted likewiſe from the brain along with the *optick nerve*; this is much thinner and tenderer than the former, and tinged on the hinder part with a black liquor. The fore part is called the *uvea*, and ſometimes the *iris*, from it's variety of colours. In it's middle is a ſmall hole called the ſight or pupil; the *iris* conſiſts of ſeveral circular concentrick muſcular fibres, which are cut acroſs at right angles by other ſtrait fibres in the manner of ſo many *radii*; by the contraction of the former the pupil is leſſened, and is inlarged by the contraction of the latter.

The third coat is uſually called the *retina*, and ſometimes the *nervous coat*, being nothing elſe but the *optick nerve*, which ſpreads itſelf in the form of a membrane over the bottom of the eye, over againſt the ſight. Theſe coats lying contiguous, form a *capſula* or bag, wherein are contained the three humours of the eye, called the *aqueous*, the *cryſtalline*, and the *vitreous*.

At a little diſtance behind the pupil is placed the cryſtalline humour, which is *convex* on both ſides, but ſomewhat flatter before than behind; it is ſupported by ſmall muſcular fibres, called the *ciliary ligaments*, which are inſerted into the edges of the cryſtalline humour at one end, and at the other, into the *tunica choroides*, and being cloſely united, form a kind of membrane, whereby the cavity of the eye is divided into two parts; in the foremoſt of which is lodged the *aqueous humour*, ſo called,

3 becauſe

because in consistence and colour it somewhat resembles water, being almost equally limpid and transparent. In the hindmost is lodged the *vitreous humour*, which has it's name from the resemblance it is supposed to bear to melted glass. LECT. XXII.

It has been generally thought by Anatomists, that the humours of the eye are of different densities; and that the crystalline is much more dense than either of the other two, but Doctor ROBINSON has informed us in his lecture on the eye, that upon weighing these humours in an hydrostatical balance, he found the *aqueous* and *vitreous* to be very nearly of the same specifick gravity; and that the specifick gravity of the crystalline, did not exceed the specifick gravity of the others, in a greater proportion than that of eleven to ten; whence it follows, that the crystalline is not of such great use in bringing the rays together, and thereby forming on the *retina* the pictures of outward objects, as it has been commonly thought to be by *optical* writers; for though in shape it resembles a double *convex lens*, and on that account is fitted to make the rays converge; yet forasmuch as it is situated between two humours, which are nearly of the same density with itself, it can have but little force on the particles of light; for they are found by experience to be refracted very little in passing out of one *medium* into another, when the difference in the densities of the *mediums* is but small.

Behind all the coats and humours is situated the *optick nerve*, which passes out of the skull through a small hole in the bottom of the *orbit* which contains the eye. O represents the *optick nerve*, SS the *sclerotica* or outermost coat, whose foremost transparent part ABF, is the *cornea*, CC is the *choroides*, the fore part whereof AP, and FP constitutes the *uvea* or *iris*, with the pupil PP in the middle; RR is the *retina*, AD and FE the *ciliary ligaments*, DE the Pl. 9. Fig. 12.

the

LECT. XXII. the *cryſtalline humour*, V V the *vitreous humour*, and W W the *watry humour*.

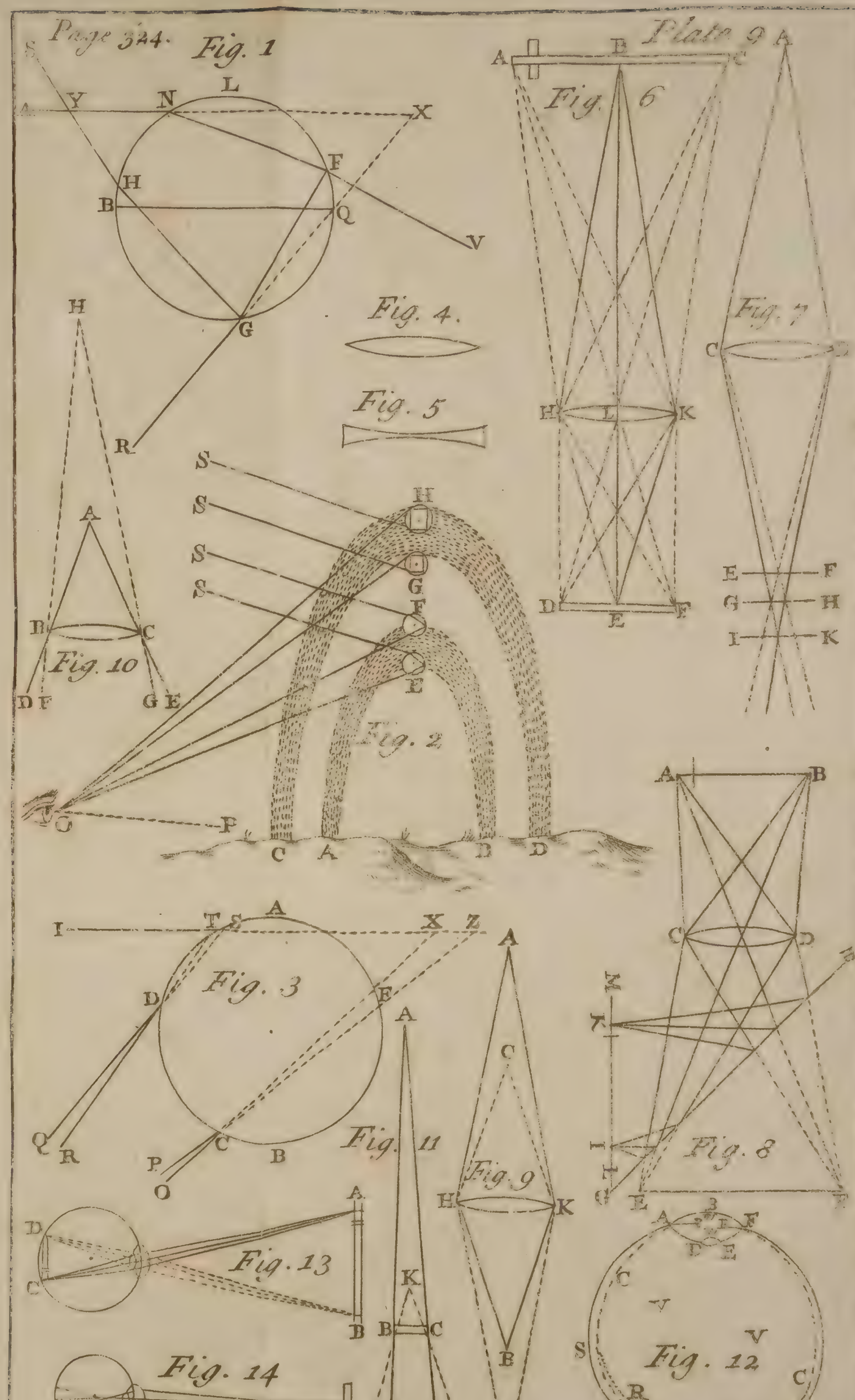
Underneath the white of the eye are inſerted into the *ſclerotica*, ſix muſcles, which take their riſe from different parts of the orbit, and are diſtinguiſhed by different names, taken from the different motions which they give the eye; their tendons ſpread themſelves over the *ſclerotica*, ſo as to terminate in the confines of the *cornea*; by which means, when the ſix muſcles act together, they preſs the ſides of the eye towards each other, whereby the eye is lengthened, and at the ſame time the convexity of the *cornea* is increaſed; both which effects are in ſome caſes abſolutely neceſſary in order to diſtinct viſion, as will appear preſently.

Having given this ſhort account of the conſtituent parts of the eye, I now proceed to lay before you the *manner of viſion*. If an object as A B, be placed at a convenient diſtance before the eye, the rays which flow from the ſeveral points of the object, and falling on the *cornea* paſs through the pupil, will be brought together by the refractive power of the eye on ſo many correſponding points of the *retina*, and there paint the image or repreſentation of the object, in the ſame manner as the images of objects placed before a *convex lens* are exhibited on white paper, placed at a proper diſtance behind.

Pl. 9. Fig. 13. Thus the rays which flow from the point A, are united on the *retina* at C, and thoſe which iſſue from B, are collected at D; and in like manner, the rays which proceed from the intermediate points of the object, are again united at ſo many intermediate points on the *retina*. On this union of the rays at the bottom of the eye, depends diſtinct

Pl. 9. Fig. 14. Pl. 10. Fig. 1. viſion; for ſhould they be united before they arrive at the *retina*, or ſhould the point of their union lie beyond the *retina*, it is evident, that the rays from each point muſt take up ſome ſpace on the *retina*, and

at page 256


Page 324.
Plate 9
Fig. 1
Fig. 2
Fig. 3
Fig. 4.
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9
Fig. 10
Fig. 11
Fig. 12
Fig. 13
Fig. 14

and of consequence, those which flow from contiguous points of the object, will be mixed and blended together on the fund of the eye, so as to exhibit a confused representation of the object.

Now forasmuch as the rays which fall upon the eye from radiating points, whose distances from the eye are different, have different degrees of divergence, the divergency of the rays increasing as the distance of the radiating point lessens, and lessening as that increases; and whereas those rays which have greater degrees of divergence, require a stronger refractive power to bring them together at a given distance, than what is requisite to make those meet which diverge less, it is manifest, that in order to see objects distinctly at different distances, the eye must have a power of increasing and lessening it's refractive force, and thereby of adapting it self to the different distances of objects; and this it does by means of the six muscles which are inserted into the *sclerotica*; for when a radiating point is placed so near, as that the rays which issue from it fall upon the eye with a considerable degree of divergence, the muscles act strongly on the eye, whereby the *cornea* is rendered more convex, and of consequence, refracts the rays with greater force; besides by the lengthening of the eye from the joint action of the muscles, the *retina* is removed to a greater distance from the *cornea*, by which contrivance, the rays are made to convene at the *retina*, notwithstanding the great degree of divergence wherewith they enter the eye. As the radiating point recedes from the eye, and the divergency of the rays of course grows less, the muscles relax themselves in order to lessen the convexity of the *cornea*, and to shorten the eye, a less convexity of the *cornea*, as also a less distance between the *cornea* and *retina*, being requisite to distinct vision in greater distances of the object than in smaller.

LECT. XXII.

Though moſt mens eyes are ſo framed as to be able to ſee diſtinctly at different diſtances, yet ſome there are which are defective in this point, as being unable to ſee any thing diſtinctly but when placed very near; and this is the caſe of their eyes who are called *myopes*, purblind, or ſhort-ſighted; in ſuch the *cornea* is too convex in proportion to the length of the eye; for which reaſon, all thoſe rays which iſſue from diſtant points, and of conſequence diverge but little, when they enter the eye, are made to convene before they reach the *retina*. As theſe men advance in years, their eyes like thoſe of other old men, for want of a due ſupply of humours, abate of their convexity and grow flatter; upon which account they begin to ſee objects diſtinctly at a diſtance, without the help of ſpectacles, and are for that reaſon deemed to have the moſt laſting eyes.

By the help of concave glaſſes, purblind perſons may ſee diſtant objects diſtinctly; for as it is the property of ſuch glaſſes to make the rays diverge, if the rays which flow from a diſtant point, and fall upon the eye with a ſmall degree of divergence, be made to paſs through a concave *lens* of a proper concavity, they will thereby be made to diverge ſo much, as that the eye, notwithſtanding the great convexity of the *cornea*, ſhall not be able to bring them together till they arrive at the *retina*.

Pl. 10. Fig. 2.

If CD be a concave *lens*, and if B be the *focus* of the rays which flow from the point A; that is, if the rays which diverge from A, paſs through the glaſs, and by the refraction which they ſuffer in their paſſage, proceed in ſuch a manner as if they had diverged from B; and if the diſtance at which a purblind perſon ſees diſtinctly with his naked eye, be equal to the diſtance of B from the glaſs, ſuch a perſon will, by the help of the glaſs CD, be able to ſee the point A diſtinctly; becauſe the rays which flow from A, after they paſs through the glaſs, fall upon his eye with the ſame degree of divergence, as

if

at page 258

if they had issued from B, the point of distinct vision. Hence it follows, that if in the *Theorem* laid down in my last lecture, for finding the *focus* of double concaves exposed to diverging rays, namely $F = \frac{RD}{R+D}$, wherein F denotes the *focus*, D the distance of the point of divergence, and R the *radius* of the convexity, we suppose F to denote the distance at which the purblind person sees distinctly without a glass, and D the distance at which he sees distinctly by the help of the glass, by clearing R we shall have $R = \frac{FD}{D-F}$; that is to say, the *radius* of the concavity of a double concave of equal concavities, which inables a purblind person to see an object distinctly, when placed beyond the reach of his naked eye, must be equal to a rectangle, under the distance at which he sees distinctly with his naked eye, and the distance at which it is required he should see distinctly by the help of the glass, divided by the difference of those distances. For instance, if a person with his naked eye can read at the distance of three inches only, and it be required to find the *radius* of such a glass as shall inable him to read at the usual distance of eighteen inches; in this case, F being equal to three inches, and D to eighteen, their product is 54; which being divided by their difference, which is 15, gives three and $\frac{3}{5}$ in the quotient, which shews, that the *radius* of the glass must be three inches and $\frac{3}{5}$ths nearly.

LECT. XXIII.

Where the distance at which it is required the purblind person shall see distinctly is infinite, or in other words, where it is so great, as that the distance to which the power of his naked eye reaches, bears no sensible proportion to it, there D—F becomes equal to D, and of course, R becomes equal to F; so that in order to see such objects as are very remote,

 purblind

LECT. XXII. purblind perſons muſt make uſe of concave glaſſes, whoſe *radii* are equal to the diſtances at which they ſee diſtinctly with their unarmed eyes.

As purblind perſons cannot ſee remote objects diſtinctly, ſo on the other hand, thoſe who are old cannot, generally ſpeaking, ſee ſuch as are nigh; the reaſon of which is, that in old men the *cornea* for want of a due ſupply of humour to plump out the eye, has not a degree of convexity ſufficient to bring the rays together on the *retina*, when they fall upon the eye with a conſiderable degree of divergence; as is the caſe of all thoſe rays which flow from points ſituated near the eye. The proper remedy for this defect is a *convex lens*, becauſe it leſſens the divergency of the rays, and brings them nearer to a parellelıſm. Pl. 10. Fig. 3. If with reſpect to the *convex lens* CD, A be the *focus* of the rays which diverge from B; that is to ſay, if the rays which flow from B and paſs through the *lens*, do afterwards proceed in ſuch a manner as if they had diverged from A, and if the diſtance at which an old man can ſee directly with his naked eye, be equal to the diſtance of A from the glaſs, he will be able, by the aſſiſtance of the glaſs, to ſee the nearer point B diſtinctly; becauſe the rays which iſſue from that point in paſſing through the glaſs, acquire the ſame degree of divergence, with thoſe which flow from A, the point of diſtinct viſion, and of conſequence, may as eaſily be brought together on the *retina*, by the refractive power of the eye; hence, if we take the *Theorem* laid down in my laſt lecture, for finding the *foci* of double *convexes* of equal convexities, and fit it to the caſe before us, where the *focus* is imaginary, by making $F = \frac{DR}{R-D}$, if then we ſuppoſe F to denote the diſtance at which an old eye ſees diſtinctly, and D the nearer diſtance at which it is required to make it ſee diſtinctly with the aſſiſt-

ance of a glaſs, by clearing R, we ſhall find it equal to $\frac{FD}{F-D}$. So that the *radius* of ſuch a double convex of equal convexities as inables an old man to ſee a nigh object diſtinctly, muſt be equal to a rectangle under the diſtance at which he ſees diſtinctly with his naked eye, and the diſtance at which he is to ſee by the help of the glaſs, divided by the difference of thoſe diſtances. To illuſtrate this by an example; ſuppoſe an old man cannot with his naked eye read at a leſs diſtance than of four feet, and it is required to aſſign the *radius* of ſpectacles which ſhall inable him to read at a diſtance of a foot and an half; in this caſe, F is four feet, and D is one and an half, and their product is ſix, which when divided by their difference, to wit, two and an half, gives $2\frac{4}{10}$ in the quotient; which ſhews, that the ſpectacles muſt be ground to a *radius* of two feet and four tenths.

If F be infinite, which is the caſe where the eye can ſee nothing but what is extremely remote, then F—D is equal to F, and of conſequence, R is equal to D; ſo that where an old man can ſee no objects diſtinctly but ſuch as are very far off, in order to ſee diſtinctly at nearer diſtances, he muſt for each diſtance uſe ſuch ſpectacle glaſſes as have their *radii* equal to the diſtance.

If D be given, then R becomes equal to $\frac{F}{F-1}$; and foraſmuch as the proportion of F to F—1 increaſes as F leſſens, R muſt do ſo too, which ſhews, that where the diſtances at which two old eyes when unarmed can ſee diſtinctly are different, in order to make them ſee diſtinctly at any leſſer given diſtance, the eye which can ſee at the ſmaller diſtance, muſt be furniſhed with a glaſs of a greater *radius* than the other. And herein lies the whole ſecret of younger and older ſpectacles, thoſe being deemed

LECT. XXII. the youngest, which are ground to the largest *radius*.

Having shewn you of what use both convex and concave glasses are in assisting defective eyes, I shall now lay before you the alterations which they produce in the appearances of objects; and first, as to *convexes*; if an object be viewed through a *convex lens*, at a less distance than the *focus*, it appears more remote and bigger than it does to the naked eye. That it must appear more remote, will be evident, if we consider what has been already proved in a former lecture, namely, that where rays fall upon a *convex lens*, from a point less distant than the *focus*, after they have passed the glass, they proceed in such a manner as if they had issued from a more distant point; and since this is the case of the rays which flow from each point in the object, the object must of consequence seem to be more distant than it is; and it must likewise appear greater; for if AB be an object exposed to a naked eye at O, it's extreme points A and B will be perceived by the eye by means of the rays AO and BO, which flow directly from those points to the eye, but if a *convex lens* as C, be interposed, the eye will no longer perceive the extremities by means of the rays AO and BO, because as they are refracted by the *lens*, they are made to concur before they can reach the eye; the eye therefore must now perceive those points by means of some other rays as AE and BD, which falling upon the glass at a greater distance from each other, are by the refractive power of the glass thrown into the directions EO and DO, and made to concur at O; so that continuing those lines directly backward as far as the object, to wit to I and H; the eye at O will perceive the extreme points of the object as situated at I and H; that is, it will perceive the object magnified. And if the eye be farther removed from the glass suppose to P, the object will appear still greater, it's extremities in that

Pl. 10. Fig. 4.

case

cafe appearing at L and K in the lines PG and PF produced. And on the other hand, if the eye continuing in it's place, the object be farther removed from the *lens*, it will appear larger; for whereas at the nearer diftance the eye perceives the extreme points of the object by means of the rays AE and BD, which fall upon the *lens* at E and D, and are thence refracted to O; when the object is at the greater diftance, it's extremities cannot be feen by means of the rays incident on the glafs at E and D; for fince the interval between the extremities continues the fame, the rays which flow from them and fall upon the *lens* at E and D, will diverge lefs at a greater diftance of the object than at a fmaller; confequently, they will concur before they reach the eye; and therefore in this cafe the extreme points of the object muft be conveyed to the eye by fome rays as aG and bF, which diverging more than the former, fall without them at G and F, whence they are refracted to the eye at O, in the lines G O and F O, which being continued backward as far as the nearer diftance of the object, to wit to L and K, fhew that the object which at the nearer diftance appeared to extend itfelf only from I to H, does at the greater diftance feem to reach from L to K, and of confequence, appears more magnified.

Pl. 10. Fig. 5.

If the object be removed beyond the *focus*, it will appear ftill greater; but whereas before it paffes the *focus* it appears diftinct, as alfo more and more diftant the farther it is removed from the glafs, when it gets beyond the *focus* it appears confufed, and the farther it is removed from the glafs, the more confufed it appears, and the nearer it feems to approach the eye, provided it's diftance from the glafs be not fo great as to make it project it's image between the eye and the glafs.

This feeming approach of the object at a time when it really recedes, and in a cafe, where, accord-

LECT. XXII. ing to the received principles of *Dioptricks*, it ought to appear at a distance, if possible, more than infinite, has very much puzzled the Writers of *Opticks*, and was looked upon as an insuperable difficulty, till Doctor BERKELEY took it into consideration in his *Essay upon Vision*, wherein, among other difficulties which he has cleared up relating to vision, he has given us a natural and satisfactory account of this. The substance of what he has there delivered concerning this matter is, that by custom and experience we are taught to judge those objects near which appear confused, because according to the ordinary course of nature, those objects, and those only, appear confused which are brought very near the eye; and therefore if an object shall at any time appear confused, though from another cause, the mind will immediately connect nearness of distance in the object, with that confusion in the appearance, as having always experienced them to go together; and the greater the confusion is, the nearer it will judge the object to be, because it has always observed the nearest distances to be attended with the greatest confusions; now if in the case before us, we suppose A to be an object placed before the *convex lens* BC, at a greater distance than the *focus*, the rays after they have passed through the glass will converge towards some point as D; if then an eye be placed at a little distance behind the glass, suppose at E, it will perceive the object confused, because as the rays fall upon it converging, they will be made to meet before they arrive at the fund of the eye, and consequently, will be scattered on the *retina*, and thereby render the appearance confused; if the eye be moved gradually backward to F, G, and D, or which is the same thing, if by carrying the object forward, the rays be made to fall upon the eye at less and less distances from the *focus*, they will be scattered more and more upon the *retina*, be-

Pl. 10. Fig. 6.

cause

cause the convergency wherewith they fall upon the eye is by so much the greater, by how much the nearer the eye is placed to the *focus* or the point D; consequently, the object as it is more and more removed from the glass, will appear more and more confused; for which reason, the mind which has been used to connect nearer distances with greater degrees of confusion, will in this case, judge the object to approach, though in reality it recedes; and what fully confirms this is, that if by placing a concave glass at a proper distance between the eye and the convex, the convergency of the rays be taken off, and the appearance thereby rendered distinct, the object will then appear at it's due distance.

If an eye be removed from a *convex lens* beyond the place where the image is projected, that is, if the eye be farther from the *lens* than is the point D, the object will appear in an inverted position, and seem to be situated between the eye and the glass; for in this case, the eye sees only the image or representation of the object, which, as I shewed in a former lecture, is projected at D in an inverted position; upon looking at the image with both eyes, it appears double, and upon shutting either eye, the image on the contrary side disappears; the reason of which is this, the eye at O perceives the image by means of the rays ODC, and therefore sees it on the same side with C, whereas the eye at P perceives it by means of the rays PDB, and on that account sees it on the same side with B; as the head is moved farther back, the distance between the two images must decrease, and at length vanish; for since the interval between the eyes continues unvaried, the rays which exhibit the image to each eye, will diverge less and less as the head is more and more removed from D, as is evident from the bare inspection of the scheme; consequently, the distance between the two images must

Pl. 10. Fig. 6.

continually decreaſe, and at laſt become ſo ſmal as to be inſenſible.

As to concave glaſſes, ſince it is their property to make the rays which flow from any point to diverge, in ſuch a manner as if they had iſſued from a point leſs diſtant, it is evident, that an object ſeen through a *concave lens* muſt appear nearer than it really is, and it muſt likewiſe appear diminiſhed; Pl. 10. Fig. 7. for the extreme points of the object AB, are ſeen by the naked eye by means of the rays AO and BO, which when the *concave lens* CD is interpoſed, are made to diverge, ſo as not to meet at O, conſequently, upon the interpoſition of the glaſs, the eye will not perceive the extremities of the object by thoſe rays, but by ſome others as AK and BL, which falling within the former, are by the refractive power of the glaſs, made to proceed in the lines KO and LO, ſo as to meet at O; wherefore continuing OK and OL backward to the object, the extremities of the object will be ſeen at E and F, that is, the object will appear to be leſs than it really is; and by the ſpreading of the rays in their paſſage through the glaſs, ſome of them are made to eſcape the eye, which if the glaſs were removed, would fall upon the pupil; for which reaſon, the object muſt appear leſs luminous; ſo that the property of concave glaſſes is to make objects appear ſmaller, nearer, and more faint and obſcure, than they do to the naked eye.

LECTURE XXIII.

OF CATOPTRICKS.

LECT. XXIII. IN this lecture, wherewith I ſhall cloſe this courſe, I ſhall explain to you the *Doctrine of* CATOPTRICKS, or that part of *Opticks* which treats of the reflexion

reflexion of light; in doing of which, I ſhall firſt ſay ſomething concerning the cauſe of that reflexion; ſecondly, I ſhall lay down two principles, which are the chief foundation of *Catoptricks*; and laſtly, I ſhall lay before you the moſt remarkable properties of plain and ſpherical mirrors.

As to the firſt, before Sir ISAAC NEWTON publiſhed thoſe wonderful and ſurpriſing diſcoveries which he made, concerning the nature and properties of light, it was an opinion generally received by the writers of *Opticks*, that the rays of light were reflected in the manner of other bodies, by ſtriking on the ſolid and impervious parts of bodies; but that great Philoſopher has fully proved this opinion to be erroneous; and has ſhewn, that the particles of light are turned back before they touch the reflecting body, by ſome power of the body which is equally diffuſed all over it's ſurface; what he has delivered concerning this matter, is to be met with in the eighth *Propoſition* of the *ſecond Book of his Opticks*, wherein, after he has offered ſeveral reaſons to prove, that light is not reflected by ſtriking againſt bodies, he at laſt expreſſes himſelf in the following manner; " Were the rays of light reflected " by impunging on the ſolid parts of bodies, their re- " flexions from poliſhed bodies could not be ſo re- " gular as they are; for in poliſhing glaſs with ſand, " putty, or tripoly, it is not to be imagined, that " thoſe ſubſtances can, by grating and fretting the " glaſs, bring all it's leaſt particles to an accurate " poliſh, ſo that all their ſurfaces ſhall be truly plain, " or truly ſpherical, and look all the ſame way, ſo " as together to compoſe one even ſurface. The " ſmaller the particles of thoſe ſubſtances are, the " ſmaller will be the ſcratches by which they con- " tinually fret and wear away the glaſs until it be " poliſhed, but be they never ſo ſmall, they can wear " away the glaſs no otherwiſe than by grating and

" ſcratching

LECT. XXIII. " ſcratching it, and breaking the protuberances, " and therefore poliſh it no otherwiſe than by bringing it's roughneſs to a very fine grain; ſo that the " ſcratches and frettings of the ſurface become too " ſmall to be viſible. And therefore, if light were " reflected by impinging on the ſolid parts of the " glaſs, it would be ſcattered as much by the moſt " poliſhed glaſs, as by the rougheſt. So then it re" mains a *Problem*, how glaſs poliſhed by fretting " ſubſtances can reflect light ſo regularly as it does; " and this *Problem* is ſcarce otherwiſe to be ſolved, " than by ſaying, that the reflexion of a ray is effect" ed, not by a ſingle point of the reflecting body, " but by ſome power of the body, which is evenly " diffuſed all over it's ſurface, and by which it acts " upon the ray without immediate contact."

Now taking it for granted, that this repelling power is the true cauſe of reflexion, if it be ſuppoſed to act upon the rays of light in lines perpendicular to the ſurface of the reflecting body; it will thence follow, that the angle of incidence, or the angle contained between the incident ray, and a line drawn perpendicular to the reflecting ſurface at the point of incidence, is equal to the angle of reflexion, or the angle contained between the ſame perpendicular and the reflected ray. For if we ſuppoſe a ray of light to move in the direction A C, towards the reflecting ſurface BCD; and if we ſuppoſe that motion to be reſolved into two, one in the direction AE, parallel to BD, and the other in the direction AB, perpendicular to BD, it is manifeſt, that of thoſe two motions, the latter only is oppoſed to the repelling force; and of conſequence, the ray after reflection, will go on in the parallel direction, with the ſame velocity it did before; and foraſmuch as the repelling force which oppoſes the perpendicular motion, acts inceſſantly, it no ſooner deſtroys the motion of the ray towards the body, but it

Pl. 10. Fig. 8.

it gives it an equal degree of motion the contrary way; that is, it throws it back with the ſame perpendicular velocity wherewith it approached. If therefore EG be taken equal to AE, and from G be let fall GD equal and parallel to AB, EG will expreſs the parallel motion of the ray after reflexion, and DG it's perpendicular motion; and the diagonal line CG, will be actually deſcribed by the ray, by vertue of it's compound motion; and from the nature of ſimilar triangles, the angle of incidence ACE, muſt be equal to ECG, the angle of reflexion; and this is the firſt of thoſe principles whereon the doctrine of *Catoptricks* is founded. The ſecond is, that every radiant point when ſeen by reflexion, appears in that place where the reflected ray meets the perpendicular, drawn from the radiant point to the reflecting ſurface; for inſtance, if from Pl. 13.
a radiant point as R, placed before the plane *ſpeculum* AB, be let fall the line REM, perpendicular Fig. 8.
to the plane of the *ſpeculum*; and if RC and CD be ſo drawn, as that the former may denote the incident ray, and the latter the reflected; and if DC be continued on, till it meets the perpendicular REM; an eye at D will perceive the radiant point, as placed at M, the point of interſection of the reflected ray, and the perpendicular; and thus it is in all caſes of reflexion, except two, wherein this principle ſeems to fail; one whereof relates to plain glaſs *ſpeculums*, and the other to concave ſpherical mirrors; the latter has been obſerved by TAQUET, Doctor BARROW, and others; but the former has not been mentioned by any one of the *optick* writers that I know of; I ſhall take notice of each in it's proper place, and proceed now to conſider the chief properties of mirrors, and firſt, of ſuch as are plain.

When an object is ſeen by reflection from a plain *ſpeculum*, it's image appears as far behind the *ſpeculum*, as the object is before; for the proof of which, let

LECT. XXIII. let R be an object placed before the plain *speculum* AB, and let it be seen by reflexion from the point C, by an eye situated somewhere in the line CD; then producing CD, till it meets the perpendicular REM, the image will, by the second principle, appear at M; now the angles of incidence and reflexion being equal, their complements are so too, that is to say, the angle RCE is equal to DCB or MCE; so that in the two right-angled triangles, the angles at C being equal, and the side EC common to both, the triangles must be equal, and the side ME, that is, the distance of the image behind the *speculum*, must be equal to RE, the distance of the object before the *speculum*; and the same thing is in like manner demonstrable, though the point of reflexion be taken different from C; for the reflected ray will constantly meet the perpendicular in the point M; whence it follows, that however the situation of the eye with respect to the mirror may be changed, yet if the object and mirror remain unmoved, the image will always appear in the same place; it likewise follows, that there cannot appear more than one image of one and the same object; but then this is to be understood with respect to such mirrors, as being opaque, have but one reflecting surface; for in looking-glasses, which by reason of their transparency, have a double reflexion in some certain positions of the eye and object, several images may be seen. Thus if A B be a looking-glass, R the flame of a candle, placed at a small distance before AH, the plane of the glass produced, an eye being placed at Q, shall see several images standing at small distances one beyond another, in the same position with the letters C, D, E, F, whereof the first and second appear bright and luminous, and the rest but faint and obscure; for the several images taken in their order from the second, grow more and more dark and obscure, till at length they

Pl. 10. Fig. 9.

Pl. 10. Fig. 10. Exp. 1.

they become too weak and feeble to affect the ſight, and of conſequence vaniſh.

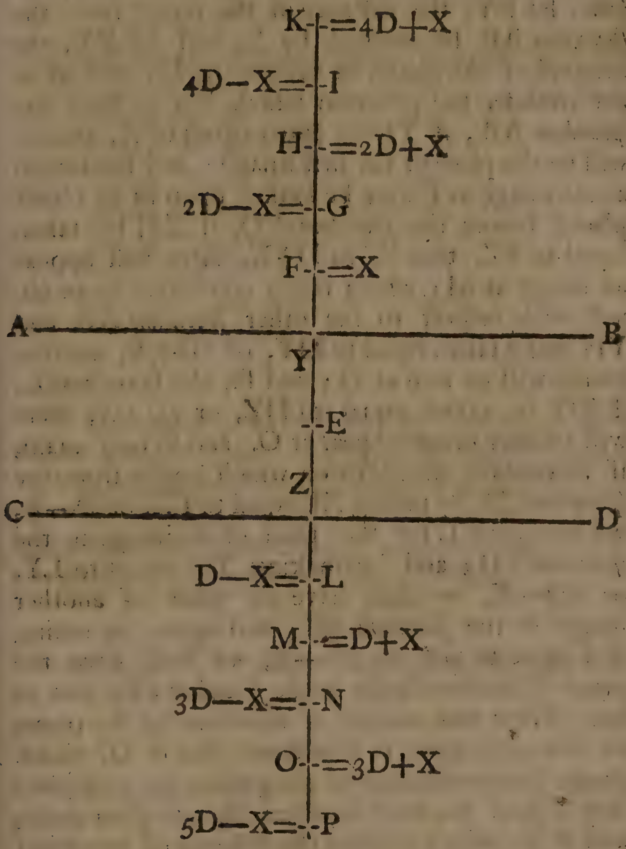
In order to account for this multiplicity of images, let ABCD be a looking-glaſs, whoſe neareſt ſurface, or that which lies next the eye is AB, and it's farther or ſilvered ſurface is DC, R the place of the candle, and Q the place of the eye, RS a line drawn from the candle perpendicular to AO and DY the two ſurfaces of the glaſs produced; the angle REA being made equal to QEB, and the line QE being produced till it cuts the perpendicular RS in T, the eye ſhall ſee the firſt image at T, by means of the reflexion from the outward ſurface AB, the ray RE being reflected to the eye from the point E. Let a ſecond ray as RG, paſs into the glaſs at G, and being refracted to the point H of the farther ſurface, let it thence be reflected to K, and there paſſing out of the glaſs, let it by refraction be carried to the eye; let then QK be produced, and the eye ſhall ſee a ſecond image ſituated in that line, and that at a little diſtance beyond the perpendicular RS; for if the rays ſuffered no refraction in paſſing in and out of the glaſs, the ſecond image would not be ſeen by means of the ray RG, but by means of the ray RH, which paſſing directly from R to H, is thence reflected directly to Q, and being produced till it cuts the perpendicular in X, would exhibit the ſecond image at X; but foraſmuch as the place of the image is changed by the refraction, and brought nearer to the glaſs, if we ſuppoſe the line QX to be moved upward about the point Q, till it coincides with the line QV, in which the ſecond image really appears, the point X muſt neceſſarily fall beyond the perpendicular, and ſo of conſequence muſt the place of the image. Let now a third ray as RF, paſs into the glaſs at F, and be refracted to L, and from thence, let it be reflected to E, and from E to M, and from M to N, where let it go out, and be refracted to the

Pl. 10. Fig. 11.

LECT. XXIII. the eye at Q; then producing QN to W, a third image will appear in that line somewhere beyond the perpendicular; for were there no refraction, the ray which after three reflexions exhibits the third image, would, when produced, cut the perpendicular in the point S; and therefore, since the line QS is raised up by the refraction, and made to coincide with the line QW, the point S, that is, the place of the third image, must fall beyond the perpendicular.

As a third image is seen by means of three reflexions, so is a fourth, by five reflexions, a fifth by seven, a sixth by nine, and so on, according to the progress of the odd numbers, every succeeding image being seen by two reflexions more than the preceding; and this is the true reason, why setting aside the first and second, which being seen each by one single reflexion, appear almost equally bright, every succeeding image appears more dim and faint than the foregoing, the rays of light being rendered more weak and feeble by reflexion.

K-



If two plane *speculums* as AB and CD, be set parallel to one another, and an object be placed any where between them as at E, the rays of light which issue from the object and fall upon each *speculum*, will be reflected backward and forward from one to the other a great number of times; by which means, there will appear in each *speculum*, a great number of images situated one behind another, in a right line perpendicular to the *speculums*, and passing through the object at E; in order to determine the

LECT. XXIII. the diſtances of the ſeveral images from the *ſpeculums*, let EY, the diſtance of the object from the *ſpeculum* AB, be denoted by X, and let ZY, the interval of the glaſſes be denoted by D; and let us firſt conſider the reflexion which begins from the *ſpeculum* AB; if YF be taken equal to X, then F will be the place of the firſt image; and foraſmuch as the image at F may be looked upon as an object placed before the *ſpeculum* CD, if ZM be taken equal to FZ, that is, to D+X, there will appear an image at M; which being conſidered as an object with reſpect to the other *ſpeculum* AB, and YH being taken equal to MY, or 2D+X, another image will be ſeen at H; and for the ſame reaſon, if ZO be taken equal to HZ, or 3D+X, there will another image appear at O, and ſo on; again, if we conſider the reflexion which begins from the *ſpeculum* CD, by taking ZL equal to EZ, or D—X, we ſhall have L for the place of an image in the *ſpeculum* CD; and by making YG equal to LY, or 2D—X, we ſhall have the place of another image in the *ſpeculum* AB; and again, by taking ZN equal to GZ, or 3D—X, we ſhall have the place of another image in the *ſpeculum* CD, and ſo on. From this manner of determining the places of ſeveral images, it is evident, that if D, which ſtands for the diſtance of the glaſſes, be multiplied into each of the even numbers taken in their order, and if X, which denotes the diſtance of the object from AB, be ſubducted from each product, and likewiſe added to each, the differences, and the ſums taken in their order, will expreſs the diſtances of the ſeveral images from the *ſpeculum* AB, the diſtance of the firſt being X; that is to ſay, the diſtance of the ſecond image will be 2D—X, of the third 2D+X, of the fourth 4D—X, of the fifth, 4D+X, and ſo on, according to the firſt Table.

TABLE I.

Distances of the several images from the speculum AB.

$$1 = X$$
$$2 = 2D-X$$
$$3 = 2D+X$$
$$4 = 4D-X$$
$$5 = 4D+X$$
$$6 = 6D-X$$
$$7 = 6D+X$$
$$8 = 8D-X$$
&c.

If D be multiplied into each of the odd numbers taken in their order, and if X be deducted from each product, and likewise added to each as before, the differences and the sums taken in their order, will express the distances of the several images from the *speculum* CD, as in the second Table.

TABLE II.

The distances of the several images from the speculum CD.

$$1 = D-X$$
$$2 = D+X$$
$$3 = 3D-X$$
$$4 = 3D+X$$
$$5 = 5D-X$$
$$6 = 5D+X$$
$$7 = 7D-X$$
$$8 = 7D+X$$
&c.

If by moving the object nearer to AB, X becomes less, then all those images whose distances are expressed by those symbols wherein X is affirmative, will come nearer to the *speculums*, whilst those whose distances are expressed by symbols

LECT. XXIII. wherein X is negative, move farther off; thus in the *speculum* AB, the first, third, fifth, seventh, and so on, will approach, and the second, fourth, sixth, eighth, and so on, will recede; so that the several images, beginning from the first, will approach and recede alternately; and on the other hand, those in the *speculum* CD, will recede and approach alternately, beginning from the first; hence, if a man puts his hand between the two *speculums*, and moves his palm towards one of them, a person looking into the other, shall see several pairs of hands, palm to palm, approaching each other.

Exp. 2.

Pl. 10. Fig. 12. Exp. 3.

If two plane *speculums* as AC and BC, be inclined to one another, so as to meet in an acute angle at C; and if an object be placed any where between them, suppose at F, an eye looking into either, shall see several images standing in the circumference of a circle, whose center is at C the concourse of the *speculums*, and it's *radius* equal to CF the distance of the object from the concourse; for if from F be drawn FD perpendicular to the *speculum* CA, and KD be made equal to FK, D will be the place of an image in the *speculum* CA; and if from D be drawn DE perpendicular to the *speculum* CB, and produced till HE is equal to DH, E will be the place of an image in the *speculum* CB, and thus by drawing perpendiculars continually from the place last found to the opposite *speculum* may the places of all the images be found which are seen by means of those reflexions, the first whereof is made from the *speculum* CA; and in the same manner, by drawing the perpendiculars FG, GL, and so on, may the places of all those images be found which are seen by means of the reflexions whereof the first is made from the *speculum* CB. Now that the points D and E are in the circumference of the circle whose *radius* is CF, I thus prove, in the triangles CFK and CDK, the sides FK and DK are

equal

equal by the conſtruction, and CK is common to both, and the angles at K are right ones, wherefore the two triangles are equal, and of conſequence CD is equal to CF; again, the triangle CDH is equal to the triangle CEH, the ſides DH and EH being by conſtruction equal, as are alſo the angles at H, wherefore CE is equal to CD, which is equal to CF, conſequently, a circle deſcribed on the center C, with the *radius* CF, will paſs through the points D and E; and by the ſame way of reaſoning it will be found to paſs through G and L, and through the extremities of all the other perpendiculars; and therefore the ſeveral images muſt of neceſſity appear in the circumference of a circle whoſe center is at the concourſe of the *ſpeculums*, and whoſe *radius* is equal to the diſtance of the object from that concourſe. From what has been ſaid it follows, that if the diſtance of the object from the concourſe of the *ſpeculums* be given, the images will ſtill appear in the circumference of the ſame circle, notwithſtanding any alteration that may be made in the angle whereat the *ſpeculums* meet; if that be inlarged, the images will be fewer in number, and at greater diſtances from one another; and on the other hand, if it be made leſs, the images will be more in number, and ſtand cloſer together, but the circle in whoſe circumference they appear, will be the ſame in both caſes; for that is not to be leſſened or inlarged otherways, than by leſſening or inlarging the diſtance of the object from the concourſe of the *ſpeculums*.

Having laid before you the chief properties of plain *ſpeculums*, I come now to conſider ſuch *ſpeculums* as are ſpherical; and they are of two ſorts, *concave* and *convex*; concerning which it muſt be obſerved, that as all the rays which fall upon them from a radiating point, are reflected in ſuch manner as to meet the perpendicular very nearly in one and the ſame point; in order to find out the *focus*,

LECT. XXIII. or the place where the reflected rays crofs one another nothing more is neceffary, but to determine the point wherein any one reflected ray meets the perpendicular; which may be done in the following manner; let A be a radiating point, expofed directly before the concave glafs FG, whofe center is C, AB a perpendicular from the radiating point to the *fpeculum*, which likewife denotes the diftance of the radiating point from the *fpeculum*, AD a ray falling on the *fpeculum* at D, whofe diftance from B is indefinitely fmall, DE the reflected ray meeting the perpendicular in E, CD a *radius* drawn to the point of incidence, and of confequence biffecting the angle ADE in the triangle ADE; fince the angle at D is biffected by the line DC, which cuts the oppofite fide, AD is to DE, as AC to CE; but forafmuch as the points D and B are fuppofed to be indefinitely near, AD is equal to AB, and ED is equal to EB, wherefore AB is to EB, as AC is to CE; that is, the diftance of the radiating point from the *fpeculum*, is to the diftance of the point E, where the reflected ray cuts the perpendicular, commonly called the *point of interfection*, as the diftance of the radiating point, leffened by the *radius*, is to the *radius*, leffened by the diftance of the point of interfection; that is, putting D for the diftance of the radiating point F for the diftance of the point of interfection and R for the *radius*, D : F : : D—R : R—F; confequently, reducing this analogy into an equation, and clearing F, F will be found equal to $\frac{DR}{2D-R}$; that is, the diftance of the point of interfection from the *fpeculum*, and confequently, the diftance of an image formed by reflexion from a concave *fpeculum*, is equal to a rectangle under the diftance of the object from the *fpeculum*, and the *radius*, divided by twice the diftance of the object leffened by the *radius*.

Pl. 10. Fig. 13.

Hence

Hence it follows, that if an object be placed before a concave *ſpeculum*, at an infinite diſtance, that is, if the diſtance be ſo great as that the *radius* of the *ſpeculum* bears no ſenſible proportion to it, the image will appear on the ſame ſide of the *ſpeculum* with the object, at the diſtance of one half the *radius* from the *ſpeculum*; for in this caſe, D being infinite, 2D—R becomes equal to 2D, and of conſequence, F is equal to R divided by 2; ſo that one half the *radius* is the leaſt diſtance at which an image can be projected from a concave *ſpeculum* on the ſame ſide with the object; and foraſmuch as the ſun's image, which, by reaſon of the immenſe diſtance of his body, is formed at the diſtance of half the *radius* from the *ſpeculum*, is there apt to burn, that place is uſually called the *focus* or *burning point*.

As the object approaches the *ſpeculum*, the image recedes; for as in one and the ſame *ſpeculum*, the *radius* is a ſtanding quantity, it is manifeſt, that as D leſſens, the proportion of DR to 2D—R muſt increaſe, conſequently, F, or the diſtance of the image from the *ſpeculum* muſt do ſo too; and when the object has approached ſo near the *ſpeculum* as to be at the center, the image will have receded ſo far as to be there likewiſe; for in this caſe, D being equal to R, 2D—R is equal to R, and of conſequence, F is equal to D; ſo that the object and it's image meet at the center of the *ſpeculum*; upon the objects paſſing from the center towards the glaſs, the image is projected beyond the center, and when the object has approached ſo near the *ſpeculum*, as to be diſtant from it but half the *radius*, the image is at an infinite diſtance; for in this caſe, D being equal to half the *radius*, 2D—R is nothing, conſequently, F, that is the diſtance of the image, is infinite; or to ſpeak more properly, the rays after reflexion proceed parallel; for which reaſon, if the flame of a candle be placed directly before a con-

LECT. XXIII. cave *ſpeculum*, at the diſtance of half the *radius*, the *ſpeculum* will ſeem to be in flames, and the reflected light will be ſo intenſe, as that by the help of it one may be able to read at a very conſiderable diſtance from the *ſpeculum*. TAQUET aſſerts, that he has read at the diſtance of no leſs than 400 feet; and to ſay the truth, the diſtance would be without limits, were it not for the atmoſphere, whoſe particles continually intercept the rays, and by ſo doing, at length totally extinguiſh the light. It ſometimes happens, that when the flame of a candle is placed in the *focus* of a concave *ſpeculum*, it's image is projected on a diſtant wall, which ſeems to invalidate the truth of what I juſt now proved concerning the paralleliſm of the rays after reflexion, but this is occaſioned by the flame's being too large to be contained totally within the *focus*, for were it ſo ſmall as to lie wholly within the *focus*, it would not project an image, but the rays after reflexion, would form a cylindrical body of light, which when projected on a diſtant wall, would have a circular figure, of an equal circumference with the *ſpeculum*.

When the diſtance of the object from the *ſpeculum* is leſs than half the *radius*, the image appears behind the *ſpeculum*; for in this caſe, 2D—R is a negative quantity, and of conſequence, ſo is F, which ſhews, that the diſtance of the image which is denoted by F, muſt be taken on the other ſide of the *ſpeculum*, with reſpect to the object; as the object moves nearer to the *ſpeculum* before, ſo likewiſe does the image behind; and when the object is ſo near as to touch the *ſpeculum*, the image does the ſame; for in this caſe, D being nothing, F, that is, the diſtance of the image from the *ſpeculum* is likewiſe nothing.

As to the poſition of the images which are ſeen by reflexion from a concave *ſpeculum*, thoſe which appear

appear on the same side of the *speculum* with the object must be inverted, and those which appear behind the *speculum* must be erect. For the proof of which, let AB be an object placed before the concave *speculum* FG, at any distance beyond the center C, in which case the image will be seen between the center and the *speculum*, suppose at DE; from A and B, the extreme points of the object, let the lines AH and BI be drawn perpendicular to the *speculum*, and of consequence crossing one another at the center; this being done, since the image is supposed to be at DE, and since every point of an image is seen in the perpendicular drawn from the corresponding point in the object, it is manifest, that D, the lowest point of the image, will correspond to A, the highest point of the object, and E, the highest point of the image, will correspond to B, the lowest point of the object, that is, the image will appear inverted. And by the same way of reasoning, if DE be the object, situated at such a distance between the *speculum* and the center, as to have it's image projected beyond the center at AB, the image must appear inverted. On the other hand, when an object as DE, is placed between the *speculum* and the center, and consequently, projects an image behind the *speculum*; for it must be observed, that the same object DE, which when situated between the center and the *speculum*, at a less distance from the center than half the *radius*, projects an image as AB beyond the center, does likewise project another image as HI, behind the *speculum*; and as the former image is visible to an eye placed beyond it, so the latter image is visible to an eye placed between the object and the *speculum*, and it must appear erect, inasmuch as the perpendicular CH, which passes through D the highest point

LECT. XXIII. of the object, does likewiſe paſs through H, the higheſt point of the image.

As to the magnitudes of an object and it's image, they are to one another in the ſame proportion with the ſquares of their diſtances from the *ſpeculum*; for if the line LCM be drawn through the center C, perpendicular to BA, DE, and HI, and conſequently, biſſecting the angle at C, if BA be the length or breadth of an object, and DE the length or breadth of it's image projected on this ſide the *ſpeculum*, then LA and OD will be half the length or breadth of the object and it's image, and the triangle CLA being ſimilar to COD, LA is to OD, and conſequently BA to ED, as LC to OC, that is, the length or breadth of the object, is to the length or breadth of it's image, as the diſtance of the object from the center of the *ſpeculum*, to the diſtance of it's image from the ſame center; but it has been proved, that as AC, the diſtance of the object from the center, is to EC, the diſtance of the image from the center, ſo is AB, the diſtance of the object from the *ſpeculum*, to EB, the diſtance of the image from the *ſpeculum*; conſequently, the length or breadth of an object, is to the length or breadth of it's image, as the diſtance of the object from the *ſpeculum*, to the diſtance of the image from the *ſpeculum*; and foraſmuch as ſimilar ſurfaces are to one another, as the ſquares of their homologous ſides, the magnitude of the object, is to the magnitude of the image, as the ſquare of the object's diſtance from the *ſpeculum*, to the ſquare of the image's diſtance. And by the ſame method of arguing, if DE be an object, whoſe image behind the *ſpeculum* is HI, the magnitude of the former, will be found to be to the magnitude of the latter, as the ſquare of KO, to the ſquare of KM. Hence it follows, that the object during it's continuance beyond the center, muſt appear larger than it's image, as

Pl. 10. Fig. 13.

Pl. 10. Fig. 14.

as being more diſtant from the *ſpeculum*, and when it is in the center, where it meets the image, it muſt appear equal to it, but being on the ſame ſide of the center with the *ſpeculum*, it muſt be leſs than it's image, which in that caſe lies beyond the center, and conſequently, is at a greater diſtance from the *ſpeculum*. LECT. XXIII.

It likewiſe follows, that the image which appears behind the *ſpeculum* is ever larger than the object; for ſince MK, the diſtance of the image behind the *ſpeculum*, is to OK, the diſtance of the object before the *ſpeculum*, as MC, the diſtance of the image from the center, to OC, the diſtance of the object from the center; and ſince in this caſe, the object is always leſs diſtant from the center than it's image, during the appearance of the image behind the *ſpeculum*, it is evident, that the image muſt appear larger than the object; but then this is to be underſtood with reſpect to ſuch images only, as are projected by objects leſs diſtant than the center; for if an object be beyond the center, an eye being cloſe to the *ſpeculum*, ſhall ſee the image at the ſame diſtance, and of an equal magnitude with the object; and in this caſe, the ſeveral parts of the image do not appear in thoſe points where the perpendiculars from the correſponding points of the object meet with the reflected rays; the reaſon of all which ſeems to be this, the portion of the *ſpeculum* which the eye makes uſe of in this caſe, is ſo exceedingly ſmall, that notwithſtanding the ſpherical figure of the *ſpeculum*, it may be looked upon as plane, and conſequently, the appearances muſt be the ſame as in other plain *ſpeculums*; that is, the image muſt appear as far behind the *ſpeculum* as the object is before it, and of the ſame magnitude with the object. Pl. 10. Fig. 14.

If an image formed on this ſide a concave *ſpeculum* be looked at with both eyes, it will appear double, provided the diſtance of the eyes from the image

LECT. XXIII. image be but ſmall, and upon ſhutting either eye, the contrary image will diſappear; for ſince the reflected rays which form the ſeveral points of an image meet and croſs one another at the image, thoſe which enter the right eye, muſt be reflected from the left ſide of the *ſpeculum*, and thoſe which fall upon the left eye, muſt be reflected from the right ſide of the *ſpeculum*, and of conſequence, one and the ſame point of the image muſt appear to the right eye, as ſituated before the left ſide of the *ſpeculum*, and to the left eye, as ſituated before the right ſide of the *ſpeculum*; that is, it muſt appear double, and the right or left image muſt vaniſh upon cloſing the contrary eye. Thus, if the point
Pl. 10. Fig. 15. C of the image AB, be looked at with both eyes, one whereof is at O, and the other at Q, the eye at O ſhall ſee it by means of the rays ON, which are reflected from N, and of conſequence, ſhall ſee it as placed before N, but the eye at Q ſeeing it by means of the rays QM, which proceed from M, ſhall ſee it as ſituated before M, for which reaſon, the point C will appear double; and what has been thus ſhewn with reſpect to the point C, may in the ſame manner be ſhewn with regard to all the other points in the image, and therefore the whole image muſt appear double, as the eyes are more and more removed from the images, they approach nearer together, and at length coincide; the reaſon of which is plain, from the bare inſpection of the figure; for ſince the interval of the eyes continues the ſame, it is evident, that when they are farther removed from the image, the rays whereby they ſee the point C muſt be reflected from parts of the *ſpeculum* leſs diſtant from one another than M and N, and the diſtance of the parts of the *ſpeculum* which reflect the rays to each eye, muſt continually leſſen as the eyes are more and more removed from the image, and at certain diſtances of

the

the eyes, must become so small as not to be sensible. And thus much concerning such spherical *speculums* as are *concave*; as to *convex speculums*, in order to determine the places of images formed by reflexion from them, let A be a radiating point, exposed directly before the *convex speculum* H K, whose center is C, AB a perpendicular from the radiating point to the *speculum*, which likewise denotes the distance of the radiating point from the *speculum*, AD a ray falling on the *speculum* at D, whose distance from B is indefinitely small, DE the reflected ray meeting the perpendicular in E, CD a *radius* drawn to the point of incidence, and of consequence bissecting the angle FDE; let the angle FCD be made equal to ECD, and let CF be continued till it meets AD produced; this being done, it is evident, that the angle at C in the triangle ACF, is bissected by the line CD, which cuts the opposite side, consequently, AC is to FC, as AD to DF; but forasmuch as D and B are supposed to be indefinitely near, AD is equal to AB, and DE to BE; and because the triangles CFD and CED are equal, DF is equal to DE, and FC is equal to CE; wherefore, AB is to BE, as AC to CE; that is, the distance of the radiating point from the *speculum*, is to the distance of the point E where the reflected ray cuts the perpendicular, which is called the *point of Intersection*, as the sum of the distance of the radiating point and the *radius*, to the *radius* lessened by the distance of the point of intersection; that is, putting D for the distance of the radiating point, F for the distance of the point of intersection, and R for the *radius* as before, $D : F :: D+R : R-F$; consequently, reducing this analogy into an equation, and clearing F, F will be found equal to $\frac{DR}{2D+R}$; that is, the distance of the point of intersection behind the

LECT. XXIII.

Pl. 10. Fig. 16.

speculum,

LECT. XXIII. *speculum*, and consequently the distance of an image behind the *speculum*, is equal to a rectangle under the distance of the object from the *speculum* and the *radius*, divided by the sum of twice the distance of the object added to the *radius*. Hence it follows, that if an object be placed so near a *convex speculum* as to touch it, it's image will do so too; for in this case, D being nothing, F is likewise nothing; as the object recedes from the *speculum*, the image goes off behind; and when the object is removed to an infinite distance, the image appears behind in the midway between the *speculum* and it's center; for in this case, D being infinite, $2D+R$ becomes $2D$, and of consequence, F is equal to $\frac{R}{2}$, so that objects seen by reflexion from convex spherical *speculums*, appear constantly behind the *speculum*, within the limits of half the *radius*; and forasmuch as the images constantly appear on the same side of the center with the objects, they must be less than the objects; for if we suppose HI to be an object placed before the *convex speculum* FG, and projecting it's image at DE, it is manifest, that the image subtends the same angle at a smaller distance, than the object does at a larger distance, and consequently, must be less; and the diproportion between the object and it's image, must increase as the object recedes, and decrease, as it approaches, because, as the object recedes from the center, the image approaches, and as that approaches, the image recedes; but as the image can never be more distant from the center than the object, it can in no case appear larger. The proportions which the magnitudes of the object and it's image bear one to another, is the same with the squares of their distances from the *speculum*, as in the case of concave *speculums*; the proof of which being exactly the same with that made use of in the case of concaves, I shall not here repeat it.

Pl. 10. Fig. 14.

As

As to the poſition of ſuch images as are ſeen by reflexion from convex ſpherical *ſpeculums*, they muſt always appear erect; for as they ever appear on the ſame ſide of the center with the objects, the perpendiculars which are drawn from the uppermoſt parts of the objects, muſt paſs through the uppermoſt parts of the images; and thoſe from the lower parts of the objects, muſt likewiſe paſs through the lower parts of the images; thus, the perpendicular HC, which comes from H, the higheſt point in the object, paſſes through D, the higheſt point of the image, and IC, which comes from I, the loweſt point of the object, paſſes through E, the loweſt point of the image; and ſo it is with regard to the perpendiculars which come from the intermediate points; ſo that the ſeveral parts of the image have the ſame ſituation with the correſponding parts of the object, and of conſequence the image appears erect.

LECT. XXIII.

Pl. 10. Fig. 14.

APPENDIX.

APPENDIX.

Of the COLLISION of NON ELASTICK AND ELASTICK BODIES.

PROBLEM I.

If two bodies be either entirely void of elaſticity or perfectly elaſtick, and one ſtrike the other directly; if A *and* B *denote the quantities of matter or weights of the two bodies,* a *and* b *their velocities before the ſtroke; and if* A *be the ſwifter body when the bodies move the ſame way, the body which has the greater motion when they move contrary ways, and the moving body when one of them is at reſt before the ſtroke; to determine the* ratio *of the bodies when their velocities before the ſtroke are given, or the* ratio *of their velocities before the ſtroke when the bodies are given; that is, to determine* $\frac{A}{B}$ *when* a *and* b *are given, or* $\frac{a}{b}$ when A *and* B *are given; ſo as that the motion of* A *before the ſtroke, ſhall be to it's motion after the ſtroke, in the given* ratio *of* m *to* 1.

To give a ſolution of this *Problem*, it is neceſſary to know the motions of A before and after the ſtroke, both when the bodies are entirely void of elaſticity, and when they are perfectly elaſtick; and likewiſe to know the motion of A after the ſtroke, when the bodies move the ſame way, when they move contrary ways, and when B is at reſt before the ſtroke. The motion of A before the ſtroke, is Aa in all caſes. And from what has been delivered by our Author, when the bodies are entirely void of elaſticity, the motion of A after the ſtroke, is

AAa-

$\frac{AAa+ABb}{A+B}$ when before the stroke the bodies move the same way, $\frac{AAa-ABb}{A+B}$ when they move different ways before the stroke, and $\frac{AAa}{A+B}$ when before the stroke B is quiescent. And when the bodies are perfectly elastick, the motions of A after the stroke, when before the stroke the bodies move the same way, contrary ways, and B is quiescent, are $\frac{2ABb+AAa-ABa}{A+B}$, $\frac{AAa-ABa-2ABb}{A+B}$, and $\frac{AAa-ABa}{A+B}$. Hence, this *Problem* contains six *Cases*, three when the bodies are entirely void of elasticity, and three when they are perfectly elastick, which *Cases* are thus solved.

When the bodies are entirely void of elasticity.

CASE I. If the bodies move the same way, Aa will be to $\frac{AAa+ABb}{A+B}$, as m to 1; whence we have $\frac{A}{B}=\frac{a-mb}{ma-a}$, and $\frac{a}{b}=\frac{mB}{A+B-mA}$.

CASE II. If the bodies move contrary ways, Aa will be to $\frac{AAa-ABb}{A+B}$, as m to 1; whence we have $\frac{A}{B}=\frac{mb+a}{ma-a}$, and $\frac{a}{b}=\frac{mB}{mA-A-B}$.

CASE III. If B be at rest before the stroke, then will Aa be to $\frac{AAa}{A+B}$, as m to 1; whence we have

have $\frac{A}{B} = \frac{1}{m-1}$. In this cafe b is nothing, and confequently $\frac{a}{b}$ is infinite.

When the bodies are perfectly elaftick.

Case IV. If the bodies move the fame way, Aa will be to $\frac{2ABb+AAa-ABa}{A+B}$, as m to 1; whence we have $\frac{A}{B} = \frac{ma+a-2mb}{ma-a}$, and $\frac{a}{b} = \frac{2mB}{mB+A+B-mA}$.

Case V. If the bodies move contrary ways, Aa will be to $\frac{AAa-ABa-2ABb}{A+B}$, as m to 1; whence we have $\frac{A}{B} = \frac{2mb+ma+a}{ma-a}$, and $\frac{a}{b} = \frac{2mB}{mA-A-B-mB}$.

Case VI. If B be at reft before the ftroke, Aa will be to $\frac{AAa-ABa}{A+B}$, as m to 1; whence we have $\frac{A}{B} = \frac{m+1}{m-1}$. In this cafe b is nothing, and confequently $\frac{a}{b}$ is infinite.

Exam. I. If the bodies be entirely void of elafticity, and move the fame way, A with a velocity of 7, and B with a velocity of 3; and A lofe half it's motion by the ftroke, or, which amounts to the fame, if the motion of A before the ftroke be

be to it's motion after, as 2 to 1. In this caſe, a, b, m, are 7, 3, 2; and $\frac{A}{B}$, which is equal to $\frac{a-mb}{ma-a}$ by *Caſe* 1, will be equal to $\frac{1}{7}$; ſo that A and B will be as 1 and 7. Here Aa, the motion of A before the ſtroke, is 7, and $\frac{AAa+ABb}{A+B}$, it's motion after the ſtroke, is $3\frac{1}{2}$; but 7 is to $3\frac{1}{2}$, as 2 to 1.

Exam. II. If the bodies be entirely void of elaſticity, and move the ſame way, if A and B be as 1 and 4, and the motion of A before the ſtroke be to it's motion after, as 3 to 1, in which caſe m will be 3; then $\frac{a}{b}$, which is equal to $\frac{mB}{A+B-mA}$ by *Caſe* 1, will be equal to $\frac{6}{1}$, ſo that a and b will be as 6 and 1. Here Aa, the motion of A before the ſtroke is 6; and $\frac{AAa+ABb}{A+B}$ it's motion after the ſtroke by *Caſe* 1, is 2; but 6 is to 2, as 3 to 1.

Exam. III. If B be at reſt before the ſtroke, and the motion of A before the ſtroke be to it's motion after, as 10 to 1, in which caſe m will be 10; then will $\frac{A}{B}$ be $\frac{1}{9}$, or A and B will be as 1 and 9. If the velocity of A before the ſtroke be expreſſed by 1, that is, if a be 1, then will Aa be 1, and $\frac{AAa}{A+B}$ b $\frac{1}{10}$; but 1 is to $\frac{1}{10}$, as 10 to 1.

It is to be obſerved, that A can never communicate all it's motion to B, except when it is infinitely greater than B, in which caſe B will become nothing. For if A communicate all it's motion to B, m will be 1; and $\frac{A}{B}$, which is as $\frac{1}{m-1}$, will be as $\frac{1}{0}$; but $\frac{1}{0}$ is infinite; and therefore A muſt be infi-

A a nitely

nitely greater than B, to lofe all it's motion by the ftroke.

Exam. IV. If the bodies be perfectly elaftick, and move the fame way with velocities which are as 3 and 2; and if the motion of A before the ftroke be to it's motion after, as 2 to 1; then will a, b, m, be 3, 2, 2; and $\frac{A}{B}$, which is as $\frac{ma+a-2mb}{ma-a}$ by *Cafe* 4, will be $\frac{1}{3}$; fo that A and B will be as 1 and 3. Here Aa, the motion of A before the ftroke, is 3; and $\frac{2ABb+AAa-ABa}{A+B}$, it's motion after the ftroke, is $\frac{3}{2}$; but 3 is to $\frac{3}{2}$, as 2 to 1.

Exam. V. If the bodies be perfectly elaftick, and move the fame way, if A and B be as 4 and 5, and the motion of A before the ftroke be to it's motion after, as 3 to 1; then will A, B, m, be 4, 5, 3; and $\frac{a}{b}$, which is as $\frac{2mB}{mB+A+B-mA}$ by *Cafe* 4, will be $\frac{5}{2}$, fo that a and b will be as 5 and 2. Here, Aa, the motion of A before the ftroke is 20; and $\frac{2ABb+AAa-ABa}{A+B}$, the motion of A after, is $\frac{60}{9}$; but 20 is to $\frac{60}{9}$, as 3 to 1.

Exam. VI. If A and B be perfectly elaftick, and B be at reft before the ftroke, if A move with a velocity of 4, and it's motion before the ftroke be to it's motion after, as 3 to 1; then will a, b, m, be 4, o, 3; and $\frac{A}{B}$, which is as $\frac{m+1}{m-1}$ by *Cafe* 6, will $\frac{4}{2}=\frac{2}{1}$; fo that A and B will be 2 and 1. Here, Aa the motion of A before the ftroke, is 8; and $\frac{AAa-ABa}{A+B}$, it's motion after the ftroke, is $\frac{8}{3}$; but 8 is to $\frac{8}{3}$, as 3 to 1.

SCHOLIUM.

SCHOLIUM.

If it be required to know the motion of B after the ſtroke in the ſix *Caſes* before mentioned, that motion may be had, from what our Author has delivered, when the weights of the Bodies, and their velocities before the ſtroke are given.

If the bodies be entirely void of elaſticity; the motion of B after the ſtroke, when before the ſtroke, the bodies move the ſame way, when they move contrary ways, or when B is quieſcent, is $\frac{BAa+BBb}{A+B}$, $\frac{BAa-BBb}{A+B}$, or $\frac{BAa}{A+B}$.

And if the bodies be perfectly elaſtick; the motions of B after the ſtroke, when before the ſtroke the bodies move the ſame way, when they move contrary ways, or when B is quieſcent, is $\frac{2BAa+BBb-BAb}{A+B}$, $\frac{2BAa-BBb+BAb}{A+B}$, or $\frac{2BAa}{A+B}$.

PROB. II. *If two bodies* A *and* B, *be given and be perfectly elaſtick, if* A *be the leſſer body, and* B *be at reſt before the ſtroke; it is required to find an intermediate body of ſuch a weight or quantity of matter, which I ſhall denote by* x, *as that,* A *ſtriking* x *at reſt, and* x *with the motion acquired by the ſtroke ſtriking* B *at reſt, the motion produced in* B *ſhall be greater than can be produced by an intermediate body of any other weight, or, in other words, that the motion in* B *ſhall be a* maximum.

The motion of x after it is ſtruck by A, is $\frac{2Aax}{A+x}$, and the motion of B after it is ſtruck by x, is $\frac{4ABax}{AB+Ax+Bx+xx}$, by *Schol. Prob.* 1.

But by supposition the motion of B is a *maximum*, and consequently it's fluxion is nothing. The fluxion therefore of $\frac{4ABax}{AB+Ax+Bx+xx}$ is nothing; that is $\frac{4A^2B^2a\dot{x}-4AB\dot{x}ax^2}{\overline{AB+Ax+Bx+xx}|^2}=0$. Consequently, $4A^2B^2a\dot{x}-4AB\dot{x}ax^2=0$; and, by dividing by $4ABa\dot{x}$, $AB-x^2=0$; and $AB=x^2$; whence x is a mean proportional between A and B.

Our Author has given a clear solution of this *Problem*, but in a different manner.

COR. I. If a number of bodies be in a continual geometrical progression, if the least of the bodies be A, the *ratio* of the increase be e, and the number of bodies n; and if A strike the second body at rest, and the second with the motion acquired strike the third body at rest, and so on to the last; the bodies, their velocities and motions, will be thus expressed.

$$\text{Bodies -- } A,\quad eA,\quad e^2A,\quad e^3A,\quad \&c.\ \overline{e}|^{n-1}A.$$

$$\text{Velocities - } a,\ \frac{2a}{1+e},\ \frac{4a}{\overline{1+e}|^2},\ \frac{8a}{\overline{1+e}|^3},\ \&c.\ \overline{\frac{2}{1+e}}\Big|^{n-1}a.$$

$$\text{Motions - } Aa,\ \frac{2Aae}{1+e},\ \frac{4Aae^2}{\overline{1+e}|^2},\ \frac{8Aae^3}{\overline{1+e}|^3},\ \&c.\ \overline{\frac{2e}{1+e}}\Big|^{n-1}Aa.$$

EXAM. I. If the number of bodies increasing in geometrick proportion be 20, and the common *ratio* of the terms be 2, n will be 20, and e be 2. The last body will be 524288 times greater than the first; the velocity of the last will be to the velocity of the first, as 1 to $2216\frac{4}{5}$; and the motion of the last will be about $236\frac{1}{2}$ times greater than the motion of the first.

EXAM. II. If the number of bodies be 100, and the common *ratio* of the progression be 2; then will n be 100,

100, and e will be 2. In this caſe, the laſt body will be above 633825300000000000000000000000 times greater than the firſt, it's velocity will be to the velocity of the firſt, as 1 to 2710220000000000000 nearly; and the motion of the laſt will be to the motion of the firſt, nearly as 2338480000000 to 1.

Cor. II. If the motion of the firſt body be to the motion of the laſt, as 1 to D, that is, if Aa be to $\left.\overline{\frac{2e}{1+e}}\right|^{n-1}$ Aa, as 1 to D, then will e be equal to

$$\frac{D^{\frac{1}{n-1}}}{2-D^{\frac{1}{n-1}}}.$$

For example, if the number of bodies be 20, and the motion of the laſt be 100000 times greater than the motion of the firſt, n will be 20, D will be 100000, and e will be 10.9746 nearly; ſo that each preceding body in the 20 bodies muſt be 10.9746 times greater than the body lying next behind it.

Cor. III. If the motion of the firſt body be to the motion of the laſt, as 1 to D, that is, if Aa be to $\left.\overline{\frac{2e}{1+e}}\right|^{n-1}$ Aa, as 1 to D, D will be equal to $\left.\overline{\frac{2a}{1+e}}\right|^{n-1}$, and, putting R for $\frac{2e}{1+e}$, and L for logarithm, we ſhall have $D=R^{n-1}$, and L, D $=$ $\overline{n-1}\times$ L, R.

For example, if e be 4, and n be 25, $\frac{2e}{1+e}$ will be $\frac{8}{5}$, the logarithm of which number is 0.2041299 $=$ L, R; and $\overline{n-1}\times$ L, R $= 4.8988795 =$ L, D. The natural number of this logarithm is 79228 nearly; ſo that in this caſe the motion of the laſt

 body

body will be nearly 79228 times greater than the motion of the firſt.

Cor. IV. If D and R be given, n may be found by being equal to $\frac{L,D+L,R}{L,R}$; for by the laſt *Corollary* $L,D=\overline{n-1}\times L,R$, and conſequently $n=\frac{L,D+L,R}{L,R}$.

For example, if D be 100000, and e be 2, in which caſe R will be $\frac{4}{3}$; then will L, D, be 5.000000, and L,R 0.1249387; and $\frac{L,D+L,R}{L,R}$, will be $41.02=n$; ſo that more than 41 bodies will be neceſſary to make the motion of the laſt 100000 times greater than the motion of the firſt.

Of the Motion *of a* Globe *in a* Fluid Medium.

Prob. III. *If the diameter and denſity of a Globe moving in a fluid* medium, *if the denſity of the* medium, *the velocity with which the* globe *ſets out, and the time of the motion, be all given; to determine, the part of the velocity which is deſtroyed by the reſiſtance of the* medium, *the remaining part of the velocity, and the ſpace deſcribed by the globe in the given time.*

Let D denote the diameter of the globe, d it's denſity, δ the denſity of the fluid *medium*, V the velocity with which the globe ſets out, t the time of the motion expreſſed in ſeconds, m the part of a diameter or number of diameters of the globe which it would deſcribe with the velocity V in the time t, and T the time in which the globe with the velocity V would *in vacuo* deſcribe a ſpace which is to $\frac{8D}{3}$ as d to δ; and then the part of the velocity

deſtroyed

deſtroyed by the reſiſtance of the medium, will be $\frac{m\delta V}{\frac{8}{3}d+m\delta}$; the remaining part of the velocity will be $\frac{\frac{8}{3}dV}{\frac{8}{3}d+m\delta}$; and the ſpace deſcribed in the *medium* in the time t, will be $\frac{8Dd}{3\delta} \times \text{Log.} \overline{1+\frac{m\delta}{\frac{8}{3}d}} \times 2.302585093$.

For Sir ISAAC NEWTON has proved, that the part of the velocity which is deſtroyed by the reſiſtance of the medium in the time t, is $\frac{Vt}{T+t}$; that the remaining part is $\frac{VT}{T+t}$; and that the ſpace deſcribed in the time t, is $TV \times \text{Log.} \overline{\frac{T+t}{T}} \times 2.302585093$. But by conſtruction, T is as $\frac{8Dd}{3\delta V}$, and V is as $\frac{mD}{t}$. And therefore, by ſubſtituting $\frac{8Dd}{3\delta V}$ and $\frac{mD}{t}$ inſtead of T and V in the foregoing expreſſions, the part of the velocity deſtroyed by the reſiſtance of the medium in the time t will be $\frac{m\delta V}{\frac{8}{3}d+m\delta}$, the remaining part of the velocity will be $\frac{\frac{8}{3}dV}{\frac{8}{3}d+m\delta}$, and the ſpace deſcribed in the time t will be $\frac{8dD}{3\delta} \times \text{Log.} \overline{1+\frac{m\delta}{\frac{8}{3}d}} \times 2.302585093$.

COR. I. If the denſity of the globe be equal to the denſity of the *medium*, that is, if d be equal to δ, the velocity deſtroyed by the reſiſtance of the *medium* in the time t, will be $\frac{mV}{\frac{8}{3}+m}$.

This *Corollary* will obtain, if the globe and the *medium* be perfectly denſe or void of pores; for by being entirely void of pores, they will have equal denſities. And ſuch a Globe moving in ſuch a *medium* the length of 3 times it's diameter, will loſe above half it's velocity; for if m be 3, $\frac{mV}{\frac{8}{3}+m}$ will be $\frac{9V}{17}$. And this will always be the velocity loſt in moving three times the length of the diameter, when the globe and the *medium* have equal denſities.

Cor. II. If a globe in moving through m times it's diameter in a fluid *medium*, loſe the n part of it's velocity; then will $n = \frac{m\delta}{\frac{8}{3}d+m\delta}$, $d = \frac{\delta \times \overline{m-nm}}{\frac{8}{3}n}$, $\delta = \frac{\frac{8}{3}dn}{m-nm}$, and $m = \frac{8nd}{3\delta-3n\delta}$. For $\frac{m\delta V}{\frac{8}{3}d+m\delta} = nV$; whence $n = \frac{m\delta}{\frac{8}{3}d+m\delta}$, $d = \frac{\delta \times \overline{m-nm}}{\frac{8}{3}n}$, $\delta = \frac{\frac{8}{3}dn}{m-nm}$, and $m = \frac{8nd}{3\delta-3n\delta}$.

Exam. I. If a globe loſe $\frac{3}{4}$ of it's velocity in moving the length of 10 times it's diameter in water, in which caſe n will be $\frac{3}{4}$, m will be 10, and δ will be 1; then d will be $\frac{5}{4}$, that is, the globe will be denſer than water in the proportion of 5 to 4.

Exam. II. If a globe 10 times as denſe as water, loſe $\frac{3}{4}$ths of it's velocity in moving 10 times it's diameter in a fluid; the denſity of that fluid will be 8 times as great as the denſity of water. In

In this caſe d is 10, m is 10, and n is $\frac{3}{4}$; and δ, which is equal to $\frac{\frac{8}{3}dn}{m-nm}$, is 8.

Exam. III. If a globe twice as denſe as water, loſe $\frac{3}{4}$ths of it's motion by moving in a fluid 14 times as denſe as water; it will ſuffer this loſs of velocity in moving the length of $1\frac{1}{7}$D. For in this caſe, d, δ, n, are 2, 14, $\frac{3}{4}$; and m, which is equal to $\frac{8nd}{3\delta-3n\delta}$, will be $\frac{8}{7}=1\frac{1}{7}$.

Exam. IV. If a perfectly ſolid globe move 24 times the length of it's diameter in a perfectly ſolid *medium*, it will loſe 9 parts in 10 of the velocity it had at the beginning of the motion. For in this caſe d is equal to δ, and m is 24; and n, which is equal to $\frac{m}{\frac{8}{3}+m}$, will be equal to $\frac{72}{80}=\frac{9}{10}$.

Exam. V. If a globe of equal denſity with water, move half the length of it's diameter in air, it will loſe the $\frac{1}{4587\frac{2}{3}}$ part of it's velocity, on ſuppoſition that the denſity of water is to the denſity of air, as 860 to 1. For in this caſe d, δ, m, are 860, 1, $\frac{1}{2}$; and n, which is equal to $\frac{m\delta}{\frac{8}{3}d+m\delta}$, will be $\frac{1}{4587\frac{2}{3}}$.

Exam. VI. If the earth moved round the ſun in a fluid *medium* of equal denſity with the air at the ſurface of the earth, it would by the reſiſtance of the *medium* loſe almoſt all it's motion in 10000 years, on ſuppoſition that the denſities of the earth, of water, and of the *medium*, are 5, 1, $\frac{1}{860}$ or in decimals 0.0011628. For the earth moves in it's

orbit

orbit with a velocity that carries it at the rate of 4893938782791 miles, or 617142343 times the length of it's own diameter in 10000 years, on supposition that the sun's horizontal parallax is $10\frac{1}{2}''$. In this case therefore d, δ, m, are 5, 0.0011628, and 617142343; and consequently n, which is equal to $\frac{m\delta}{\frac{8}{3}d+m\delta}$, will be a $\frac{717713}{717727}$th part, which is nearly the whole, of it's present velocity.

By the *French* measures, a degree of a great circle of the earth contains 342366 *Paris* feet, or 365403·3158 *English* feet, on supposition that a *Paris* foot is to an *English* foot, as 1142 to 1070. And consequently the diameter of the earth, supposing the earth to be spherical, will be 41870881 *English* feet, or 7930 miles. The mean distance of the sun from the earth, reckoning the parallax at $10\frac{1}{2}''$ seconds, is about 19644.2675 semidiameters of the earth, or 77889520.6375 miles; consequently the circumference of the earth's orbit is 489393878.2791 miles, which the earth describes in one year, or 29558161.6 seconds of time.

Exam. VII. If the earth move in an *Æther* 700000 rarer than the air at the surface of the earth, it will lose about $\frac{1}{14}$th part of it's present velocity in 10000 years; for in this case d, δ and m, are 5, 0.00000000166, and 617142343; and consequently n, which is equal to $\frac{m\delta}{\frac{8}{3}d+m\delta}$ will be equal to a $\frac{1.02445628938}{14.35778962271}$th part, that is $\frac{1}{14}$th part of the present velocity very nearly.

And if the earth move 100000 years in this *Æther*, it will lose almost half of it's present motion in that time.

Exam.

Exam. VIII. If we ſuppoſe the earth to loſe the $\frac{1}{100}$th part of it's preſent velocity by moving in an *ætherial medium* for 400000 years, in which time it will have deſcribed 24685693680 times it's diameter, the denſity of the *medium* will be above 200 millions of times leſs than the denſity of the air at the ſurface of the earth. For in this caſe d, n, m, are 5, 0.01, 24685693680, and conſequently δ, which is equal to $\frac{\frac{8}{3}dn}{m-nm}$, will be $\frac{0.1333333}{24438836743.2000000} = \frac{1}{183366000000}$. But the denſity of water being 1, the denſity of air is $\frac{1}{860}$; and conſequently, the denſity of the air at the ſurface of the earth will be to the denſity of this *medium*, as above 213200000 to 1.

Of the Motion *of* Wheels *over* Obſtacles.

Prob. IV. *If a wheel moving on an horizontal plane, meet with an immoveable obſtacle in it's way over which it is to be drawn by a force fixed to it's center; if the weight and diameter of the wheel, the height of the obſtacle, and the direction of the force drawing the wheel, be all known; thence to determine the force that is ſufficient to draw the wheel over the obſtacle.*

Let GPME be the wheel, ND the horizontal plane on which it moves from N towards D, EF the obſtacle over which it is to be drawn; let the wheel arrive at the obſtacle, and touch it's top E; and there let it be ſuppoſed to ſtand preſſing the horizontal plane at G with it's whole weight. Draw OEK a tangent to the wheel in the point E, draw the diameter ACG perpendicular to the horizontal plane, and produce it till it meet the tangent in O; from E draw the *radius* EC; draw EH perpendicular

cular to AG; and mr, MC, perpendicular to EC, and consequently parallel to the tangent OK; and lastly, draw the *radius* Cm; if the whole weight of the wheel be expressed by CO, in the direction of which line that weight acts when the wheel is wholly supported by the horizontal plane at G, that weight may be resolved into two others CE and OE, acting according to the directions of those lines, the weight CE pressing against the top of the immoveable obstacle, and being wholly sustained by it, and the weight OE drawing the wheel down in a direction parallel to the tangent OEK. Let W denote the whole weight of the wheel, r it's *radius*, h the height of the obstacle, and x the part of the whole weight which draws the wheel down in a direction parallel to OEK; and then we shall have this analogy; as x is to W, so is OE to CO, or HE to CE, from the similarity of the triangles CEO, and CEH; whence $x = \frac{W \times HE}{r}$; but HE from the nature of the circle, is equal to $\sqrt{AH \times HG}$, or to $\sqrt{AH \times EF}$, that is, in symbols, to $\sqrt{2rh - hh}$; and therefore $x = \frac{W \times \sqrt{2rh - hh}}{r}$. A force just equal to this weight, and acting in direct opposition to it, that is, drawing the wheel upward in the direction CM parallel to OK, will just be able to make the wheel rest on E the top of the obstacle, without suffering any part of it's weight to rest on the horizontal plane at G. This force must be increased to produce the same effect, if it act in any other direction than that of CM. For let it draw the wheel in the direction Cm, m lying between E and M, and then the force acting in this direction may be resolved into 2 forces, which will be as Cr and rm, whereof Cr draws the wheel directly against E the top of the obstacle, and

and ſo is loſt, and mr draws it upwards in a direction parallel to OK. But mr is leſs than Cm or CM, and to become equal to it, and conſequently, ſufficient to ſupport the wheel againſt the top of the obſtacle without ſuffering any part of it's weight to reſt on the horizontal plane, it muſt be increaſed in the *ratio* of Cm or CM to rm, that is, putting s for the ſine of the angle which the direction of the force makes with CE, in the *ratio* of r to s; but the force rm cannot be increaſed, but the whole force CM muſt be increaſed in the ſame proportion. And therefore the force $\frac{W \times \sqrt{2rh - hh}}{r}$ muſt be increaſed in the proportion of r to s, and then, putting F for the force, acting in the direction Cm, which is juſt ſufficient to ſupport the wheel on the obſtacle without ſuffering it to preſs on the plane ND, $F = \frac{W \times \sqrt{2rh - hh}}{s}$; and the ſmalleſt addition to this force will make it draw the wheel over the obſtacle.

Since the reſiſtance given by the obſtacle, is equal to the force that is juſt ſufficient to make the wheel reſt on the obſtacle without ſuffering any part of it's weight to preſs on the plane of the horizon, that is, putting R for the reſiſtance given by the obſtacle, ſince R is equal to F; R will be equal to $\frac{W \times \sqrt{2rh - hh}}{s}$.

It is to be obſerved, that the direction of the force muſt lie between CE and CA; for if the force draw the wheel in the direction CE it will be wholly ſpent upon the obſtacle, and not in the leaſt contribute to draw the wheel over it; and if it draw the wheel directly upwards from C to A, it will not make it to preſs againſt the obſtacle, and conſequently,

quently, however great we may ſuppoſe it to be, can never draw it over it.

Cor. I. If the direction of the moving force change continually, paſſing from CE to CM, and thence to CP, the ſine of the angle which the line of direction makes with CE, will increaſe in the paſſage of that line from CE to CM, and decreaſe in it's paſſage from CM to CP; but as s increaſes or leſſens, $\frac{W \times \sqrt{2rh - hh}}{s}$ will leſſen or increaſe; and conſequently the force F will leſſen in the paſſage of the line of it's direction from CE to CM, and thence increaſe in the paſſage of that line to CA. So that the force will be leaſt when it acts in the direction CM, in which caſe the whole force will be employed in drawing the wheel over the obſtacle; whereas in all other directions, part of the force will be loſt by drawing directly againſt the top of the obſtacle. Hence the moſt advantageous direction of the force, will be that which makes a right angle with CE, in which caſe s will be equal to r, and $F = \frac{W \times \sqrt{2rh - hh}}{r}$.

Cor. II. If the height of the obſtacle be given, in which caſe h will be as 1, and the force draw the wheel in the direction CM parallel to OK; then F will be as $\frac{W \times \sqrt{2r - 1}}{r}$.

If the *radii* of 4 wheels, be 1, 2, 3, 4, then will $\frac{\sqrt{2r - 1}}{r}$, be 1, $\frac{\sqrt{3}}{2}$, $\frac{\sqrt{5}}{3}$, $\frac{\sqrt{7}}{4}$, that is, as the numbers 1000, 866, 745, 661; and the forces requiſite to ſupport theſe wheels on the point E, ſo as not to ſuffer any part of their weight to reſt on the horizontal plane, will be as their weights multiplied

into

into theſe numbers reſpectively. The force requiſite to ſupport the firſt wheel, will be as it's weight multiplied into 1000, the force requiſite to ſupport the ſecond wheel as it's weight multiplied into 866; and ſo of the reſt. And if the weights of all the wheels be equal, the forces neceſſary to ſupport them, and conſequently the reſiſtances given by the obſtacle to which theſe forces are equal, will be as the numbers 1000, 866, 745, 661. So that in wheels of a given weight, the leſſer the wheel is, the greater will be the reſiſtance which is given to it by an obſtacle of a given height.

Cor. III. If the height of the obſtacle be indefinitely ſmall and given, in which caſe the tangent OK will coincide with the horizontal plane ND, and the point E coincide with the point G; and if the force draw the wheel in a direction parallel to OK or ND; then will F be as $W \times \frac{\sqrt{2r}}{r}$, or, becauſe 2 is a given quantity, as $\frac{W}{\sqrt{r}}$; and if the weight of the wheel be given, F will be as $\frac{1}{\sqrt{r}}$.

If the *radii* of 4 wheels of equal weights be, 1, 2, 3, 4, and the wheels be drawn on a ſmooth plane parallel to the horizon; the forces neceſſary to put them in motion, when they draw in directions parallel to that plane, will be as 1, $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{4}}$, that is, as the numbers 1000, 707, 577, 500. And therefore, of wheels drawn on the plane of the horizon by forces acting in directions parallel to that plane, leſſer wheels will require a greater force to put them in motion than greater.

Cor.

Cor. IV. If the height of the obſtacle be proportional to the *radius* of the wheel, and if the force draw the wheel in a direction parallel to OK; that is, if h be as r, and F be as $\frac{W \times \sqrt{2rh-hh}}{r}$; then will the force, and conſequently the reſiſtance given by the obſtacle, be as the weight of the wheel; for $\frac{\sqrt{2rh-hh}}{r}$ will be as $\frac{\sqrt{2rr-rr}}{r}$, that is as 1; and therefore F will be as W.

Cor. V. If the direction of the force drawing the wheel be parallel to the horizontal plane, that is, if mC be parallel to ND; then will the force that is requiſite to ſuſtain the wheel on the point E, be $\frac{W \times \sqrt{2rh-hh}}{r-h}$. For in this caſe the angle mCE is equal to the angle CEH, and conſequently, their ſines are equal, that is, s is equal to CH, which in ſymbols is r—h. And therefore F, which univerſally is as $\frac{W \times \sqrt{2rh-hh}}{s}$, is in this caſe as $\frac{W \times \sqrt{2rh-hh}}{r-h}$.

If the height of the obſtacle be given, in which caſe h will be as 1, then will F be as $\frac{W \times \sqrt{2r-1}}{r-1}$.

If the *radii* of 4 wheels of equal weight, be 1, 2, 3, 4; then will F with reſpect to theſe four wheels, be as $\frac{1}{0}$, $\frac{\sqrt{3}}{1}$, $\frac{\sqrt{5}}{2}$, $\frac{\sqrt{7}}{3}$, that is, as infinite, 1732, 1128, 882. The height of the obſtacle is equal to the *radius* of the firſt wheel, inaſmuch as I have ſuppoſed them both to be as 1; and conſequently the force muſt be infinite to make the wheel

wheel reſt againſt E, and hinder any part of it's weight from preſſing on the horizontal plane at G.

Cor. VI. The force is to the weight of the wheel, as the ſine of the angle ECH is to the ſine of the angle which the line of direction of the force makes with EC; that is $\frac{F}{W} = \frac{\sqrt{2rh - hh}}{s}$.

If the force be one half of the weight of the wheel, that is, if F be one half of W, $\sqrt{2rh - hh}$ will be one half of s; if F be equal to W, $\sqrt{2rh - hh}$ will be equal to s; and if F be as W, $\sqrt{2rh - hh}$ will be as s.

Of the Motion *of* Water *through* Orifices *and* Pipes.

Prob. V. *To determine the motion of water running out of a hole made in the bottom of a veſſel.*

Sir Isaac Newton has given a general ſolution of this *Problem* in the following paragraph, which is contained in *prop.* 36. *prob.* 8. *lib.* 2.

" Sit ACDB vas cylindricum, AB ejus orificium Pl. 11.
" ſuperius, CD fundum horizonti parallelum, EF Fig. 2.
" foramen circulare in medio fundi, G centrum fo-
" raminis, et GH axis cylindri horizonti perpen-
" dicularis. Et finge cylindrum glaciei APQB ejuſ-
" dem eſſe latitudinis cum cavitate vaſis, et axem eun-
" dem habere, et uniformi cum motu perpetuo de-
" ſcendere, et partes ejus quam primum attingunt
" ſuperficiem AB liqueſcere, et in aquam converſas
" gravitate ſuâ defluere in vas, et cataractam vel
" columnam aquæ ABNFEM cadendo formare, et
" per foramen EF tranſire, idemque adæquate im-
" plere. Ea vero ſit uniformis velocitas glaciei
" deſcendentis ut et aquæ contiguæ in circulo AB,
" quam aqua cadendo et caſu ſuo deſcribendo alti-

" tudinem IH acquirere poteſt; et jaceant IH et
" HG in directum, et per punctum I ducatur recta
" KL horizonti parallela et lateribus glaciei occur-
" rens in K et L. Et velocitas aquæ effluentis
" per foramen EF ea erit quam aqua cadendo ab I
" et caſu ſuo deſcribendo altitudinem IG acquirere
" poteſt. Ideoque per theoremata Galilæi erit
" IG ad IH in duplicata ratione velocitatis aquæ
" per foramen effluentis ad velocitatem aquæ in cir-
" culo AB, hoc eſt, in duplicata ratione circuli
" AB ad circulum EF; nam hi circuli ſunt reci-
" proce ut velocitates aquarum quæ per ipſos, eodem
" tempore et æquali quantitate, adæquatè tranſeunt.
" De velocitate aquæ horizontem verſus hic agitur.
" Et motus horizonti parallelus quo partes aquæ ca-
" dentis ad invicem accedunt, cum non oriatur a
" gravitate, nec motum horizonti perpendicula-
" rem a gravitate oriundum mutet, hic non conſi-
" deratur. Supponimus quidem quod partes aquæ
" aliquantulum cohærent, et per cohæſionem ſuam
" inter cadendum accedant ad invicem per motus
" horizonti parallelos, ut unicam tantum efforment
" cataractam et non in plures cataractas dividantur:
" ſed motum horizonti parallelum, a cohæſione illâ
" oriundum, hic non conſideramus.

This *Theory* Sir Isaac corrected by experiments, proved it in ſix different caſes, and drew ſeveral corollaries from it. The reaſon why a correction was neceſſary will be ſhewn in the *Scholium*. And the truth of his and other corollaries flowing from this theory, will more eaſily appear by expreſſing the foregoing proportions of the velocities in ſymbols; to do which, let A denote the *area* of the circle AB, a the *area* of the hole EF, H the line HG which is the perpendicular height of the water in the veſſel above the hole, x the height IH, from which water or any other body muſt fall by the force of gravity from a ſtate of reſt, to acquire the velocity of the water in AB, V the velocity of water

water in it's paſſage through the hole EF, and v it's velocity in the ſurface AB; and then the proportions will be thus expreſſed, $H+x \,.\, x :: V^2 \,.\, v^2 :: A^2 \,.\, a^2$; whence, $\sqrt{\overline{H+x}} \,.\, \sqrt{x} :: V \,.\, v :: A \,.\, a$.

Cor. I. The height from which a body muſt fall to acquire a velocity equal to the velocity of the water in the ſurface AB, is equal to $\frac{v^2H}{V^2-v^2}$, or $\frac{a^2H}{A^2-a^2}$. For by inverſion and diviſion of proportion, $x \,.\, H :: v^2 \,.\, V^2-v^2 :: a^2 \,.\, A^2-a^2$; whence $x = \frac{v^2H}{V^2-x^2} = \frac{a^2H}{A^2-a^2}$. But x denotes IH. And therefore $IH = \frac{v^2H}{V^2-v^2} = \frac{a^2H}{A^2-a^2}$.

Cor. II. The perpendicular height of the water in the veſſel, denoted by H, is equal to $\frac{IH \times \overline{V^2-v^2}}{v^2}$, or $\frac{IH \times \overline{A^2-a^2}}{a^2}$, by *Cor.* 1.

Cor. III. The height from which a body muſt fall to acquire a velocity equal to that with which the water flows through the hole, is equal to $\frac{V^2H}{V^2-v^2}$, or $\frac{A^2H}{A^2-a^2}$. For by diviſion of proportion, $H+x = IG \,.\, H :: V^2 \,.\, V^2-v^2 :: A^2 \,.\, A^2-a^2$, whence $IG = \frac{V^2H}{V^2-v^2} = \frac{A^2H}{A^2-a^2}$.

Cor. IV. The perpendicular height of the water in the veſſel, denoted by H, is equal to $\frac{IG \times \overline{V^2-v^2}}{V^2}$, or to $\frac{IG \times \overline{A^2-a^2}}{A^2}$, by *Cor.* 3.

Cor. V. If the *area* of the ſurface be equal to the *area* of the hole, H will be nothing in compariſon of IH and IG which will be equal. For if A be equal to a, H will be nothing by *Cor.* 2. and IH and IG will be equal and infinite by *Cor.* 1, and *Cor.* 3.

The truth of this *Corollary* may likewiſe appear from the nature of gravity. For if A be equal to a, V muſt be equal to v. But V can never be equal to v while there is any acceleration of the motion of the water in it's deſcent through the veſſel, as there will always be till H becomes nothing in compariſon of the equal lines IH and IG, which in this caſe muſt be conſidered as infinite.

Cor. VI. If a be greater than A, in which caſe $A^2 - a^2$ will be negative, H will be negative, by *Cor.* 4; and IG, and conſequently V, will be affirmative, by *Cor.* 3. But a negative perpendicular height of the water in the veſſel, and an affirmative velocity of the water flowing through the hole, require an inverſion of the veſſel or a turning of it's bottom upwards; by which inverſion the hole will become the upper orifice, and the upper orifice the hole; a will become A, and A become a; and the velocity will be affirmative, that is, the water will move downwards, as it ought to do from the nature of gravity. Farther, when a is greater than A, the veſſel will be conical with it's wider end downwards; but from the nature of gravity, water poured in at the top or narrower end of ſuch a veſſel, will deſcend in a cylindrical column, which will not fill the baſe, as the foregoing account of this motion requires; and therefore, to give this caſe the conditions required, there muſt be an inverſion of the veſſel.

Cor. VII. If the hole be ſmall, and the ſurface of the water infinitely large, both a and v may be

be confidered as o with refpect to A and V; confequently IH will be o, by *Cor.* 1. and IG will be equal to H, by *Cor.* 3.

In this cafe, and this only, the fuperficial parts of the water have no velocity at the very beginning of the motion, but begin to defcend from a ftate of reft, as quiefcent bodies do when the fupport is taken away. In all other cafes, in which a and v have fome magnitudes when compared with A and V, the fuperficial parts of the water fet out with fome velocity, and do not begin to defcend, on the water's beginning to flow through the hole, as heavy bodies near the furface of the earth begin to defcend from a ftate of reft.

Cor. VIII. If the *ratio* of the furface to the hole be given, as it will be when each of them continues the fame, or when both of them change in the fame proportion; the velocity in the furface will be proportional to the velocity through the hole, and both will be proportional to the velocity which would be acquired by a body in falling though a height equal to the perpendicular height of the water in the veffel. If $\frac{A}{a}$ be given, $\frac{V}{v}$ will be given; and confequently v will be as V. And fince $\frac{A}{a}$ is given, $\frac{a^2}{A^2-a^2}$, and $\frac{A^2}{A^2-a^2}$ will both be given; and confequently both IH and IG will be as H, by *Cor.* 1. and *Cor.* 3. But v and V are as $\sqrt{IH}$ and $\sqrt{IG}$. And therefore, both v and V will be as $\sqrt{H}$.

By this *Corollary*, when A and a continue invariable, and the heights of the water in the veffel are 1, 4, and 16 feet; the velocities in AB and EF will be as 1, 2, and 4. But bodies placed at fmall diftances from the furface of the earth, do all begin to defcend with the fame velocity very near-

 ly,

ly, as has been proved by experiments. And therefore the ſuperficial parts of the water in this caſe, begin to deſcend in a very different manner, or with very different velocities from that with which a heavy body placed at thoſe heights, begins to deſcend from a ſtate of reſt. The velocity in AB is regulated by the velocity in EF, and the velocity in EF is always meaſured by $\sqrt{H}$, when $\frac{A}{a}$ is given.

Cor. IX. The velocity of the water in the ſurface AB is always the $\frac{a}{A}$ part of the velocity through the hole, that is, v is the $\frac{a}{A}$ part of V, or in other words, $v = \frac{aV}{A}$. When a is nothing in proportion to A, as we may ſuppoſe it to be, when a is very ſmall, and A exceedingly great, then will v be no ſenſible part of V, that is, it will be nothing; and conſequently, the ſuperficial parts of the water will in this caſe begin their motion, as heavy bodies do, from a ſtate of reſt.

Cor. X. The whole motion of the deſcending column AMEFNB, is equal to the motion of a cylinder of water, whoſe baſe is a, whoſe altitude is H, and whoſe velocity is V, that is, to the motion $aH \times V$. For Va is equal to vA, that is, the motion of the water in EF is equal to it's motion in AB; and from the nature of the deſcending column, each of them is equal to the motion in any ſection of the column parallel to EF or AB; and conſequently, the motion in all the ſections, ſuppoſing them to be indefinitely many, that is, the whole motion of the deſcending column, will be equal to the motion in the hole multiplied into the number of

of ſections, that is to $Va \times H$, or $aH \times V$. This property has been proved by Dr. JURIN.

COR. XI. The force which can generate the whole motion of the water running out of the hole, is equal to the weight of a cylinder of water whoſe baſe is a, and altitude is 2IG, by *Cor.* 3; that is, equal to the weight of a cylinder of water, whoſe magnitude is $2aH \times \frac{A^2}{A^2 - a^2}$. For in the ſame time, in which the water running out is equal to this cylinder, this cylinder, by falling from the height IG by the force of it's gravity, will acquire a velocity equal to that with which the water runs out. But when the quantities of matter and velocities of two bodies are equal, their motions, and conſequently the forces which can generate thoſe motions in equal times, will likewiſe be equal. And therefore the force which can generate the whole motion of the water running out of the hole, is equal to the weight of a cylinder of water whoſe magnitude is $2aH \times \frac{A^2}{A^2 - a^2}$.

COR. XII. The weight of the deſcending column AMEFNB is equal to the weight of a cylinder of water, whoſe baſe is a and whoſe height is $\frac{2HA}{A+a}$, that is, whoſe magnitude is $2aH \times \frac{A}{A+a}$. For let IO be a mean proportional between IH and IG, and then, $\sqrt{IH} . \sqrt{IG} :: IH . IO :: IO . IG :: a . A$; and, by diviſion of proportion, $HO . IH :: OG . IO$; and by alternation and compoſition, $HO + OG . 2HO :: IH + IO . 2IH :: a + A . 2a$. But, by *Cor.* 11, in the time a drop of water falls by it's own gravity from I to G, the quantity of water diſcharged by the hole will be equal to $a \times 2IG$, or $A \times 2IO$; and in the time the drop

 deſcends

descends from I to H, the quantity of water passing through the surface AB, and discharged by the hole, will be equal to $A \times 2IH$; and the difference of these quantities, namely $A \times 2HO$, will be the quantity discharged in the time the falling drop descends from H to G, which quantity is the column AMEFNB; for in the time the drop descends from H to G, the superficial parts of the water, setting out with the velocity of the drop at H, and descending freely and without resistance, will reach the hole. And therefore, all the water in the vessel will be to the water in the column AMEFNB, as $A \times H$ is to $A \times 2HO$, or as $H = HO + OG$ to $2HO$; or as $a + A$ to $2a$; whence, putting Q for the quantity of water in the descending column, $A \times H \,.\, Q :: A + a \,.\, 2a$; and consequently, $Q = 2aH \times \frac{A}{A+a}$.

This *Corollary* may be proved in another manner, thus. The cataract is the difference of the two hyperboloids KAMEFBL and KABL, supposing the assymptote KL to be infinitely extended both ways, and the *area* AB to be infinite; but by fluxions, as Dr. JURIN has shewn, the hyperboloid KAMEFNBL is equal to $2a \times \overline{H+x}$, or to $\frac{2A^2x}{a}$, because H is equal to $\frac{A^2x - a^2x}{a^2}$ by *Cor.* 2; and the hyperboloid KABL, is equal to $2Ax$, and the difference of the two is $\frac{2A^2x}{a} - 2Ax = \frac{2A^2x - 2Aax}{a}$. All the water in the vessel is AH, or, by substituting $\frac{A^2x - a^2x}{a^2}$ in the room of H, $\frac{A^3x - Aa^2x}{a^2}$; and consequently, the water in the vessel

veſſel is to the water in the cataract, as $\frac{A^3x - Aa^2x}{a^2}$ is to $\frac{2A^2x - 2Aax}{a}$, that is, after due reduction, as $A+a$ is to $2a$. Therefore $AH . Q :: A+a . 2a$: whence, $Q = 2aH \times \frac{2A}{A+a}$.

Cor. XIII. The weight of all the water in the veſſel, is to the weight of that part of it which is ſuſtained by the bottom, as the ſum of the circles AB and EF is to their difference. For, ſince $A \times H . Q :: A + a . 2a$, by *Cor.* 12, $A \times H . A \times H - Q :: A+a . A+a-2a = A-a$ by diviſion of proportion.

Cor. XIV. The weight of the water which the bottom ſuſtains is to the weight of the cataract, as the difference of the circles AB and EF, to twice the leſſer circle EF. For $A \times H . Q :: A+a . 2a$, by *Cor.* 12. And by diviſion of proportion, $A \times H - Q . Q :: A + a - 2a = A - a . 2a$.

Cor. XV. The weight of water which the bottom ſuſtains, is to the weight of water perpendicularly incumbent thereon, as the circle AB, is to the ſum of the circles AB and EF. For the weight of water which the bottom ſuſtains is $A \times H - Q = AH - \frac{2aHA}{A+a}$, by *Cor.* 12, $= \frac{A^2H - aAH}{A+a}$; and the weight perpendicularly incumbent on the bottom is $\overline{A - a} \times H = AH - aH$. But $\frac{A^2H - aAH}{A+a} . AH - aH :: A^2 - aA . A^2 - a^2 :: A . A+a$ by dividing by $A-a$.

Cor. XVI. The quantity of water in the deſcending column, is to the quantity perpendicularly incumbent

incumbent on the hole, as twice the circle AB, is to the ſum of the circles AB and EF. For the quantity of water in the deſcending column is $2aH \times \frac{A}{A+a}$. But $2aH \times \frac{A}{A+a} . aH :: \frac{2A}{A+a} . 1 :: 2A . A+a.$

Hence, when a is nothing, as we may ſuppoſe it to be when A is infinitely great, the deſcending column will be equal in magnitude to 2aH, as Dr. JURIN has ſhewn it to be by determining it's magnitude by fluxions.

COR. XVII. The weight of the deſcending column, is to the weight of water which can generate the whole motion of the water running out of the hole, as the difference of the circles AB and EF, is to the greater circle AB. For, putting F for the force or weight which can generate the whole motion of the water running out of the hole, and ſuppoſing Q to denote the weight of the deſcending column, we ſhall have F equal to the weight of a quantity of water whoſe magnitude is $2aH \times \frac{A^2}{A^2-a^2}$, by *Cor.* 11, and Q equal to the weight of a quantity, whoſe magnitude is $2aH \times \frac{A}{A+a}$, by *Cor.* 12. And therefore, $Q . F :: 2aH \times \frac{A}{A+a} . 2aH \times \frac{A^2}{A^2-a^2} :: 1 . \frac{A}{A-a} :: A-a . A.$ Hence $Q = \frac{F \times \overline{A-a}}{A}$ and $F = \frac{QA}{A-a}$; and conſequently, the force which can generate the whole motion of the water running out of the hole, will always exceed the weight of the deſcending column, except when a becomes 0, as we may ſuppoſe it to do, when it is very ſmall, and A exceedingly great.

COR.

Cor. XVIII. The force which can generate the whole motion of the water running out of the hole, is to the weight of water perpendicularly incumbent on the hole, as twice the ſquare of the greater circle AB, to the difference of the ſquares of the circles AB and EF. For the force which can generate the whole motion of the water running out of the hole, is the weight of $2aH \times \frac{A^2}{A^2-a^2}$ quantity of water, by *Cor.* 11. and the weight of water perpendicularly incumbent on the hole, is the weight of the cylinder aH. But $2aH \times \frac{A^2}{A^2-a^2}$. $aH :: 2A^2 . A^2-a^2$. In the ſame *ratio* is the whole motion of the effluent water to the motion of the water in the cataract.

Cor. XIX. If in the middle of the hole be placed a little circle PQ parallel to the horizon, whoſe center is G, and if the *area* of this circle be called o; the weight of water which it ſuſtains during the efflux of the water through the ring ſurrounding it, is to the weight of half the cylinder oH, as a to $a-\frac{1}{2}o$; if R denote the weight ſuſtained, R is to $\frac{oH}{2}$, as a to $a-\frac{1}{2}o$, and R is equal to $\frac{aoH}{2a-o}$. For if we ſuppoſe A to be contracted till it becomes equal to a, in which caſe IH will be infinite, by *Cor.* 1. the water, notwithſtanding this, will deſcend about the column PQH which the little circle ſuſtains with velocities, which are every where in the ſubduplicate *ratio* of the diſtance from KL, and likewiſe in the reciprocal *ratio* of the ſeveral ſections through which it paſſes; conſequently, the cataract AEPHQFB, is equal to the difference of the two hyperboloids PEAKLBFQH and Pl. 11. Fig. 3.

and AKLB. But the hyperboloid PEAKLBFQH $= 2a - 2o \times \overline{H+x} = 2aH - 2oH + 2ax - 2ox$; and the hyperboloid AKLB is $2ax$; and the difference of the two is $2aH - 2oH - 2ox$, which is the cataract AEPHQFB. The *ratio* of all the water in the veſſel to this annular cataract, is $\frac{aH}{2aH-2oH-2ox}$. But from the nature of the motion of the deſcending water, a is to a–o, as $\sqrt{H+x} \,.\, \sqrt{x}$, whence $H = \frac{2aox - oox}{a^2 - 2ao + oo}$. The foregoing *ratio*, when this value of H is ſubſtituted in it's room, will, after due reduction, become $\frac{2a-o}{2a-2o}$. Therefore aH, the whole quantity of water in the veſſel, is to the annular cataract, as $2a-o$ to $2a-2o$; whence the annular cataract is $\frac{2a^2H - 2aoH}{2a-o}$, which being ſubducted from aH, leaves $\frac{aoH}{2a-o}$ for the quantity ſuſtained by the little circle o. Conſequently, $R = \frac{aoH}{2a-o}$; and $R \,.\, \frac{oH}{2} :: a \,.\, a - \frac{1}{2}o$.

SCHOLIUM.

Upon examining this motion by experiments, Sir Isaac Newton found the velocity of the water in it's paſſage through the hole to be leſs than it ought to be, if the water in the veſſel deſcended from the ſurface to the hole freely and without reſiſtance, in the proportion of 1 to $\sqrt{2}$. For he obſerved the vein of the effluent water, and found it to contract and grow narrower, to the diſtance of about a diameter of the hole below it, at which place he meaſured the diameter of the vein, and found it to be leſs than the diameter of the hole in the proportion of

of 21 to 25, and consequently, the *area* of a section of the vein at that place to be less than the *area* of the hole, in the proportion of 441 to 625, that is, of 1 to $\sqrt{2}$. But as the vein contracts the velocity increases. And therefore, at the distance of a diameter of the hole below it, the velocity will be greater than in the hole in the proportion of $\sqrt{2}$ to 1. If IG be four feet or 48 inches, and the diameter of the hole be 1 inch, 1 added to 48 will make the height from the place where the velocity is greatest to be 49 inches; and if the velocities of the discending column in the hole and that place, were truly measured by the subduplicate *ratios* of those heights, as they would be if the water descended freely and without resistance, they would be nearly equal, being as the numbers 69 and 70. And therefore, the velocity of the water in the hole is less than it would be if it was proportional to $\sqrt{IG}$, in the *ratio* of 1 to $\sqrt{2}$. This diminution of velocity can be owing to nothing but the lateral motion of the descending water, retarding it's perpendicular motion downwards, and making it less than it otherwise would be, in the said *ratio* of 1 to $\sqrt{2}$. Hence, the velocity with which the water flows through the hole, is very nearly equal to the velocity which a body, by falling freely and without resistance from a state of rest at I, would acquire in descending through $\frac{1}{2}$ IG. For the velocity acquired in falling through $\frac{1}{2}$ IG, is to the velocity acquired in falling through IG, as 1 to $\sqrt{2}$.

Pl. 11. Fig. 2.

According to Sir ISAAC NEWTON, a body falling *in vacuo* from a small height above the surface of the earth, will describe $193\frac{1}{3}$ inches, or $16\frac{1}{9}$ feet in one second minute of time, and will have acquired a velocity at the end of the fall, which being continued uniform, would carry it through twice that space, that is, $386\frac{2}{3}$ inches or $32\frac{2}{9}$ feet, in an equal

equal time. But uniform velocities are as the ſpaces deſcribed by them in the ſame time, and the velocities acquired by a body falling in *vacuo* through the ſpaces $16\frac{1}{9}$, and GI or $\frac{A^2H}{A^2-a^2}$, are in the ſubduplicate *ratios* of thoſe ſpaces; and therefore $32\frac{2}{9}$. $V :: \sqrt{16\frac{1}{9}} . \sqrt{\frac{A^2H}{A^2-a^2}}$. Whence, $V = 8.02773 \sqrt{\frac{A^2H}{A^2-a^2}}$ feet, $= 96.33276 \sqrt{\frac{A^2H}{A^2-a^2}}$ inches. And leſſening theſe meaſures of the velocity of the water flowing through the hole, the *ratio* of 1 to $\sqrt{2}$, that is, dividing each by 1.414, we ſhall $V = 5.6773196 \sqrt{\frac{A^2H}{A^2-a^2}}$ feet, $= 68.1278352 \sqrt{\frac{A^2H}{A^2-a^2}}$ inches. Theſe are the true meaſures of the velocity of the water in it's paſſage through the hole, which velocity is therefore ſuch as carries it at the rate of $5.6773196 \sqrt{\frac{A^2H}{A^2-a}}$ feet, or $68.1278352 \sqrt{\frac{A^2H}{A^2-a^2}}$ inches, in a ſecond minute of time. Theſe expreſſions may be ſhortened if A be conſiderably greater than a, for in all ſuch caſes $\frac{A^2H}{A^2-a^2}$ will be ſo nearly equal to H, that $\frac{A^2}{A^2-a^2}$ may be ſafely rejected; and then the foregoing meaſures of the velocity will become, $5.6773196 \sqrt{H}$ feet, or $68.1278352 \sqrt{H}$ Inches. To ſhew the truth of this by an example, let A be 100 ſquare inches, and a 1 ſquare inch, and then $\frac{A^2H}{A^2-a^2}$ will be $\frac{10000H}{9999}$; if H be 4 feet or 48 inches, $\frac{10000H}{9999}$ will be 48,0048 inches, which is only greater than 48 by 48 parts

parts of an inch divided into 10000. The excefs is fo fmall, that it may be fafely rejected.

Another true meafure of the velocity of the water flowing through the hole, will be had by dividing the quantity of water difcharged, by the *area* of the hole and time of the difcharge taken together; the quantity of water difcharged being expreffed in cubick inches, the *area* of the hole in fquare inches or parts of a fquare inch, and the time of the difcharge in feconds. Let Q denote the quantity difcharged, d the diameter of the hole, and t the time of the difcharge, and then V will be meafured by $\frac{Q}{at} = \frac{Q}{0.7853981 6d^2t}$ inches, which will be the fpace defcribed in one fecond of time.

This meafure is equal to the former, that is, $\frac{Q}{0.78539816d^2t} = 68.1278352\sqrt{H}$; and confequently, $Q = 53.5074764d^2t\sqrt{H}$ cubick inches; or $13555.227d^2t\sqrt{H}$ grains; becaufe a cubick inch of water weighs $253\frac{1}{3}$ grains. If W denote the weight of water difcharged, then will $W = 13555.32d^2t\sqrt{H}$ grains.

In order to know, whether the velocities of water flowing through circular holes of different diameters, when placed at the fame perpendicular diftance from the furface of the water, be all equal; what relation the velocity of water flowing through a hole, bears to the velocity of water flowing through an horizontal pipe of an equal diameter, inferted into the fide of a veffel at an equal perpendicular diftance from the furface of the water; and under what circumftances the meafure of the velocity laid down in my *Animal Œconomy* obtains, I fay, in order to know thefe things, I caufed a proper *apparatus* to be made, and from the experiments made with it, I compofed the following Tables.

TABLE

TABLE I.

t	H	d	W	w	$\frac{w}{W}$
10	4	$\frac{1}{10}$	2711	2944	1086
		$\frac{4}{10}$	43377	47040	1084
		$\frac{5}{10}$	67776	72960	1076
		$\frac{8}{10}$	173507	178560	1029
	2	$\frac{1}{10}$	1917	2087	1088
		$\frac{4}{10}$	30672	33600	1095
		$\frac{5}{10}$	47925	51840	1082
		$\frac{8}{10}$	122688	128400	1046

TABLE II.

d	l	w	d	l	w	d	l	w
$\frac{2}{10}$	0	12736	$\frac{4}{10}$	0	47040	$\frac{8}{10}$	0	178560
	d	14385		d	54720		d	204720
	2d	14400		2d	56160		2d	224640
	3d	13792		3d	52800		3d	217440
	4d	13728		4d	52220		4d	212160
	5d	13663		5d	51600		5d	203520
	10d	12683		10d	47040		16d	188160
							23d	178560

The firſt Table contains, in the firſt column, under t, the time of the diſcharge in ſeconds; in the ſecond column, under H, the perpendicular heights of the water above the hole in *London* feet; in the third, the diameters of the hole in parts of an inch; in the fourth, under W, the weights of water in grains, which ought to have been diſcharged by the theory or foregoing rule; in the fifth, under w, the weights of water in grains which were diſcharged by experiment, each weight being a mean taken from five or ſix experiments; and in the ſixth column, under $\frac{w}{W}$, the

the *ratio* of the weight diſcharged by experiment, to the weight which ought to have been diſcharged by the theory.

The ſecond Table conſiſts of three parts, and each part of three columns. The firſt column of each part, contains the diameter of the pipe in parts of an inch; the ſecond contains the lengths of the pipe in the terms of the diameter, beginning with the hole, which may be conſidered as a pipe of an infinitely ſmall length expreſſed by o; and the third column contains the weights in grains diſcharged in 10 ſeconds, each weight being a mean taken from particular experiments. The holes and pipes were all at the perpendicular diſtance of 4 feet from the ſurface of the water, ſo that here t was 10″, and H four feet.

TABLE III.

d	H	l	W	w	$\frac{w}{W}$	H	W	w	$\frac{w}{W}$
$\frac{1}{10}$	2	1	2180	2180	1000	$\frac{1}{2}$	1090	982	901
		2	1541	2080	1349		770	922	1196
		3	1258	2057	1634		629	877	1393
		4	1090	1874	1719		545	762	1398
		5	980	1759	1804		490	720	1469
		6	890	1690	1899		445	665	1494
		7	824	1564	1898		412	620	1505
		8	770	1520	1972		385	585	1519
		9	727	1440	1982		363	553	1522
		10	689	1410	2045		344	525	1523
		12	629	1320	2098		314	470	1493
		14	582	1225	2102		291	430	1476
		16	545	1163	2134		272	383	1405
		18	514	1086	2113		257	350	1362
		20	487	1030	2113		243	320	1313
		24	445	866	1946		222	260	1168
		25	436	860	1972		218	253	1160
		28	412	844	2048		206	230	1116
		32	385	758	1967		192	202	1048
		36	363	659	1814		181	185	1018
		48	314	509	1618				
		60	281	421	1496				
		72	257	345	1342				

TABLE IV.

d	H	l	W	w	$\frac{w}{W}$	H	W	w	$\frac{w}{W}$
$\frac{2}{10}$	2	1	12332	10040	814	$\frac{1}{2}$	6166	5018	814
		2	8720	9270	1063		4360	4630	1062
		3	7120	8820	1238		3560	4400	1235
		4	6166	8570	1389		3083	4270	1385
		5	5515	8240	1494		2758	4040	1465
		6	5034	7840	1557		2517	3880	1541
		7	4661	7580	1626		2330	3766	1616
		8	4360	7360	1688		2180	3668	1682
		9	4111	7150	1739		2055	3570	1737
		10	3900	6950	1782		1950	3414	1751
		16	3083	5776	1873		1541	2955	1918
		25	2466	4785	1940		1233	2460	1995
		36	2055	4048	1970		1027	2120	2064
		49	1762	3480	1975		881	1730	1963
		64	1542	3250	2108		771	1326	1720
		81	1370	3062	2235		685	1120	1635
		97	1252	2700	2156		626	940	1502

The third and fourth Tables confift each of two parts correfponding to different perpendicular heights of the water in the veffel, and different diameters of the pipes. In both Tables, H denotes the perpendicular height of the water in the veffel above the pipe in feet; l the length of the pipe in inches; W the weight in grains which ought to be difcharged by the firft *Propofition* of my *Animal Œconomy*; w the weight in grains which was difcharged by experiment; and $\frac{w}{W}$ the *ratio* of the weight difcharged by experiment to the weight which ought to have been difcharged by that *Propofition*. The diameter, of all the pipes in the third Table was $\frac{1}{10}$ of an inch, and of all the pipes in the fourth Table $\frac{2}{10}$

of

of an inch. And the time of the discharge was 10 seconds in all the experiments of both tables.

The quantity or weight of water which ought to be discharged by the first *Proposition* of the *Animal Œconomy*, may be thus found. I there proved, that the velocity of water flowing through a pipe, is as $\sqrt{\frac{F}{dl}}$. But if the force which can generate the motion of water flowing through a pipe lying parallel to the horizon, be equal to the force which can generate the motion of water flowing through a hole of an equal diameter with the pipe, when placed at an equal perpendicular distance from the surface of the water; F, by *Cor.* 11. of this *Problem*, will be as $2d^2H$, on supposition that the *area* of the hole is extreamly small in comparison of the *area* of the surface of the water. And therefore the velocity of water flowing through a pipe lying parallel to the horizon, is as $\sqrt{\frac{2dH}{l}}$. The weight of water discharged, is as the orifice of the pipe, the time of the discharge, and velocity, taken together; that is, as $d^2t\sqrt{\frac{2dH}{l}}$. And therefore, W is as $d^2t\sqrt{\frac{2dH}{l}}$.

A pipe of $\frac{1}{10}$ of an inch in diameter, and 1 inch in length, discharged 2180 grains of water in 10 seconds, when it was inserted into the side of the vessel at the perpendicular distance of 2 feet from the surface. In this case therefore, d, t, H, l, were 0.1, 10, 2, 1; and $d^2t\sqrt{\frac{2dH}{l}}$ was equal to 0.06326. Hence we may find W in other cases by this analogy; 2180 : 0.06325 : : W : $d^2t\sqrt{\frac{2dH}{l}}$; whence $W=48746.3\ d^2t\sqrt{\frac{dH}{l}}$.

In the first part of the third Table, W is $\frac{2180}{\sqrt{l}}$,

 and

and in the firſt part of the fourth Table, $\frac{12332}{\sqrt{l}}$;
and W in the ſecond part of each Table is one half of W in the firſt part.

OBSERVATIONS *on the* TABLES.

OBS. I. By the firſt Table the diſcharges by experiment are nearly proportional to the diſcharges by the theory, that is, w is nearly proportional to W, or $\frac{w}{W}$ is nearly the ſame, whatever be the diameter of the hole, provided the time of the diſcharge, and the perpendicular height of the water in the veſſel above the hole, be given. The diſcharges by experiment were all ſomething larger than the diſcharges by the theory, which might be partly owing to the pouring in of the water at the top of the veſſel, in order to keep the veſſel conſtantly full during the time of the diſcharge; for the pouring, though it was done gently, might a little increaſe the velocity wherewith the water ran out of the hole.

OBS. II. By the ſecond Table, the weight of water diſcharged, and conſequently the velocity, increaſes from the hole till the length of the pipe becomes equal to about twice it's diameter, that is, till l becomes equal to about 2d, and is greater there than at any other length of the pipe. The greateſt velocities in theſe pipes in proportion to the velocities through their reſpective holes, are as the numbers 1130, 1215, 1258 to 1000.

OBS. III. From the length of twice the diameter, that is from the length 2d, the velocity leſſens continually on increaſing the length of the pipe, and becomes equal to the velocity through the hole when the length of the pipe becomes equal to about $22.3657d\sqrt{d}$ inches. For, by the ſecond Table the velocities of the water flowing through the pipes, were nearly equal to the velocities through their reſpective holes, when the lengths of the pipes, were

were 10d, 16d and 23d, that is 2 inches, 6.4 inches, and 18.4 inches. But 2, 6.4, and 18.4, are nearly as 1, 2.8, and 8, the ſeſquiplicate *ratios* of 1, 2 and 4, and 1, 2 and 4, are as the diameters $\frac{2}{10}$, $\frac{4}{10}$ and $\frac{8}{10}$. And therefore the velocities of the water flowing through the pipes, were nearly equal to the velocities through their reſpective holes, when the lengths of the pipes were in the ſeſquiplicate *ratios* of their diameters. The diameter of the ſmalleſt pipe being $\frac{2}{10}$ of an inch, $d\sqrt{d}$ is 0.0894; and if d be of any other magnitude, and l be the length of a pipe of that diameter through which the water flows with a velocity equal to that with which it flows through it's correſponding hole, we ſhall have this proportion; as 2 is to 0.0894, ſo is l to $d\sqrt{d}$, whence $l = 22.3657 d\sqrt{d}$.

Obs. IV. By the third and fourth Tables, the quantity of water diſcharged by experiment in proportion to the quantity which ought to have been diſcharged by the theory, that is $\frac{w}{W}$, increaſes gradually till the pipe comes to be of a certain length, and after that it decreaſes gradually on increaſing the length of the pipe. In the two parts of the third Table this *ratio* was greateſt, when the lengths of the pipes in inches were about 20 and 10, and it was greateſt in the two parts of the fourth Table, when the lengths of the pipes were 81 and 36. But from the courſe of the numbers expreſſing $\frac{w}{W}$ in the ſecond part of the fourth Table, I think this *ratio* would have been greater in a pipe of 40 inches in length, than in the one I uſed of 36, and therefore ſhall ſuppoſe that it would have been greateſt at the lengths of 81 and 40. Conſequently, putting x for the length of the pipe in inches, at which this *ratio* is greateſt, x will be as $\sqrt{H}$ when d is given, and as d^2 when H is given, and when neither d nor H is given, as $d^2\sqrt{H}$. Hence we may

form a rule for finding the length of the pipe, at which this *ratio* ſhall be a *maximum*; for it was a *maximum* in a pipe of $\frac{1}{10}$th of an inch in diameter, when it's length was 20 inches, and the perpendicular height of the water in the veſſel 2 feet. In this caſe therefore x, d and H, are 20, 0.1, and 2, and $d^2\sqrt{H}$ is 0.01414; and in other caſes, x may be found by this analogy; as 20 is to 0.01414, ſo is x to $d^2\sqrt{H}$; whence x is equal to $1414d^2\sqrt{H}$. To ſee whether this rule be univerſal, and obtain in pipes of greater diameters, and at greater diſtances from the ſurface of the water, I ſhall ſuppoſe d and H to be 0.5 and 3, as in our Author's Table p. 227, and then $1414d^2\sqrt{H}$ will be about 600 inches or 50 feet, which length is twice as great as it was in reality; for the *ratio* was a *maximum* by that Table, when the length of the pipe was 25 feet; ſo that the value of x here determined ſeems to obtain only in pipes of ſmall diameters.

Obs. V. By the third and fourth Tables, the quantity diſcharged by experiment in proportion to the quantity which ought to have been diſcharged by the theory, that is $\frac{w}{W}$ does not differ much in pipes whoſe lengths are within certain limits. $\frac{w}{W}$, in the pipes, whoſe lengths were 6 and 32 in the firſt part of the third Table, is leſs than in the pipe where this *ratio* is a *maximum*, in the proportions of 100 to 112 and 110, and the difference of $\frac{w}{W}$ and the *maximum* is ſtill leſs in pipes of all other lengths between 6 and 32; ſo that in this part of the Table, 6 and 32 are the limits, at and within which there is a near agreement between theory and experiment. $\frac{w}{W}$ in the pipes whoſe lengths are 4 and 16 in the ſecond part of this

Table,

Table, is lefs than in the pipe where this *ratio* is a *maximum*, in the proportion of about 100 to 109 and 108, and it is ftill lefs in pipes of all other lengths within thefe limits. And $\frac{w}{W}$ in the pipes whofe lengths are 9 and 64 in the fecond part of the fourth Table, for the pipes were not carried to fuch lengths as were neceffary to fettle the limits in the firft part, is lefs than the *maximum* in the proportion of 100 to 118 and 120; and it is ftill lefs in pipes of all other lengths within thefe limits.

Obs. VI. By the third and fourth Tables, the quantity of water difcharged by experiment always exceeds the quantity which ought to be difcharged by the theory; it was near double within the limits of the firft part of the third Table and fecond part of the fourth, and greater in the fecond part of the third Table in the proportion of 5713 to 3858. If we fuppofe it to be double within the limits, in pipes of all lengths, then will w be equal to 2 W, or to $97492.6d^2t\sqrt{\frac{dH}{l}}$ grains, W being equal to $48746.3d^2t\sqrt{\frac{dH}{l}}$ grains, as was fhewn above.

Prob. VI. *If the diftance of an object from a double convex* lens *whofe furfaces are fpherical, if the* radii *of both the fpherical furfaces, the thicknefs of the* lens, *and the fines of incidence and refraction, be all given; thence to determine the diftance behind the* lens *of the principal* focus *or concourfe of the rays iffuing from the object and falling perpendicularly, or very nearly fo, on that furface of the* lens *which is turned towards the object.*

Let MN be a *lens*, E and e the centers of it's fpherical furfaces MCN and MDN, Q an object placed directly before the *lens*, Qq a line drawn from the object perpendicular to the furfaces of the *lens*, and confequently paffing through the centers e and E; let the point A be indefinitely near to C, in which cafe QA and QC may be looked upon as equal; Pl. 11. Fig. 5.

equal; let q be the *focus* or concourse of the rays QA and QC after the first refraction by the surface MCN, and z their *focus* or concourse after the second refraction by the surface MDN. Put D for QC the distance of the object from the *lens*, r for the *radius* CE, ρ for the *radius* eD, x for Dq, the distance of the *focus* behind the *lens*, after the first refraction, and z for Dz it's distance behind the *lens* after the second refraction; and lastly, let I and R denote the sines of incidence and refraction of the rays passing out of air or any other *medium* into the first surface MCN, and consequently R and I the sines of incidence and refraction in their passage out of the second surface MDN into air or that other *medium*.

Pl. 11. Fig. 4. To determine z, we must first determine the measure of x in known terms, to do which draw AF perpendicular to Qq, EI perpendicular to QAI the incident ray produced, and ER perpendicular to the refracted ray Aq; and then, from the similarity of the two triangles QAF and QIE, and also of the triangles qAF and qER, and from QA being equal to QC, and qA equal to qC, we shall have AF equal to $\frac{QC\times EI}{QE}$, or, in symbols, to $\frac{DI}{D+r}$, by the two first triangles, and by the two last triangles, equal to $\frac{Cq\times ER}{Eq}$, or, in symbols, to $\frac{Rx}{x-r}$. Consequently $\frac{DI}{D+r}$, is equal to $\frac{Rx}{x-r}$, and $x=\frac{DIr}{DI-DR-Rr}$.

Pl. 11. Fig. 5. Having found the measure of x or Cq in known terms, z or Dz may be thus determined. For that measure, that is for $\frac{DIr}{DI-DR-Rr}$, put A, to za and qa produced draw the perpendiculars eI and eR, and draw am perpendicular to Qq. And then, from

from the ſimilarity of the triangles qam and qRe, and alſo of the triangles zam and zIe, and from qa being equal to qD, and za equal to zD, am will be equal to $\frac{eR \times qD}{qe}$ from the firſt triangles, and equal to $\frac{eI \times zD}{ze}$ from the ſecond. eR is the ſine of incidence of the ray Aa falling on the ſecond ſurface MDN; and eI the ſine of it's refraction; and therefore eR will be I, and eI will be R. qD is equal to $qC - CD = A - t$, putting t for CD the thickneſs of the *lens*; and qe is equal to $qC + eC = qC + eD - CD = A + \rho - t$; conſequently $\frac{R \times \overline{A - t}}{A + \rho - t} = \frac{Iz}{z + \rho}$; and from this equation $A = \frac{I\rho z + Rtz + Rt\rho - Itz}{Rz + R\rho - Iz}$. But A denotes $\frac{DIr}{DI - DR - Rr}$.

And therefore $\frac{I\rho z + Rtz + R\rho t - Itz}{Rz + R\rho - Iz} = \frac{DIr}{DI - DR - Rr}$.

By clearing z in this equation, we ſhall have $z = \frac{DIRr\rho + RRr\rho t + DRR\rho t - DIR\rho t}{DII\rho + 2DIRt - DIIt - DIR\rho - DRRt - IRr\rho - RRrt + IRrt - DIRr + DIIr}$

To give this equation a more ſimple form, divide both numerator and denominator by $I - R$; and then, the numerator will become $\frac{R}{I - R} \times IDr\rho + \frac{R}{I - R} \times Rr\rho t - DR\rho t$, or, by putting B inſtead of $\frac{R}{I - R}$, $BIDr\rho + BRr\rho t - DR\rho t$, and the denominator will become $IDr + ID\rho - IDt + RDt + Rrt - BIr\rho$; and the equation will be reduced to another form, and ſtand thus.

$$z = \frac{BIDr\rho + BRr\rho t - DR\rho t}{IDr + ID\rho - IDt + RDt + Rrt - BIr\rho}.$$

This is Dr. HALLEY's univerſal *Theorem* for finding the principal *focus* of rays falling diverging on a double

a double *convex lens*, published in the *Philosophical Transactions*.

If the rays, instead of falling diverging, fall parallel on a double convex *lens*, as they will nearly do, when the object is at an immense distance from the *lens*, D in this case may be considered as infinite; and consequently, all the terms in which D is not found, may be thrown out of the equation, and then $z = \frac{BIDr\rho - DR\rho t}{IDr + ID\rho - IDt + RDt} = \frac{BIr\rho - R\rho t}{Ir + I\rho - It + Rt}$.

And lastly, if the rays fall converging on a double convex *lens*, the signs of all the terms in which D is found must be changed; for when the rays fall converging, the point behind the *lens* to which they tend at their incidence, must be considered as the place of the object, which, from it's being differently situated with respect to the *lens* from what it is when the rays fall diverging, requires the signs of all the terms in which D is found to be changed, which being done, we shall have

$$z = \frac{DR\rho t + BRr\rho t - BIDr\rho}{IDt - IDr - ID\rho - RDt + Rrt - BIr\rho}.$$

These are the three general *Theorems* for finding the principal *focus* of rays falling, diverging, parallel, or converging on a double convex *lens*.

If the *lens* be made of glass, as *lenses* usually are, and the object be placed in air, then, since the sine of incidence of a ray passing out of air into glass, is to the sine of refraction, as 3 to 2, I, R and B will be 3, 2, and 2; and the foregoing general *Theorems* for finding the *foci* of rays falling diverging, parallel, and converging, on a double convex glass, will be $z = \frac{6Dr\rho + 4r\rho t - 2D\rho t}{3Dr + 3D\rho - 3Dt + 2Dt + 2rt - 6r\rho}$, $z = \frac{6r\rho - 2\rho t}{3r + 3\rho - 3t + 2t}$, and

z =

$$z = \frac{2D\rho t + 4r\rho t - 6Dr\rho}{3Dt - 3Dr - 3D\rho - 2Dt + 2rt - 6r\rho}.$$

And if the *radii* be equal, and the thicknefs of the glafs be neglected or confidered as o, then will thefe *Theorems* ftand thus, $z = \frac{Dr}{D-r}$, $z = r$, and $z = \frac{-Dr}{-D-r}$.

If the *lens* be a double concave glafs, the *radii* of whofe two fpherical furfaces are equal, and if the thicknefs of the *lens* be confidered as o, the *radii* will lie on different fides of the *lens* with refpect to the object from what they did before, and confequently, the figns of the *radii* muft be changed; and then the laft *Theorems*, in which the *radii* were fuppofed to be equal, and the thicknefs of the glafs was neglected or confidered as o, will ftand thus, $z = \frac{-Dr}{D+r}$, $z = -r$, and $z = \frac{Dr}{r-D}$. By thefe *Theorems*, z is always negative when the rays fall upon the double concave, diverging, or parallel, and when they fall converging it is negative when D is greater than r. When z is negative, the *focus* falls on the fame fide of the glafs with the object, contrary to what it does in all cafes of a double convex *lens*, excepting that of diverging rays, when the diftance of the object is lefs than the *radius*, or D is lefs than r. For in that cafe, z, which is equal to $\frac{Dr}{D-r}$, will be negative.

By this *Problem* we may determine how far a radiating point muft be diftant from the eye, to have the principal *focus* of the rays iffuing from it placed in the *retina*, on fuppofition that the coats and humours of the eye are unchangeable as to their figures, magnitudes and denfities.

Let ABGz reprefent a human eye, in which ABG is the *cornea*, AMCNGB the cavity containing the aqueous humour, MCND the cryftalline humour, and Pl. 11. Fig. 6.

and AMDNGz the cavity containing the vitreous humour. According to Doctor JURIN, the *radii* of the spherical surfaces of the *cornea* and of the crystalline humour, that is, of the spherical surfaces ABG, MCN and MDN, are in 10th parts of an inch, 3.3294, 3.3081, and 3.5056; and the distance of the *cornea* from the anterior part of the crystalline, the thickness of the crystalline, the distance of the posterior part of the crystalline from the *retina*, and the distance of the *cornea* from the *retina*, are in the same parts of an inch, 1.0358, 1.8525, 6.2617 and 9.15. Let Q be the radiating point, q the principal *focus* of the rays by the first refraction of the aqueous humour, by virtue of which refraction they fall converging on the crystalline, and let z be their *focus* after their refractions by the crystalline and vitreous humours. By taking the specifick gravities of the humours of the eye, I have found that the specifick gravities of the aqueous and vitreous humours are very nearly equal, and each much the same with that of water; and that the specifick gravity of the crystalline is greater than the specifick gravity of water, in the proportion of about 11 to 10. For the mean specifick gravities of 5 crystalline humours of oxen's eyes, and of 3 crystalline humours of sheep's eyes, were 11134 and 11033, the specifick gravity of water being 10000, and the mean of these two means, is 11083, which I shall suppose to be the specifick gravity of the crystalline humour of a human eye. But the refractive power of the crystalline is very nearly proportional to it's density, and the sine of incidence of rays passing out of the aqueous humour into the crystalline, is to the sine of refraction, very nearly as 21 to 20, as I shall shew in the *Scholium*. And consequently, I will be 21, and R will be 20. From these measures I now proceed to determine the distance of a radiating point from the *cornea*, that is, the distance of Q from B, so as that the

focus

focus of the rays, iſſuing from it and falling diverging on the *cornea*, may by the refractive powers of the aqueous, cryſtalline, and vitreous humours, be placed in the *retina* at z. By the refraction of the aqueous humour, the rays fall on the cryſtalline with ſuch a degree of convergence as would make them unite at q. In the univerſal *Theorems* therefore for finding the principal *focus* of rays falling converging on a double convex *lens*, Cq is D, Dz equal to 6.2617 is z, the *radius* of MCN is r, the *radius* of MDN is ρ, CD the thickneſs of the cryſtalline is t, and I and R are 21 and 20. And by clearing D in that *Theorem*, we ſhall have $D = \frac{BIr\rho z + BRr\rho t - Rrtz}{BIr\rho + Itz - Irz - I\rho z - R\rho t - Rtz} = 10.3102 =$ Cq. And $Cq + BC = 11.346 = Bq$.

In the *Theorem* for finding x, Bq is x, QB is D, I is 4, and R 3, the ſine of incidence of rays paſſing out of air into water or into the aqueous humour, being to the ſine of refraction, as 4 to 3, and the *radius* of the *cornea* is 3.3294 10th parts of an inch; conſequently, D is 57.48, that is, about 5 inches and 3 quarters. So that ſuppoſing the eye to be unchangeable, a radiating point placed at the diſtance of $5\frac{3}{4}$ inches from it, will have it's image placed in the *retina*.

SCHOLIUM.

" Let AB repreſent the refracting plane ſurface Pl. 11. Fig. 7.
" of any body, and IC a ray incident obliquely on
" the body at C, ſo that the angle ACI may be
" infinitely little, and let CR be the refracted ray.
" From a given point B perpendicular to the refracting ſurface erect BR meeting the refracted
" ray CR in R, and if CR repreſent the motion of the
" refracted ray, and this motion be diſtinguiſhed into
" two motions CB and BR, whereof CB is parallel to
" the refracting plane, and BR perpendicular to it:
" CB ſhall repreſent the motion of the incident ray,

I " and

" and BR the motion generated by the refraction." Newt. *Opt. Prop.* 10. p. 245.246. CBR is equal to the angle of incidence, and CRB is equal to the angle of refraction; conſequently, if R be made the center, and a circle be ſuppoſed to be drawn with the *radius* CR, CR will be the ſine of the angle of incidence, and CB the ſine of the angle of refraction; and, putting I and R for thoſe ſines, we ſhall have this analogy, I . R : : CR . CB. Hence $\frac{I^2-R^2}{R^2}=\frac{CR^2-CB^2}{CB^2}$, or $\frac{I^2}{R^2}-1=\frac{BR^2}{CB^2}$. But the motion of the ray at it's incidence repreſented by CB, is given; and therefore, $\frac{I^2}{R^2}-1$ is as BR^2. But by the aforeſaid propoſition BR^2 expreſſes the refractive force, and is nearly as the denſity of the body; as Sir I. Newton found, by computing BR^2 from the ſines I and R in ſeveral bodies, and then comparing it with their reſpective denſities. And conſequently, putting D for the denſity of the body, $\frac{I^2}{R^2}-1$ is as D, and $\frac{I}{R}$ as $\sqrt{D+1}$. In paſſing out of air into water $\frac{I}{R}$ is $\frac{4}{3}$, and, the denſity of water being 10000, $\sqrt{D+1}$ is 10004: And in paſſing out of air into the cryſtalline, whoſe denſity is to that of water as 11083 to 10000, $\sqrt{D+1}$ is 10528. Therefore in paſſing out of air into the cryſtalline $\frac{I}{R}$ will be $\frac{7}{5}$; for 10004 . 10528 : : $\frac{4}{3}$. $\frac{42112}{30012}=\frac{7}{5}$ very nearly. $\frac{I}{R}$ in paſſing out of the aqueous humour into the cryſtalline, will be compounded of the *ratios* $\frac{3}{4}$ and $\frac{7}{5}$, by the ſecond *Theorem* of the *Opticks*; p. 113, and therefore $\frac{I}{R}$ will be equal to $\frac{21}{20}$; or I will be to R, as 21 to 20.

FINIS.

Fig. 1.

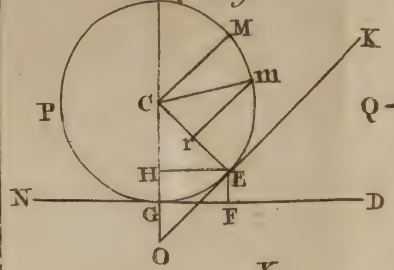


Fig. 4.

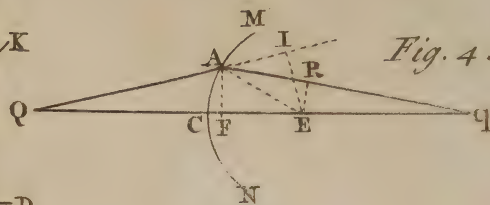


Fig. 3.

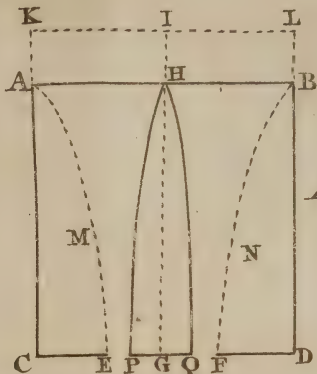


Fig. 2.

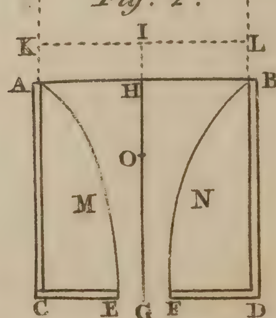


Fig. 7.

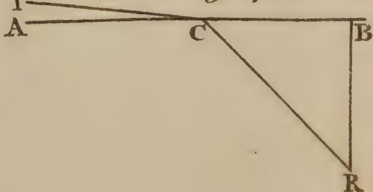


Fig. 6.

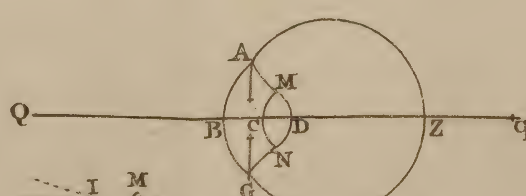
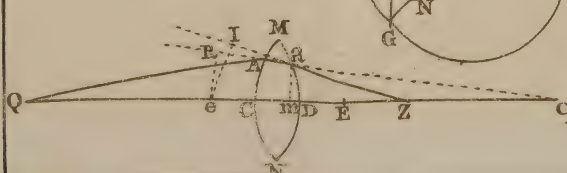


Fig. 5.



BOOKS printed for J. Nourse, at the *Lamb* without *Temble-Bar.*

I.

THE philoſophical Hiſtory and Memoirs of the Royal Academy of Sciences at *Paris:* Or, an Abridgment of all the Papers relating to Natural Philoſophy, which have been publiſhed by the Members of that Illuſtrious Society, from the Year 1699 to 1720. With many curious Obſervations relating to the natural Hiſtory and Anatomy of Animals, *&c.* Illuſtrated with Copper Plates. The Whole tranſlated and abridged, by *John Martyn*, F. R. S. Profeſſor of Botany in the Univerſity of *Cambridge*; and *Ephraim Chambers*, F. R. S. Author of the Univerſal Dictionary of Arts and Sciences. In five Volumes.

II.

The Doctrine of Annuities and Reverſions, deduced from general and evident Principles: With uſeful Tables, ſhewing the Values of ſingle and joint Lives, *&c.* at different Rates of Intereſt. To which is added, a Method of inveſtigating the Value of Annuities by Approximation, without the help of Tables. The Whole explained in a plain and ſimple Manner, and illuſtrated by great variety of Examples. By *Thomas Simpſon.*

III. Eſſays

III.

Essays on several curious and useful Subjects, in speculative and mixed Mathematicks. Illustrated by a Variety of Examples, in which the most difficult Problems of the first and second Book of Sir *Isaac Newton*'s *Principia* are explained. A Work very useful to young Gentlemen who are studying that illustrious Author. By *Thomas Simpson.*

IV.

Memoirs of the Royal Society; being a new Abridgment of the Philosophical Transactions: Giving an Account of the Undertakings, Studies, and Labours of the Learned and Ingenious in many considerable Parts of the World; from the first Institution of that illustrious Society in the Year 1665, to the Year of our Lord 1739 inclusive. The Whole carefully abridged from the Originals, and the Order of Time regularly observed, with a Translation of the *Latin* Tracts, and the theoretical Parts applied to practical Uses; also an Explanation of the Terms of Art as they occur in the Course of the Work. Being a Work of general Use to the Publick, and worthy the Perusal of all Mathematicians, Artificers, Tradesmen, *&c.* for their Improvement, in various Branches of Business. By Mr. *Baddam.* In ten Volumes. Illustrated with a great Variety of Copper Plates.